Lesson 10: Designing simulations

Goals

- Describe a multi-step experiment that could be used to simulate a compound event in a real-world situation, and justify (orally and in writing) that it represents the situation.
- Perform a simulation to estimate the probability of a compound event, and explain (orally and in writing) how the simulation could be improved.

Learning Targets

- I can design a simulation to estimate the probability of a multi-step real-world situation.

Lesson Narrative

In this lesson, students see that the probability of compound events can also be estimated using simulations. The last activity in this lesson is a culmination of all the work students have done with probability in this unit, as each group works to design a simulation for a different situation. Students strategically choose tools like dice, spinners, blocks, etc., to represent the chance experiments in the situations they are given. This is also an opportunity for students to practise communicating precisely about how their simulation is conducted and what their outcomes represent.

Building On

- Use brackets in numerical expressions, and evaluate expressions with these symbols.
- Giving quantitative measures of centre (median and/or mean) and variability (interquartile range and/or range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Addressing

- Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Instructional Routines

- Discussion Supports
- Number Talk

Required Materials

Coins
any fair two-sided coin

Compasses
Dice
cubes with sides numbered from 1 to 6

Paper bags
Paper clips
Pre-printed slips, cut from copies of the blackline master

Designing Simulations 1. A man has 5 grandchildren, 4 girls and 1 boy. He thinks this is unusual. If the probability that any child born will be a girl is $\frac{1}{2}$, what is the probability that a person who has 5 grandchildren will have exactly 4 granddaughters? Is this case unusual? Explain.
Designing Simulations 2. To be on the safe side, three detectors were installed in a factory room to make sure that if there was a fire, at least one of them would signal a warning. The company that manufactured the smoke detectors indicated that, based on their testing, the probability that any one of the smoke detectors will work correctly is 0.75 (meaning that it works 75% of the time in the long run). This also means that there is a 25% chance that if there is smoke or a fire, the detector will not work! What is the probability that if there was smoke in the factory, none of the 3 detectors would work? Does this probability indicate a safety problem for the factory? Explain.
Designing Simulations 3. An automobile factory has a reputation for assembling high quality cars. However, several new cars were shipped out to dealers that had a problem with the brakes. It is estimated that approximately 10% of the cars assembled at this factory have defective brakes. Five of these cars are shipped to a dealership near your school. What is the probability that none of the 5 cars will have defective brakes? Should the dealership be concerned? Explain.
Designing Simulations 4. Your class is planning to collect data at a wildlife refuge centre for the next 5 days. The staff at the refuge centre indicated that there is a 40% chance of seeing an eagle during any one of the days of your visit. What is the probability that if your class visits the refuge for 5 days, you will see an eagle two or more days during your 5-day visit at the refuge centre? Your teacher also indicated that if you see 2 or more eagles during the 5 days, your class will be able to name one of the eagles as part of a fundraiser. Do you think you have a good chance of being able to name an eagle? Explain.
Designing Simulations 5. At a small animal emergency hospital, there is a 20% chance that an animal brought into the hospital may need to stay overnight. The hospital only has enough room to accommodate 2 animals per night. On a particular day, five animals were brought into the hospital. What is the

probability that at least 3 of the animals may need to stay overnight? If seeing five animals per day is typical for this hospital, do you think the hospital is usually able to accommodate all of the animals that might have to stay overnight? Explain.

Protractors
Clear protractors with no holes and with radial lines printed on them are recommended.

Scissors
Multi-link cubes
Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Required Preparation

Every 3 students need 2 coins for the Breeding Mice activity.

Print and cut up questions from the Designing Simulations blackline master. Use one question for every 3 students. Groups will need access to dice, protractors, rulers, compasses, paper clips, bags, multi-link cubes, and scissors to simulate their scenarios.

Student Learning Goals

Let's simulate some real-life scenarios.

10.1 Number Talk: Division

Warm Up: 5 minutes

The purpose of this number talk is to elicit strategies and understandings students have for division, particularly when the quotient is the same for different expressions. These understandings will be helpful for students as they are finding mean in upcoming lessons.

While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find the value of each expression mentally.

$(4.2 + 3) \div 2$

$(4.2 + 2.6 + 4) \div 3$

$(4.2 + 2.6 + 4 + 3.6) \div 4$

$(4.2 + 2.6 + 4 + 3.6 + 3.6) \div 5$

Student Response

- 3.6; Strategies vary. Possible strategies: $4.2 \div 2 + 3 \div 2$ or $7.2 \div 2$.
- 3.6; Strategies vary. Possible strategies: Since there are three addends with an additional 3.6 in the parentheses and the divisor is one greater than in the previous problem, the result is the same as in the first problem (or $10.8 \div 3$).
- 3.6; Strategies vary. Possible strategies: Since you are adding an additional 3.6 to the addends from the previous problem and there are now four addends with a divisor of 4, the result will be the same as in the previous problem (or $14.4 \div 4$).
- 3.6; Strategies vary. Possible strategies: Since you are adding an additional 3.6 to the addends from the previous problem and there are now five addends with a divisor of 5, the result will be the same as in the previous problem (or $18 \div 5$).

