Lesson 3: Rational and irrational numbers

Goals

- Comprehend the term “irrational number” (in spoken language) to mean a number that is not rational and that $\sqrt{2}$ is an example of an irrational number.
- Comprehend the term “rational number” (in written and spoken language) to mean a fraction or its opposite.
- Determine whether a given rational number is a solution to the equation $x^2 = 2$ and explain (orally) the reasoning.

Learning Targets

- I know what an irrational number is and can give an example.
- I know what a rational number is and can give an example.

Lesson Narrative

In previous lessons, students learned that square root notation is used to write the side length of a square given the area of the square. For example, a square whose area is 17 square units has a side length of $\sqrt{17}$ units.

In this lesson, students build on their work with square roots to learn about a new mathematical idea, *irrational numbers*. Students recall the definition of *rational numbers* and use this definition to search for a rational number x such that $x^2 = 2$. Students should not be left with the impression that looking for and failing to find a rational number whose square is 2 is a *proof* that $\sqrt{2}$ is irrational; this exercise is simply meant to reinforce what it means to be irrational and to provide some plausibility for the claim. Students are not expected to prove that $\sqrt{2}$ is irrational in KS3, and so ultimately must just accept it as a fact for now.

In the next lesson, students will learn strategies for finding the approximate location of an irrational number on a number line.

Building On

- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
- Write and evaluate numerical expressions involving whole-number exponents.

Addressing

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Building Towards

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Instructional Routines

- Algebra Talk
- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Collect and Display
- Discussion Supports
- Think Pair Share

Required Materials

Tracing paper

Required Preparation

It would be useful throughout this unit to have a list of perfect squares for easy reference. Consider hanging up a poster that shows the 20 perfect squares from 1 to 400. It is particularly handy in this lesson.

Student Learning Goals

Let's learn about irrational numbers.

3.1 Algebra Talk: Positive Solutions

Warm Up: 5 minutes

The purpose of this warm-up is for students to review multiplication of fractions in preparation for the main problem of this lesson: estimating solutions to the equation $x^2 = 2$. For example, $\frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$, which is a value close to 2 so $\frac{3}{2}$ is a value close to $\sqrt{2}$. For this activity it is best if students work with fractions and do not convert any numbers to their decimal forms. Answers expressed in decimal form aren't wrong, but if students work with decimal forms, they will miss out on the purpose of this warm-up.

While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem.

Instructional Routines

- Algebra Talk
- Discussion Supports

Launch

Ask students, "Could 8 be a solution to $x^2 = 49$? Why or why not?"

Display one problem at a time. Give students 30 seconds of quiet think time for each problem, and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find a positive solution to each equation:

$x^2 = 36$

$x^2 = \frac{9}{4}$

$x^2 = \frac{1}{4}$

$x^2 = \frac{49}{25}$

Student Response

- 6
- $\frac{3}{2}$
- $\frac{1}{2}$
- $\frac{7}{5}$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. At the end of the discussion, ask students to explain what they know about multiplication of fractions that helped them find the value of x^2 in each problem.

To involve more students in the conversation, consider asking:

- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve for the value of x in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

3.2 Three Squares

10 minutes

This activity is the first of three activities in which students investigate the value of $\sqrt{2}$. In this activity, students draw three squares on small grids, building on their earlier work. Two of the three possible squares have easily countable side lengths since they line up along the grid lines. The third likely possibility, that of a tilted square with vertices at the midpoint of each side of the grid, has sides equal to the length of a diagonal of a 1 unit square.

Monitor for students who:

- compare lengths directly by either creating a grid ruler or by tracing a segment with tracing paper and bringing it side by side with another segment. Most likely these students will say that the side length of the tilted square is around 1.5 units (or possibly, a little bit less than 1.5 units).
- who recall the square root notation from the previous lesson and expressed the side length of the tilted square as $\sqrt{2}$. Since the area of the tilted square is 2 square units, we can express its side length as $\sqrt{2}$ units.

