Lesson 5: Negative exponents with powers of 10

Goals

- Describe (orally and in writing) how exponent rules extend to expressions involving negative exponents.
- Describe patterns in repeated multiplication and division with 10 and $\frac{1}{10}$, and justify (orally and in writing) that $10^{-n} = \frac{1}{10^n}$.

Learning Targets

- I can use the exponent rules with negative exponents.
- I know what it means if 10 is raised to a negative power.

Lesson Narrative

Sometimes in mathematics, extending existing theories to areas outside of the original definition leads to new insights and new ways of thinking. Students practise this here by extending the rules they have developed for working with powers to a new situation with negative exponents. The challenge then becomes to make sense of what negative exponents might mean. This type of reasoning appears again in KS4 when students extend the rules of exponents to make sense of exponents that are not integers.

In analogy to positive powers of 10 that describe repeated multiplication by 10, this lesson presents negative powers of 10 as repeated multiplication by $\frac{1}{10}$, leading ultimately to the rule $10^{-n} = \frac{1}{10^n}$. Students use repeated reasoning to generalise about negative exponents. Students create viable arguments and critique the reasoning of others when comparing and contrasting, for example, $(10^{-2})^3$ and $(10^2)^{-3}$. With this understanding of negative exponents, all of the exponent rules created so far are seen to be valid for any integer exponents.

Building On

- Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the place value of the digits when a decimal value is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Addressing

- Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

Instructional Routines

- Collect and Display

- Discussion Supports
- Number Talk

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Create a visual display for $10^{-n} = \frac{1}{10^n}$. For an example of how the rule works, consider showing $10^{-3} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10^3}$.

Student Learning Goals

Let's see what happens when exponents are negative.

5.1 Number Talk: What's That Exponent?

Warm Up: 10 minutes

The purpose of this Number Talk is to elicit strategies and understandings students have for dividing powers. These understandings help students develop fluency and will be helpful later in this lesson when students investigate negative exponents. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem. It is expected that students won't know how to approach the final question. Encourage them to make their best guess based on patterns they notice.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Allow students to share their answers with a partner and note any discrepancies.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Solve each equation mentally.

$$\frac{100}{1} = 10^x$$

$$\frac{100}{x} = 10^1$$

$$\frac{x}{100} = 10^0$$

$$\frac{100}{1\,000} = 10^x$$

Student Response

- $x = 2$. Possible strategies: Dividing 100 by 1 gives 100, and 100 is equal to 10^2.
- $x = 10$. Possible strategies: 10^1 is equal to 10, and 100 divided by 10 is also equal to 10.
- $x = 100$. Possible strategies: 10^0 is equal to 1, so the left side of the equation must be $\frac{100}{100}$.
- $x =$ -1. Possible strategies: Looking on the pattern on the right sides of the equations suggests the exponent may be -1.When the numerator is larger than the denominator, the exponent is positive. When the numerator is the same size as the denominator, the exponent is 0. In this case, the numerator is smaller than the denominator, so it appears the exponent should be negative.

Activity Synthesis

Ask students to share what they noticed about the first three problems. Record the equations with the solutions written in place:

$$\frac{100}{1} = 10^2$$

$$\frac{100}{10} = 10^1$$

$$\frac{100}{100} = 10^0$$

Ask students to describe any patterns they see, and how they would continue the patterns for the last problem:

$$\frac{100}{1\,000} = 10^x$$

Tell them that in this lesson, we will explore negative exponents.

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ."

Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

5.2 Negative Exponent Table

10 minutes

Students extend their understanding of exponents to include negative exponents and explain patterns in the placements of the decimal point when a decimal is multiplied by 10 or $\frac{1}{10}$. Students use repeated reasoning to recognise that negative powers of 10 represent repeated multiplication by $\frac{1}{10}$ and generalise to the rule $10^{-n} = \frac{1}{10^n}$.

A table is used to show different representations of decimals, fractions, and exponents. The table is horizontal to mimic the structure of decimals. As students work, notice the strategies they use to go between the different representations of the given powers of 10. Ask students who use contrasting strategies to share later.

Instructional Routines

- Collect and Display

Launch

Tell students to complete the table one row at a time to see the patterns most clearly. Ask a student to read the first question aloud. Select a student to explain the idea of a “multiplier” in this context. Give students 5–7 minutes to work. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may think that, for example, $\frac{1}{1\,000\,000} = 10^{-7}$ because the number 1,000,000 has 7 digits. Ask these students if it is true that $\frac{1}{10} = 10^{-2}$.

