< Damped Harmonic Oscillation >

Damped oscillations.ggb

Linear damped.ggb

< Theory >

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0 \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

- $\gamma > 2\omega_0$ (over damping) $\quad x(t) = e^{-\gamma t/2}\left[x_0 \cosh(\beta t) + \dfrac{v_0 + \gamma x_0/2}{\beta}\sinh \beta t\right] \quad \beta = \dfrac{\sqrt{\gamma^2 - 4\omega_0^2}}{2}$
- $\gamma = 2\omega_0$ (critical damping) $\quad x(t) = x_0 e^{-\gamma t/2}\left[1 + \left(\dfrac{v_0}{x_0} + \dfrac{\gamma}{2}\right)t\right]$
- $\gamma < 2\omega_0$ (under damping) $\quad x(t) = x_0 e^{-\gamma t/2}\dfrac{\cos(\omega_1 t + \phi)}{\cos(\phi)}$

  - $\omega_1 = \omega_0\sqrt{1 - \left(\dfrac{\gamma}{2\omega_0}\right)^2}$
  - $\phi = -\tan^{-1}\left(\dfrac{v_0}{\omega_1 x_0} + \dfrac{\gamma}{2\omega_1}\right)$

    $= -\cos^{-1}(\cdots)$

    $= -\sin^{-1}(\cdots)$

- x_0 : Initial position
- v_0 : Initial velocity

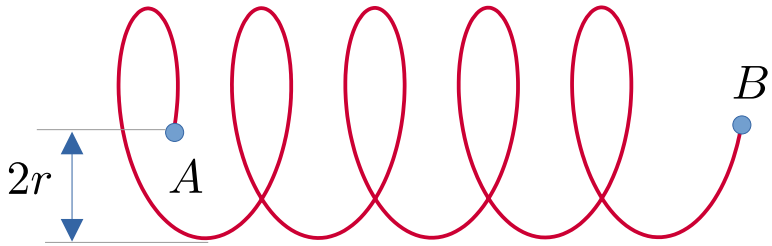
< Spring >

Spring (geo)

$$\begin{cases} x = A_x + [Lt + r + r\cos(\pi - \omega t)e_1 - 2r\sin(\pi - \omega t)e_2] \\ y = A_y + [Lt + r + r\cos(\pi - \omega t)e_2 + 2r\sin(\pi - \omega t)e_1] \end{cases} \qquad 0 \le t \le 1 \qquad \omega = 2n\pi$$

$$\begin{cases} A = (A_x, A_y) \\ B = (B_x, B_y) \end{cases} \qquad L = |A - B| = \text{Distance}(A, B) \qquad e_1 = x\frac{B - A}{L} \qquad e_2 = y\frac{B - A}{L}$$

Curve(x(A) + (L t + r + r cos(π - w t)) e_1 - 2r sin(π - w t) e_2, y(A) + (L t + r + r cos(π - w t)) e_2 + 2r sin(π - w t) e_1, t, 0, 1)



Damped oscillations.ggb

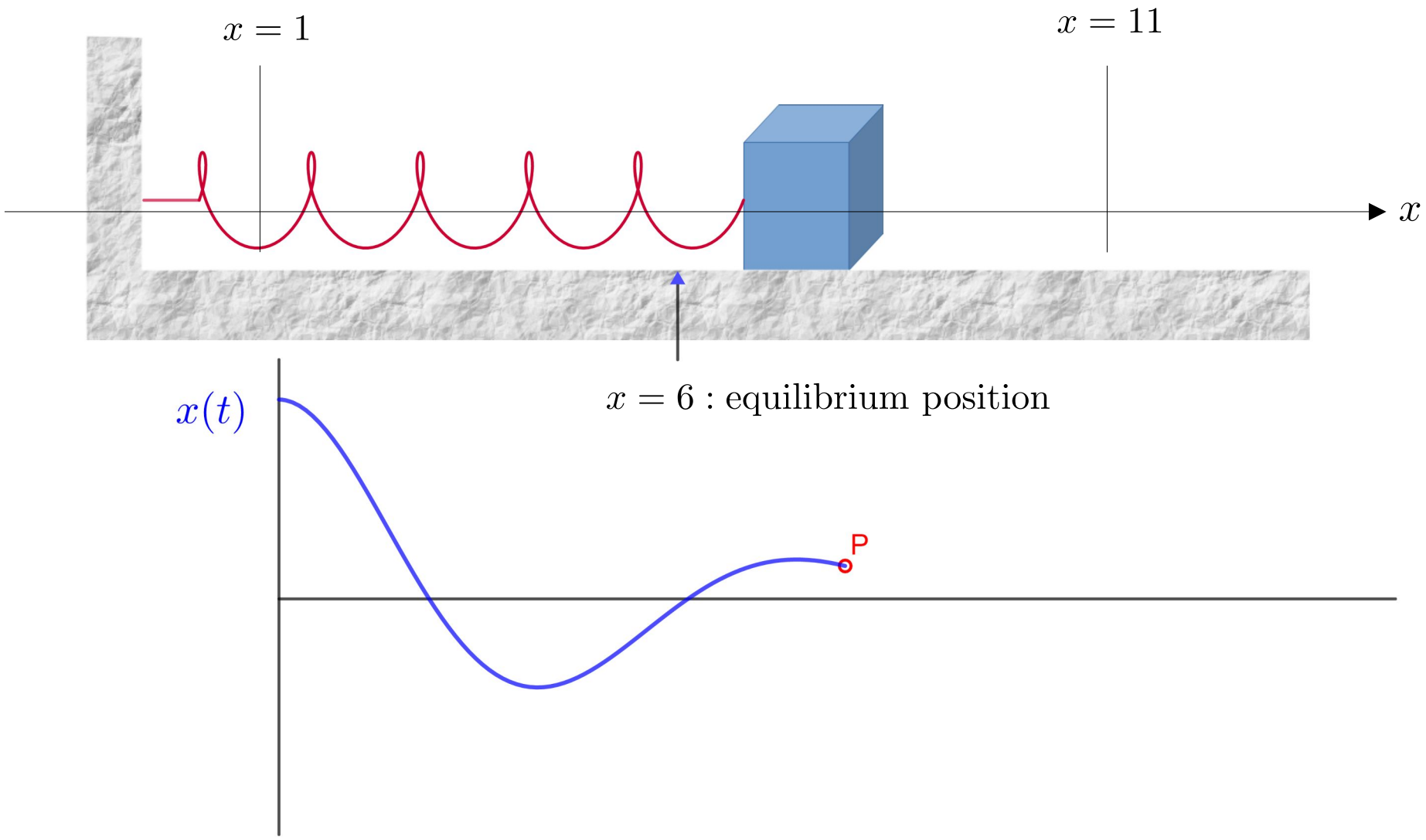
$w0 : \omega_0$ $x0 : x_0$

$\text{gam} : \gamma$ $v0 : v_0$ $\text{vtime} : t$

vtime = 7.1

w0 1 x0 5

gam 0.5 v0 0



Linear damped.ggb