If we use tracing paper or create a ruler scaled the same way as the grid, it can be seen that $\sqrt{2}$ is a bit less than 1.5. In the next activity, students will look for a more precise value of $\sqrt{2}$.

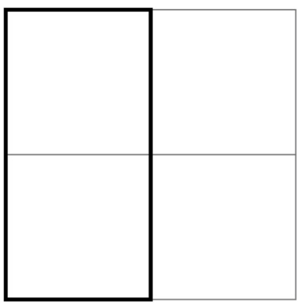
Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports

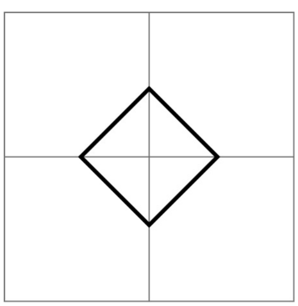
Launch

Provide access to tracing paper. Display the image from the task statement for all to see, and note that there are 9 vertices in each 2-by-2 grid. Students are asked in this activity to draw “squares of different sizes with vertices aligned to the vertices of the grid,” so make sure they can interpret this correctly. It may be helpful to draw a few non-examples.

This has vertices aligned to the grid, but is not a square:



This looks like a square, but doesn’t have vertices aligned to the grid:

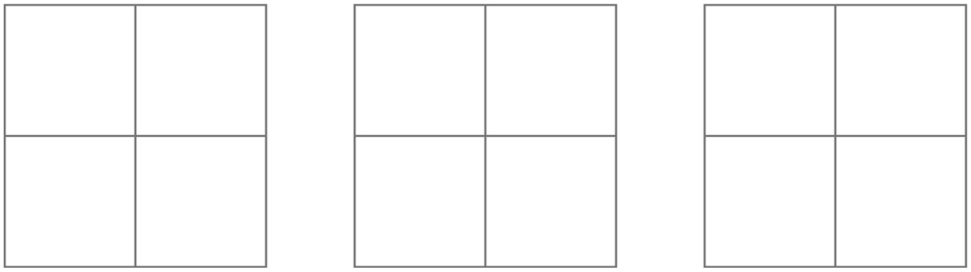


Representation: Internalise Comprehension. Provide a range of examples and counterexamples of squares with vertices that align to vertices on a grid. Consider using the provided examples in the Launch. Ask student volunteers to justify the reasoning for each.
Supports accessibility for: Conceptual processing

Anticipated Misconceptions

Some will say the side length of the tilted square is 1 unit, because a common misconception is that the diagonal of a square has a length of 1 unit. Ask students if the square has the same area as a grid square.

Student Task Statement



1. Draw 3 squares of different sizes with vertices aligned to the vertices of the grid.

2. For each square:

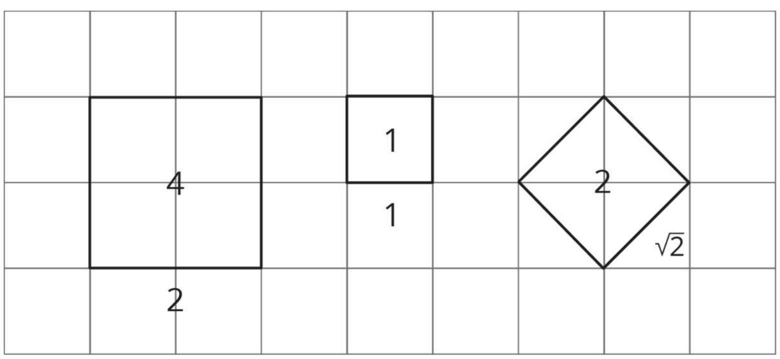
    a. Label the area.

    b. Label the side length.

    c. Write an equation that shows the relationship between the side length and the area.