Student Task Statement

Complete the table to explore what negative exponents mean.

× 10 × 10 × 10 × 10 × 10 × 10

using exponents	10^3	10^2	10^1				
as a decimal	1000.0			1.0		0.01	
as a fraction		$\frac{100}{1}$		$\frac{1}{1}$			$\frac{1}{1000}$

×? ×? ×? ×? ×? ×?

1. As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?
2. How does a multiplier of 10 affect the placement of each digit in the product? How does the other multiplier affect the placement of each digit in the product?
3. Use the patterns you found in the table to write 10^{-7} as a fraction
4. Use the patterns you found in the table to write 10^{-5} as a decimal.
5. Write $\frac{1}{100\,000\,000}$ using a single exponent.
6. Use the patterns in the table to write 10^{-n} as a fraction.

Student Response

using exponents	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
as a decimal	1 000.0	100.0	10.0	1.0	0.1	0.01	0.001
as a fraction	$\frac{1\,000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1\,000}$

1. As you move towards the right, the multiplier is $\frac{1}{10}$
2. Multiplying by 10 results in each digit shifting one place to the left. Multiplying by $\frac{1}{10}$ results in each digit shifting one place to the right.
3. $10^{-7} = \frac{1}{10\,000\,000}$ or $\frac{1}{10^7}$
4. $10^{-5} = 0.00001$

5. $\frac{1}{100\,000\,000} = 10^{-8}$

6. $10^{-n} = \frac{1}{10^n}$

Activity Synthesis

One important idea is that multiplying by 10 increases the exponent, thus multiplying by $\frac{1}{10}$ decreases the exponent. So negative exponents can be thought of as repeated multiplication by $\frac{1}{10}$, whereas positive exponents can be thought of as repeated multiplication by 10. Another key point is the effect that multiplying by 10 or $\frac{1}{10}$ has on the placement of each digit.

Ask students to share how they converted between fractions, decimals, and exponents. Record their reasoning for all to see. Here are some possible questions to consider for whole-class discussion:

- "Do you agree or disagree? Why?"
- "Did anyone think of this a different way?"
- "In your own words, what does 10^{-7} mean? How is it different from 10^7?"

Introduce the visual display for $10^{-n} = \frac{1}{10^n}$ and display it for all to see throughout the unit. For an example that illustrates the rule, consider displaying $10^{-3} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10^3}$.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections. For example, annotate the patterns students notice to help synthesise how negative exponents can be thought of as repeated multiplication by $\frac{1}{10}$, whereas positive exponents can be thought of as repeated multiplication by 10.
Supports accessibility for: Visual-spatial processing Conversing, Representing, Writing: Collect and Display. During the whole-class discussion, capture the vocabulary and phrases students use to describe their strategies for converting between fractions, decimals, and exponents. Listen for students who make connections between repeated multiplication and the placement of each digit. Record this language onto a visual display that can be referenced in future discussions. This routine provides feedback to students that supports sense-making and increases meta-awareness of mathematical language.
Design Principle(s): Support sense-making; Maximise meta-awareness

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5.3 Follow the Exponent Rules

20 minutes

This activity requires students to make sense of negative powers of 10 as repeated multiplication by $\frac{1}{10}$ in order to distinguish between equivalent exponential expressions. If students have time, instruct them to write the other expressions in each table as a power of 10 with a single exponent as well. Look for students who have productive debate regarding the interpretation of, for example, $(10^2)^{-3}$ versus $(10^{-2})^3$ and ask them to share their reasoning later.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Give students 15 minutes of partner work time followed by a whole-class discussion. Ask students to explain their reasoning to their partner as they work. If there is disagreement, tell students to work to reach an agreement.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisational skills in problem solving. For example, present one question at a time.
Supports accessibility for: Organisation; Attention

Anticipated Misconceptions

Some students may struggle to distinguish between $(10^{-2})^3$ and $(10^2)^{-3}$. Breaking down each expression into parts that emphasise repeated multiplication will help to illustrate the difference. For the first expression, $10^{-2} = \frac{1}{10} \times \frac{1}{10}$. The outer exponent of 3 means that $\frac{1}{10} \times \frac{1}{10}$ is multiplied repeatedly 3 times. Similarly for the second expression, $10^2 = 10 \times 10$ and the outer exponent of -3 means that the *reciprocal* of 10×10 is multiplied repeatedly 3 times. In the end, both expressions are equal to 10^{-6}.