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "What did you notice about the answers to these questions?"
  - "How could we use the number we are dividing by each time to explain why the answers are all the same?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

10.2 Breeding Mice

10 minutes (there is a digital version of this activity)

In this activity, students revisit the idea of simulating real-life situations with chance experiments. In the next activity, they will design and run their own simulations for situations that involve multiple steps. Here, students are asked to use their understanding of experiments that have multiple steps to simulate a single part of a larger simulation. Namely, flipping two coins represents a single offspring from a pair of mice. Since the outcome probabilities of the simulation and the real-life situation are the same, this is another option for creating simulations that represent real-life scenarios.

Launch

Arrange students in groups of 3. Provide 2 coins for each group.

Ask students: “When flipping two coins, what is the probability of both landing heads up?” $\left(\frac{1}{4}\right)$

For context, it might be helpful to explain that mice are often used in science experiments since they have similar genetics to humans, but are easier to maintain. Setting up a mating to work with a new generation of mice with specific combinations of genes can be costly and time consuming, so it can help to simulate some outcomes before actually beginning the experiment. The word “offspring” refers to children.

Give students 5–7 minutes for group work followed by a whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisational skills in problem solving. For example, pause to check for understanding of determining the number of outcomes after 3–5 minutes of work time.
Supports accessibility for: Organisation; Attention

Student Task Statement

A scientist is studying the genes that determine the colour of a mouse’s fur. When two mice with brown fur breed, there is a 25% chance that each baby will have white fur. For the experiment to continue, the scientist needs at least 2 out of 5 baby mice to have white fur.

To simulate this situation, you can flip a coin twice for each baby mouse.

- If the coin lands heads up both times, it represents a mouse with white fur.

- Any other result represents a mouse with brown fur.

1. Simulate 3 litters of 5 baby mice and record your results in the table.

	mouse 1	mouse 2	mouse 3	mouse 4	mouse 5	Do at least 2 have white fur?
simulation 1						
simulation 2						
simulation 3						

2. Based on the results from everyone in your group, estimate the probability that the scientist's experiment will be able to continue.

3. How could you improve your estimate?

Student Response

1. Answers vary.

2. Answers vary. Sample response: I estimate the probability to be $\frac{3}{9}$ since 3 of our 9 simulations had more than 2 white offspring.

3. If we did more trials, I expect the estimate to improve.

Are You Ready for More?

For a certain pair of mice, the genetics show that each offspring has a probability of $\frac{1}{16}$ that they will be albino. Describe a simulation you could use that would estimate the probability that at least 2 of the 5 offspring are albino.

Student Response

Answers vary. Sample response: Flip 4 coins. If they are all heads, that offspring is albino. Do this 5 times to represent the 5 offspring. Repeat this process many times to simulate many groups of 5 offspring and estimate the probability based on the cumulative fraction of groups that have at least 2 albino offspring.

Activity Synthesis

The purpose of the discussion is for students to articulate why the simulation is appropriate and think about other methods of simulating the same situation.

Consider asking these questions for discussion:

- "How could we get a better estimate than what you got in your group?" (Repeat the experiment many more times or combine the data from the class.)
- Collect data from the class to find a better estimate. (For reference, the actual probability is $\frac{47}{128} \approx 0.37$.)
- "Notice that we used a two-part experiment (flipping two coins) to represent a single thing (one offspring). Why was this ok to do?" (The probability of getting HH on two coins is the same as the probability of getting a single offspring with white fur.)
- "Can you think of another method that would work to simulate a single offspring?" (A spinner with 25% of the circle labelled "white" and 75% labelled "brown." One white block and three brown blocks in a bag.)

10.3 Designing Simulations

20 minutes

In this activity, each group is assigned a situation for which they will design and perform a simulation to estimate the probability. Students will give a short presentation on the methods and results of their simulation for the class after they have designed and run the simulation. Students will need to attend to precision as well as present arguments for the simulation method they chose. At this stage, students have experienced a large number of simulation methods and should be able to design their own to represent the situations using the appropriate tools.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 3. Assign each group a question slip from the blackline master. Provide access to dice, compasses, protractors, rulers, paper bags, coloured multi-link cubes, scissors, and coins. Give students 15 minutes for group work followed by a whole-class discussion.

Student Task Statement

Your teacher will give your group a paper describing a situation.

1. Design a simulation that you could use to estimate a probability. Show your thinking. Organise it so it can be followed by others.