Student Response

1. Answers vary. Sample response:



2. Sample response:

    a. The area is 2 square units.

    b. The side length is $\sqrt{2}$ units.

    c. $(\sqrt{2})^2 = 2$.

Activity Synthesis

First, select students who tried to compare the lengths directly by either creating a grid ruler or by tracing a segment with tracing paper and bringing it side by side with another segment during the discussion. Highlight the use of rotating figures to align sides in order to compare their lengths. Most likely these students will say that the side length of the tilted square is around 1.5 units (or possibly, a little bit less than 1.5 units).

Then, select students who recall the square root notation from the previous lesson and expressed the side length of the tilted square as $\sqrt{2}$. Since the area of the tilted square is 2 square units, its side length can be expressed as $\sqrt{2}$ units. This activity shows that $\sqrt{2}$ is around 1.5. In the next activity, students will look for a more precise value of $\sqrt{2}$.

Speaking, Listening: Discussion Supports. As students share their answer for the area and side length of the tilted square, press for details in students' reasoning by asking how they know the area is 2 square units. Listen for and amplify the language students use to describe either the "decompose and rearrange" or the "surround and subtract" method for finding the area of the tilted square. Then ask students to explain why the side length of the tilted square must be $\sqrt{2}$. This will support rich and inclusive discussion about strategies for finding the area and side length of a tilted square.
Design Principle(s): Support sense-making

3.3 Looking for a Solution

10 minutes

This activity is the second of three activities in which students investigate the value of $\sqrt{2}$. As a result of the previous activity, students should believe that $\sqrt{2}$ is about 1.5, maybe a little bit less.

Students should also understand that $\sqrt{2}$ is a number that we can multiply by itself and get a 2, so this is true:

$\sqrt{2} \times \sqrt{2} = 2$

The goal of this activity is to achieve a more precise value of $\sqrt{2}$ than "about 1.5, maybe a little less." In other words, an exact solution to $x^2 = 2$ is required. In this activity, students will consider whether certain candidates are solutions to this equation.

In the whole-class discussion that follows, *rational number* is defined.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Remind students that 1.5 is equivalent to $\frac{3}{2}$. Students in groups of 2. Give 2–3 minutes quiet work time followed by partner then a whole-class discussion.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: “First, I _____ because . . .”, “I noticed _____ so I . . .”, “Why did you . . .?”, “I agree/disagree because . . .”
Supports accessibility for: Language; Social-emotional skills Writing, Speaking, Listening: Stronger and Clearer Each Time. After students have had time to think about whether the numbers are solutions to the equation $x^2 = 2$, ask them to write a brief explanation. Invite students to meet with 2–3 other students for feedback. Display prompts for feedback that will help students strengthen their ideas and clarify their language. For example, “What does it mean for a number to be a solution to an equation?” and “How do you know this number is or is not a solution to the equation?” Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students revise both their ideas and their verbal and written output.
Design Principles(s): Optimise output (for explanation); Maximise meta-awareness

Student Task Statement

Are any of these numbers a solution to the equation $x^2 = 2$? Explain your reasoning.

- 1
- $\frac{1}{2}$
- $\frac{3}{2}$
- $\frac{7}{5}$

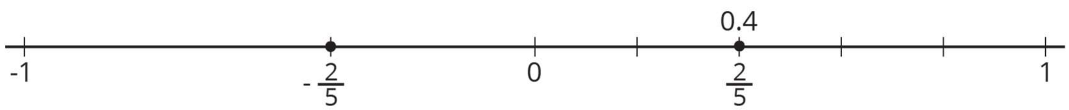
Student Response

1. No, because $1^2 = 1$, not 2.
2. No, because $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.
3. No, because $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$. ($\frac{9}{4} > 2$, since $2 = \frac{8}{4}$.)
4. No, because $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$. ($\frac{49}{25} < 2$, since $2 = \frac{50}{25}$.)