Student Task Statement

1. a. Match each exponential expression with an equivalent multiplication expression:

$(10^2)^3$

$(10^2)^{-3}$

$(10^{-2})^3$

$(10^{-2})^{-3}$

$\frac{1}{(10\times10)} \times \frac{1}{(10\times10)} \times \frac{1}{(10\times10)}$

$\left(\frac{1}{10}\times\frac{1}{10}\right)\left(\frac{1}{10}\times\frac{1}{10}\right)\left(\frac{1}{10}\times\frac{1}{10}\right)$

$\frac{1}{\frac{1}{10}\times\frac{1}{10}}\times\frac{1}{\frac{1}{10}\times\frac{1}{10}}\times\frac{1}{\frac{1}{10}\times\frac{1}{10}}$

$(10\times 10)(10\times 10)(10\times 10)$

b. Write $(10^2)^{-3}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

2. a. Match each exponential expression with an equivalent multiplication expression:

$\frac{10^2}{10^5}$

$\frac{10^2}{10^{-5}}$

$\frac{10^{-2}}{10^5}$

$\frac{10^{-2}}{10^{-5}}$

$\frac{\frac{1}{10}\times\frac{1}{10}}{\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}}$

$\frac{10\times 10}{10\times 10\times 10\times 10\times 10}$

$\frac{\frac{1}{10}\times\frac{1}{10}}{10\times 10\times 10\times 10\times 10}$

$\frac{10\times 10}{\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}}$

b. Write $\frac{10^{-2}}{10^{-5}}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

3. a. Match each exponential expression with an equivalent multiplication expression:

$10^4\times 10^3$

$10^4\times 10^{-3}$

$10^{-4}\times 10^3$

$10^{-4}\times 10^{-3}$

$(10\times 10\times 10\times 10)\times(\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10})$

$\left(\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\right)\times\left(\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\right)$

$\left(\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\right)\times(10\times 10\times 10)$

$(10 \times 10 \times 10 \times 10) \times (10 \times 10 \times 10)$

b. Write $10^{-4} \times 10^3$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

Student Response

1\. a.

$(10^2)^3$	$(10 \times 10)(10 \times 10)(10 \times 10)$
$(10^2)^{-3}$	$\frac{1}{(10\times10)} \times \frac{1}{(10\times10)} \times \frac{1}{(10\times10)}$
$(10^{-2})^3$	$\left(\frac{1}{10} \times \frac{1}{10}\right)\left(\frac{1}{10} \times \frac{1}{10}\right)\left(\frac{1}{10} \times \frac{1}{10}\right)$
$(10^{-2})^{-3}$	$\frac{1}{\frac{1}{10}\times\frac{1}{10}} \times \frac{1}{\frac{1}{10}\times\frac{1}{10}} \times \frac{1}{\frac{1}{10}\times\frac{1}{10}}$

b. $(10^2)^{-3} = 10^{2\times(-3)} = 10^{-6}$ or $(10^2)^{-3} = \frac{1}{(10\times10)} \times \frac{1}{(10\times10)} \times \frac{1}{(10\times10)} = \frac{1}{10^6} = 10^{-6}$.

2\. a.

$\frac{10^2}{10^5}$	$\frac{10\times10}{10\times10\times10\times10\times10}$
$\frac{10^2}{10^{-5}}$	$\frac{10\times10}{\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}}$
$\frac{10^{-2}}{10^5}$	$\frac{\frac{1}{10}\times\frac{1}{10}}{10\times10\times10\times10\times10}$
$\frac{10^{-2}}{10^{-5}}$	$\frac{\frac{1}{10}\times\frac{1}{10}}{\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}}$

b. $\frac{10^{-2}}{10^{-5}} = 10^{-2-(-5)} = 10^{-2+5} = 10^3$ or $\frac{10^{-2}}{10^{-5}} = \frac{\frac{1}{10}\times\frac{1}{10}}{\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}} = \frac{\frac{1}{10^2}}{\frac{1}{10^5}} = \frac{1}{10^2} \times \frac{10^5}{1} = \frac{10^5}{10^2} = 10^3$.

3\. a.