2. Explain how you used the simulation to answer the questions posed in the situation.

Student Response

Answers vary. Some sample responses:

1. Flip 5 coins. Heads represents a girl and tails represents a boy. If the coins land with 4 heads and 1 tail it will match the situation. After doing this 10 times, 1 of them had this result, so the probability should be about 0.1. It is fairly unusual.

2. Spin 3 spinners that each have 75% of the circle marked as "working" and 25% as "not working." If all three of the spinners land on "not working," the fire is undetected. After doing this 10 times, this never happened. Since it it possible, the estimate should be between 0 and 0.1. It is not a safety problem for the factory since fires are so rare and when there is a fire, at least one of the detectors will usually work.

3. In a bag, there are 9 slips of paper that say "brakes work" and 1 that says "defective brakes." Draw one paper out, replace it, and repeat for each of the five cars. If all 5 cars have working brakes, the dealer is ok. After doing this 10 times, this happened 6 times. The dealership should be concerned since having defective brakes is a big problem and the probability is only about 0.6 that none of the cars have this problem.

4. In a bag there are 6 slips of paper that say "eagle" and 4 that say "no eagle." Draw one paper out, replace it, and repeat for each of the five days. If at least 2 days have an eagle, we will get to name one. After doing this 10 times, this happened 7 times. There is a probability of about 0.7 that the class will get to name an eagle.

5. In a bag there are 2 slips of paper that say "stay overnight" and 8 that say "release." Draw one paper out, replace it, and repeat for each of the five animals. If at least 3 of the animals have to stay overnight, the hospital will have a problem. After doing this 10 times, this happened once. There is a probability of about 0.1 that the hospital will not have enough space for the animals.

Activity Synthesis

Ask each group to share their situation, their method of simulating the situation, and their results. Students should explain why their chosen method works to simulate the situation they were given. In particular, all important outcomes should be represented with the same probability as stated in the situation.

If all groups that have the same situation use the same simulation method, ask for ideas from the class about alternate methods that could be used for the situation.

For reference, the computed probabilities for each situation are:

1. $\frac{5}{32} \approx 0.16$
2. $\frac{1}{64} \approx 0.02$
3. $\frac{59049}{100000} \approx 0.59$
4. $\frac{2072}{3125} \approx 0.66$
5. $\frac{181}{3125} \approx 0.06$

Representation: Internalise Comprehension. Use colour and annotations to illustrate student thinking. As students show their representations of simulations and explain their reasoning, use colour and annotations to scribe their thinking on a display of each problem so that it is visible for all students.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Listening, Speaking: Discussion Supports. To help students explain why their chosen method works, provide sentence frames such as: "This simulation was designed so that . . ." and "The simulation and the actual event have the same probabilities because . . . ." As partners are sharing, encourage the listener to press for more explanation by asking: "Can you explain why the probabilities of the simulation and actual event match?" or "Is there another simulation that could provide the same probability?" This will help students practice justifying their simulation from their interpretation of the situation.
Design Principle(s): Maximise meta-awareness; Cultivate conversation

Lesson Synthesis

Consider these questions for discussion:

- "What are some things you had to consider when designing your simulation?" (Among other things, the probability of the actual portion of the event should match the probability of the associated simulated event.)
- "What did you learn from the simulations the other groups did?"
- "Were the results of any of the simulations surprising?"
- "Why would it make sense to design and run a simulation rather than repeat the actual experiment multiple times?" (When the actual experiment is costly in time or resources or cannot be controlled or repeated. E.g. testing the ripeness of fruit as testing destroys the product.)

10.4 The Best Power-Up

Cool Down: 5 minutes

Many modern games incorporate random numbers to simulate parts of the game. In this cool-down, students are asked to use the results from a computerised random number generator to simulate a particular result.

Student Task Statement

Elena is programming a video game. She needs to simulate the power-up that the player gets when they reach a certain level. The computer can run a program to return a random integer between 1 and 100. Elena wants the best power-up to be rewarded 15% of the time.

Explain how Elena could use the computer to simulate the player getting the best power-up at least 2 out of 3 times.

Student Response

Answers vary. Sample response: Elena could have the computer generate 3 random integers between 1 and 100. If at least 2 of the numbers are between 1 and 15, then the player got the best power-up at least twice. She could repeat this process many times and estimate the probability as the proportion of trials for which at least 2 of the numbers are between 1 and 15.

Student Lesson Summary

Many real-world situations are difficult to repeat enough times to get an estimate for a probability. If we can find probabilities for parts of the situation, we may be able to simulate the situation using a process that is easier to repeat.

For example, if we know that each egg of a fish in a science experiment has a 13% chance of having a mutation, how many eggs do we need to collect to make sure we have 10 mutated eggs? If getting these eggs is difficult or expensive, it might be helpful to have an idea about how many eggs we need before trying to collect them.