Activity Synthesis

Select students to share their reasoning for why each number is not a solution to $x^2 = 2$. If students do not point it out, make sure that they notice that $\frac{3}{2}$ is not a terrible approximation for $\sqrt{2}$, since $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$, and $\frac{9}{4}$ is only a bit larger than 2. $\frac{7}{5}$ is an even better approximation for $\sqrt{2}$, since $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$, which is just a little bit smaller than $\frac{50}{25}$ (which equals 2). If you have posted a list of perfect squares in the room for reference, refer to this list during the discussion, because that list will come in handy when students work on the next activity.

This is where we want to define *rational number*. A **rational number** is a fraction or its opposite. Remember that a fraction is a number on the number line that you get by dividing the unit interval into b equal parts and finding the point that is a of them from 0. We can always write a fraction in the form $\frac{a}{b}$ where a and b are whole numbers (and b is not 0), but there are other ways to write them. For example, 0.4 is a fraction because it is the point on the number line you get by dividing the unit interval into 10 equal parts and finding the point that is 4 away from 0. (You get the same point if you divide the unit interval into 5 equal parts and find the point that is 2 away from 0.) When we first learned about fractions, we didn't know about negative numbers. Rational numbers are fractions, but they can be positive or negative. So, $-\frac{2}{5}$ is also a rational number.



Here are some examples of rational numbers: $\frac{7}{4}$, 0, $\frac{6}{3}$, 0.2, $-\frac{1}{3}$, -5, $\sqrt{9}$, $-\frac{\sqrt{16}}{\sqrt{100}}$ Can you see why they are each "a fraction or its opposite?"

Because fractions and ratios are closely related ideas, fractions and their opposites are called RATIOnal numbers.

3.4 Looking for $\sqrt{2}$

10 minutes

This activity is the third of three activities in which students investigate the value of $\sqrt{2}$. In the previous activity, students saw that $\frac{7}{5}$ is a pretty good approximation for $\sqrt{2}$. $\sqrt{2}$ is a number when multiplied by itself equals 2, and $\frac{7}{5}$ multiplied by itself is pretty close to 2, it's $\frac{49}{25}$. In the discussion of the previous activity, students also learned (or were reminded of) the definition of rational number: a fraction or its opposite.

In this activity, students have a chance to look for some more rational numbers that are close to $\sqrt{2}$. The idea is that they will look, for a while, for a fraction that can be multiplied by itself where the product is exactly 2. Of course, they won't find one, because there is no such number. In the whole-class discussion that follows, *irrational number* is defined.

Instructional Routines

- Collect and Display

Launch

For this activity, it is best if students do not have access to a calculator with a square root button. A calculator that shows 9 decimal places will tell you that $\sqrt{2} = 1.414213562$ (which is not true) and that $1.414213562^2 = 2$ (which is also not true). (To convince students that the calculator is lying to them, you have to make them multiply 1.414213562 by itself by hand, so better to sidestep the issue for now.) It *would* be handy for students to refer to a list of perfect squares while working on this activity, so consider posting such a list and drawing students' attention to it.

Before they get started, remind students that a rational number is a fraction or its opposite, for example, $\frac{9}{8}$. Let them know that terminating decimals are also rational, for example, $0.7 = \frac{7}{10}$.

As students work, it is possible they will focus on numbers in decimal form when searching for a rational number close to $\sqrt{2}$. If you see this, encourage students to remember their work in the previous activity and the example of $\frac{7}{5}$, whose square, $\frac{49}{25}$, was very close to 2. Are there any other values like $\frac{7}{5}$ that might be even closer to $\sqrt{2}$?

Since students could search indefinitely for a solution to the last problem with no success, ask students to stop their work in order to leave 3–4 minutes for a whole-class discussion.

Representation: Develop Language and Symbols. Display or provide charts with symbols and meanings. For example, display the term rational number with its definition and examples from the previous activity.
Supports accessibility for: Conceptual processing; Memory Conversing, Reading: Collect and Display. As students work in pairs on the task, circulate and listen to as they discuss strategies for finding rational numbers that are close to $\sqrt{2}$. Record the words, phrases, and quantities students use on a visual display. Invite students to review the display, and ask questions to clarify the meaning of a word or phrase. For example, a phrase such as: "There is no number that is equal to $\sqrt{2}$" can be restated as "We could not find a rational number that is equal to $\sqrt{2}$." Encourage students to refer back to the visual display during whole-class discussions throughout the lesson and unit. This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

A **rational number** is a fraction or its opposite (or any number equivalent to a fraction or its opposite).

1. Find some more rational numbers that are close to $\sqrt{2}$.

2. Can you find a rational number that is exactly $\sqrt{2}$?

Student Response

1. Answers vary. Sample responses:

   - $\frac{10}{7}$, because $\left(\frac{10}{7}\right)^2 = \frac{100}{49}$
   - $\frac{17}{12}$, because $\left(\frac{17}{12}\right)^2 = \frac{289}{144}$
   - $\frac{13}{9}$, because $\left(\frac{13}{9}\right)^2 = \frac{169}{81}$
   - $\frac{141}{100}$, because $\left(\frac{141}{100}\right)^2 = \frac{19\,881}{10\,000}$

2. No.

Are You Ready for More?

If you have an older calculator evaluate the expression $\left(\frac{577}{408}\right)^2$ and it will tell you that the answer is 2, which might lead you to think that $\sqrt{2} = \frac{577}{408}$.

1. Explain why you might be suspicious of the calculator's result.
2. Find an explanation for why $408^2 \times 2$ could not possibly equal 577^2. How does this show that $\left(\frac{577}{408}\right)^2$ could not equal 2?
3. Repeat these questions for $\left(\frac{1414213562375}{1000000000000}\right)^2 \neq 2$,an equation that even many modern calculators and computers will get wrong.

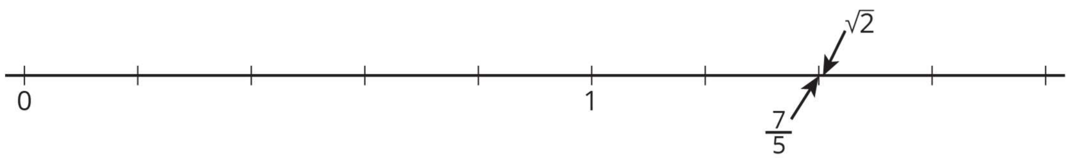
Student Response

One reason to be suspicious is that the calculator might have rounded the answer, so it may just be that $\left(\frac{577}{408}\right)^2$ is *close* to 2. In fact, we can argue for certain that it is not equal to 2. The equation $577^2 = 2 \times 408^2$ could not possibly be true since the left-hand side of that expression is odd, but the right-hand side is even. But now it could not possibly be that $\left(\frac{577}{408}\right)^2 \neq 2$, since after multiplying both sides by 408^2 this would mean $577^2 = 2 \times 408^2$. This exact argument also works for the second example, as $1\,414\,213\,562\,375^2 \neq 2 \times 1\,000\,000\,000\,000^2$ for the same reason. In fact, by breaking into cases depending on which of the numerator and denominator is even or odd, this line of thinking leads to a complete proof that no fraction at all can be squared to get the number 2, proving that $\sqrt{2}$ is irrational.

Activity Synthesis

Ideally, students were given time to look for a rational number that is a solution to $x^2 = 2$. Select students with particularly close values to share their number with the class and what strategy they used to find it. Applaud students for their perseverance, but confess that no such number exists because it is an *irrational number*.

Display a number line for all to see such as the one shown here. Tell students that an **irrational number** is a number that is not rational. That is, it is a number that is not a fraction or its opposite. $\sqrt{2}$ is one example of an irrational number. It has a location on the number line, and its location can be narrowed down (it's a tiny bit to the right of $\frac{7}{5}$), but $\sqrt{2}$ cannot be found on a number line by subdividing the unit interval into b parts and taking a of them. We have to define $\sqrt{2}$ in a different way, such as the side length of a square with area 2 square units.