We could simulate this situation by having a computer select random numbers between 1 and 100. If the number is between 1 and 13, it counts as a mutated egg. Any other number would represent a normal egg. This matches the 13% chance of each fish egg having a mutation.

We could continue asking the computer for random numbers until we get 10 numbers that are between 1 and 13. How many times we asked the computer for a random number would give us an estimate of the number of fish eggs we would need to collect.

To improve the estimate, this entire process should be repeated many times. Because computers can perform simulations quickly, we could simulate the situation 1 000 times or more.

Lesson 10 Practice Problems

Problem 1 Statement

A rare and delicate plant will only produce flowers from 10% of the seeds planted. To see if it is worth planting 5 seeds to see any flowers, the situation is going to be simulated. Which of these options is the best simulation? For the others, explain why it is not a good simulation.

a. Another plant can be genetically modified to produce flowers 10% of the time. Plant 30 groups of 5 seeds each and wait 6 months for the plants to grow and count the fraction of groups that produce flowers.

b. Roll a standard dice 5 times. Each time a 6 appears, it represents a plant producing flowers. Repeat this process 30 times and count the fraction of times at least one number 6 appears.

c. Have a computer produce 5 random digits (0 through 9). If a 9 appears in the list of digits, it represents a plant producing flowers. Repeat this process 300 times and count the fraction of times at least one number 9 appears.

d. Create a spinner with 10 equal sections and mark one of them “flowers.” Spin the spinner 5 times to represent the 5 seeds. Repeat this process 30 times and count the fraction of times that at least 1 “flower” was spun.

Solution

Using the computer is the best simulation. Using another plant will probably be costly and take a long time, so it is not a good simulation. Rolling the standard dice does not match the probability for each seed, so it will not produce a good simulation. The spinner idea would work as a simulation, but it had only 30 trials instead of 300 for the computer.

Problem 2 Statement

Jada and Elena learned that 8% of students have asthma. They want to know the probability that in a team of 4 students, at least one of them has asthma. To simulate this, they put 25 slips of paper in a bag. Two of the slips say "asthma." Next, they take four papers out of the bag and record whether at least one of them says "asthma." They repeat this process 15 times.

- Jada says they could improve the accuracy of their simulation by using 100 slips of paper and marking 8 of them.
- Elena says they could improve the accuracy of their simulation by conducting 30 trials instead of 15.

a. Do you agree with either of them? Explain your reasoning.

b. Describe another method of simulating the same scenario.

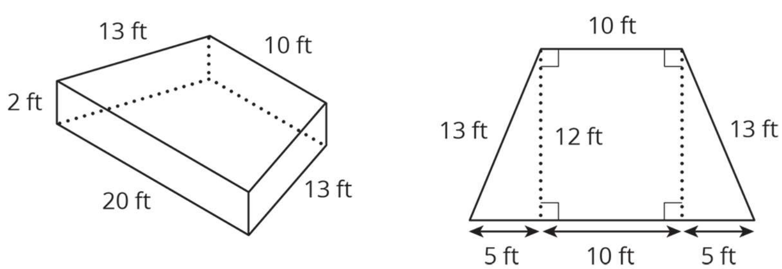
Solution

a. I agree with Elena, but not with Jada. Jada's suggestion would have the same probability of a success as the original simulation, so it would work, but would not produce more accuracy. More trials help keep uncommon outcomes from having a big impact on the estimated probability.

b. Answers vary. Sample response: Use a random digit list. For each trial, take 4 pairs of digits (00 through 99). Repeat the simulation many times. Use the proportion of times the pairs 01 through 08 appeared in the outcomes to estimate the probability that at least one student on the team has asthma.

Problem 3 Statement

The figure on the left is a trapezium based prism. The figure on the right represents its base. Find the volume of this prism.



Solution

360 ft^3. The area of the trapezium shaped base is 180 ft^2, the height is 2 ft, and $180 \times 2 = 360$.

Problem 4 Statement

Match each expression in the first list with an equivalent expression from the second list.

A. $(8x + 6y) + (2x + 4y)$

B. $(8x + 6y) - (2x + 4y)$

C. $(8x + 6y) - (2x - 4y)$

D. $8x - 6y - 2x + 4y$

E. $8x - 6y + 2x - 4y$

F. $8x - (-6y - 2x + 4y)$

1. $10(x + y)$

2. $10(x - y)$

3. $6\left(x - \frac{1}{3}y\right)$

4. $8x + 6y + 2x - 4y$

5. $8x + 6y - 2x + 4y$

6. $8x - 2x + 6y - 4y$

Solution

- A: 1
- B: 6
- C: 5
- D: 3
- E: 2
- F: 4