Just looking for rational solutions to $x^2 = 2$ and not finding any is not a proof that $\sqrt{2}$ is irrational. But for the purposes of this course, students are asked to take it as a fact that $\sqrt{2}$ is irrational. In students' future studies, they may have opportunities to understand or write a proof that $\sqrt{2}$ is irrational.

While the activities focus specifically on $\sqrt{2}$, there is nothing particularly special about this example. The square root of a whole number is either a whole number or irrational, so $\sqrt{10}$, $\sqrt{67}$, etc., are all irrational. Beyond KS3 level but worth having on hand for the sake of discussion is that rational multiples of irrational numbers are also irrational, so numbers like $5\sqrt{7}$, -$\sqrt{45}$, and $\frac{\sqrt{5}}{3}$ are also irrational.

Lesson Synthesis

The purpose of this discussion is to explicitly point out that while we have collected some evidence that supports the claim that $\sqrt{2}$ is irrational, we have not actually proved this claim. Here are some possible questions for discussion:

- "If I told you that there are no purple zebras, and you spent your whole life travelling the world and never saw a purple zebra, does it mean I was right?" (No, it is possible you just failed to find a purple zebra.)
- "So if we spent our whole lives testing different fractions and never quite got one whose square is 2, does that mean there are no such fractions?" (No, maybe you just haven't found it yet.)

Tell students that we haven't learned enough to prove for sure that $\sqrt{2}$ is not equivalent to a fraction. For now, we just have to trust that there are numbers on the number line that are not equivalent to a fraction and that $\sqrt{2}$ is one of them. However, it is possible to get very close estimates with fractions.

3.5 Types of Solutions

Cool Down: 5 minutes

Student Task Statement

1. In your own words, say what a rational number is. Give at least three different examples of rational numbers.

2. In your own words, say what an irrational number is. Give at least two examples.

Student Response

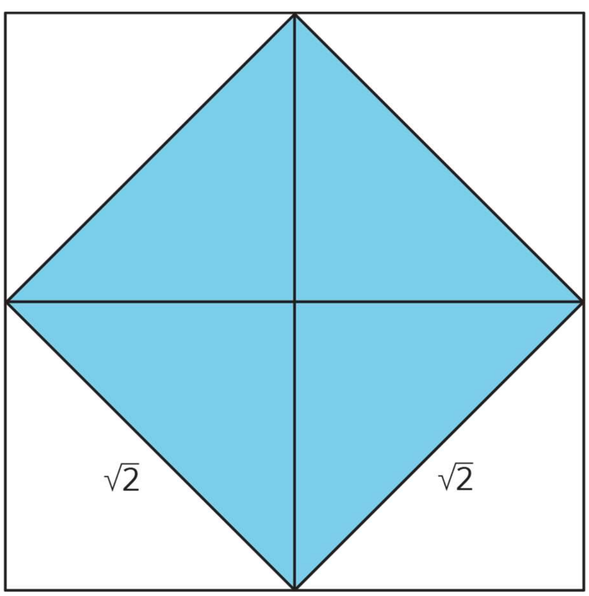
Answers vary. Sample response:

1. A rational number is a fraction, like $\frac{1}{2}$, or its opposite, like $-\frac{1}{2}$. Something like 3.98 is rational too because it is equal to $\frac{398}{100}$.

2. An irrational number is one that is not rational. $\sqrt{2}$ and π are two examples.

Student Lesson Summary

In an earlier activity, we learned that square root notation is used to write the length of a side of a square given its area. For example, a square whose area is 2 square units has a side length of $\sqrt{2}$ units, which means that $\sqrt{2} \times \sqrt{2} = 2$.

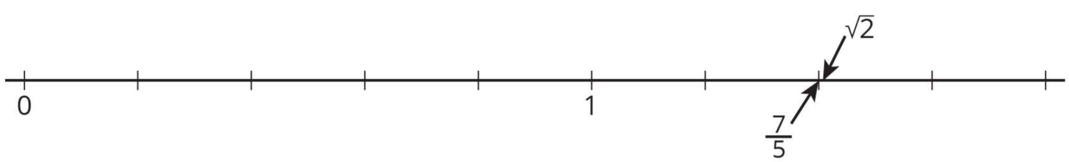


A square whose area is 25 square units has a side length of $\sqrt{25}$ units, which means that $\sqrt{25} \times \sqrt{25} = 25$. Since $5 \times 5 = 25$, we know that $\sqrt{25} = 5$.

$\sqrt{25}$ is an example of a rational number. A **rational number** is a fraction or its opposite. Remember that a fraction $\frac{a}{b}$ is a point on the number line found by dividing the segment from 0 to 1 into b equal intervals and going a of those intervals to the right of 0. We can always write a fraction in the form $\frac{a}{b}$ where a and b are whole numbers (and b is not 0), but there are other ways to write them. For example, we can write $\sqrt{25} = \frac{5}{1}$. You first learned about fractions in earlier years, and at that time, you probably didn't know about negative numbers. Rational numbers are fractions, but they can be positive or negative. So, -5 is also a rational number. Because fractions and ratios are closely related ideas, fractions and their opposites are called RATIOnal numbers.

Here are some examples of rational numbers: $\frac{7}{4}$, 0, $\frac{6}{3}$, 0.2, -$\frac{1}{3}$, -5, $\sqrt{9}$, -$\frac{\sqrt{16}}{\sqrt{100}}$ Can you see why they are each examples of "a fraction or its opposite?"

An **irrational number** is a number that is not rational. That is, it is a number that is not a fraction or its opposite. $\sqrt{2}$ is an example of an irrational number. It has a location on the number line, and its location can be approximated by rational numbers (it's a tiny bit to the right of $\frac{7}{5}$), but $\sqrt{2}$ can not be found on a number line by dividing the segment from 0 to 1 into b equal parts and going a of those parts away from 0 (if a and b are whole numbers).



$\frac{17}{12}$ is also close to $\sqrt{2}$, because $\left(\frac{17}{12}\right)^2 = \frac{289}{144}$. $\frac{289}{144}$ is very close to 2, since $\frac{288}{144} = 2$. But we could keep looking forever for solutions to $x^2 = 2$ that are rational numbers, and we would not find any. $\sqrt{2}$ is not a rational number! It is irrational.

In your future studies, you may have opportunities to understand or write a proof that $\sqrt{2}$ is irrational, but for now, we just take it as a fact that $\sqrt{2}$ is irrational. Similarly, the square root of any whole number is either a whole number ($\sqrt{36} = 6$, $\sqrt{64} = 8$, etc.) or irrational ($\sqrt{17}$, $\sqrt{65}$, etc.). Here are some other examples of irrational numbers: $\sqrt{10}$, -$\sqrt{3}$, $\frac{\sqrt{5}}{2}$, π

Glossary

- irrational number
- rational number

Lesson 3 Practice Problems

1. **Problem 1 Statement**

   Decide whether each number in this list is *rational* or *irrational*.

   $\frac{-13}{3}$, 0.1234, $\sqrt{37}$, -77, -$\sqrt{100}$, -$\sqrt{12}$

   Solution

   Rational: $\frac{-13}{3}$, 0.1234,-77,-$\sqrt{100}$; Irrational: $\sqrt{37}$,-$\sqrt{12}$

2. **Problem 2 Statement**

   Which value is an exact solution of the equation $m^2 = 14$?

   a. 7

   b. $\sqrt{14}$

   c. 3.74

   d. $\sqrt{3.74}$

   Solution B

3. **Problem 3 Statement**

   A square has vertices (0,0), (5,2), (3,7), and (-2,5). Which of these statements is true?

   a. The square's side length is 5.

   b. The square's side length is between 5 and 6.

   c. The square's side length is between 6 and 7.

   d. The square's side length is 7.

   Solution B

4. **Problem 4 Statement**

   Rewrite each expression in an equivalent form that uses a single exponent.

   a. $(10^2)^{-3}$

   b. $(3^{-3})^2$

   c. $3^{-5} \times 4^{-5}$

   d. $2^5 \times 3^{-5}$

Solution

a. 10^{-6} (or equivalent)

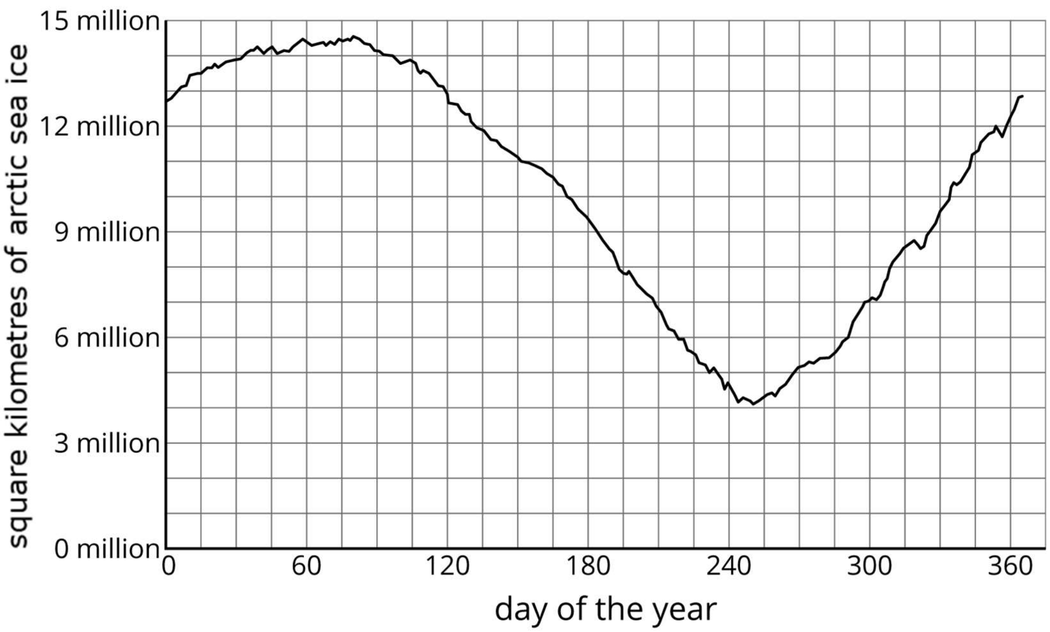
b. 3^{-6} (or equivalent)

c. 12^{-5} (or equivalent)

d. $\left(\frac{2}{3}\right)^5$ (or equivalent)

5. **Problem 5 Statement**

The graph represents the area of arctic sea ice in square kilometres as a function of the day of the year in 2016.



a. Give an approximate interval of days when the area of arctic sea ice was decreasing.

b. On which days was the area of arctic sea ice 12 million square kilometres?

Solution

a. Answers vary. Correct responses should be close to "day 75 to day 250."

b. Days 135, 350, and 360

6. **Problem 6 Statement**

The secondary school is hosting an event for KS4 students but will also allow some KS3 students to attend. The headteacher approved the event for 200 students and decided the number of KS3 students should be 25% of the number of KS4 students. How many KS3 students will be allowed to attend? If you get stuck, try writing two equations that each represent the number of KS3 students and KS4 students at the event.

Solution

40 KS3 students. Sample reasoning: Solve the system $s + j = 200$, $j = 0.25s$ (or equivalent), where s represents the number of KS4 students and j represents the number of KS3 students.