$10^4 \times 10^3$	$(10 \times 10 \times 10 \times 10) \times (10 \times 10 \times 10)$
$10^4 \times 10^{-3}$	$(10 \times 10 \times 10 \times 10) \times (\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10})$
$10^{-4} \times 10^3$	$(\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}) \times (10 \times 10 \times 10)$
$10^{-4} \times 10^{-3}$	$(\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}) \times (\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10})$

b. $10^{-4} \times 10^3 = 10^{-4+3} = 10^{-1}$ or $10^{-4} \times 10^3 = (\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}) \times (10 \times 10 \times 10) = \frac{10^3}{10^4} = \frac{1}{10} = 10^{-1}$.

Are You Ready for More?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game.

Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, "I knew I'd be the only one left, since I went first."

1. Record this game on paper a few times with different numbers of players. Does the person who starts always win?

2. Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

Student Response

1. No. If you play with five players, for example, the fifth person will win.

2. The person who starts will win if the number of people is a power of two. Otherwise, someone else will win.

Activity Synthesis

Select students who had disagreements during the activity to share what they disagreed about and how they came to an agreement. Consider asking:

- "How is $(10^{-2})^3$ different from $(10^2)^{-3}$? How are they the same?"
- "Do you agree or disagree? Why?"
- "Could you restate __'s reasoning in a different way?"

It is important for students to understand that the exponent rules work even with negative exponents. To see why, the whole class discussion must make a clear connection between the exponent rules and the process of multiplying repeated factors that are 10 and $\frac{1}{10}$. Contrast the expanded version of $(10^{-2})^3$ and $(10^2)^{-3}$. For $(10^{-2})^3$, there are 3 factors that are 10^{-2}, where 10^{-2} is two factors that are $\frac{1}{10}$, so $(10^{-2})^3 = \left(\frac{1}{10} \times \frac{1}{10}\right)\left(\frac{1}{10} \times \frac{1}{10}\right)\left(\frac{1}{10} \times \frac{1}{10}\right) = \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = 10^{-6}$. For $(10^2)^{-3}$, there are 3 factors that are $\frac{1}{10^2}$, so $(10^2)^{-3} = \left(\frac{1}{10 \times 10}\right)\left(\frac{1}{10 \times 10}\right)\left(\frac{1}{10 \times 10}\right) = \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = 10^{-6}$.Both $(10^{-2})^3$ and $(10^2)^{-3}$ are equal to 10^{-6} in the same way that $-2 \times 3 = 2 \times -3 = -6$.

Listening, Conversing: Discussion Supports. As students share the disagreements they had with their partner, press for details in students' statements by requesting that students elaborate on an idea or give an example of their process of reconciliation. For example, ask students, "How did you explain your reasoning to your partner in a way that convinced them?" In this discussion, it may be useful to revoice student ideas to model mathematical language use by restating a statement in order to clarify, apply appropriate language, or involve more students.
Design Principle(s): Maximise meta-awareness

Lesson Synthesis

The purpose of the discussion is to take a step back in order to see that negative exponents are not something new and different, but rather a natural part of the decimal place value system that we have been exploring for years.

Remind students that in primary school, we learned about our place value system and saw that it was possible to write very large numbers in a very small space because of positional notation. We learned that the value of a number is the sum of the numbers of each base 10 unit (ones, tens, hundreds, and so forth). We sometimes wrote things like $456 = 4 \times 100 + 5 \times 10 + 6 \times 1$. We can express this with exponents as $456 = 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$. We now extend the discussion of place value by considering ways to write very small numbers in a manner consistent with what we did with large numbers. Ask students:

- "How would you write 2 796 as a sum with powers of 10?" ($2 \times 10^3 + 7 \times 10^2 + 9 \times 10^1 + 6 \times 10^0$)
- "What about small numbers? How do you write the place value units of 0.1, 0.01, and 0.001 with powers of 10?" (10^{-1}, 10^{-2}, 10^{-3})
- "Then how would you write 0.2796 as a sum with powers of 10?" ($2 \times 10^{-1} + 7 \times 10^{-2} + 9 \times 10^{-3} + 6 \times 10^{-4}$)
- "Think about the meaning of exponents. How is 10^3 related to 10^{-3}?" (Exponents tell us to repeatedly multiply by a base. Whether the base is 10 or $\frac{1}{10}$, the structure of repeated multiplication is the same. 10^3 is multiplication by 10 repeated 3 times and 10^{-3} is multiplication by $\frac{1}{10}$ repeated 3 times.)
- "Who would need to work with very large numbers? Who would need to work with very small numbers?" (Astronomers might need to work with very large numbers. Biologists, physicists, engineers and others might need to work with very small numbers.)

5.4 Negative Exponent True or False

Cool Down: 5 minutes

Student Task Statement

Mark each of the following equations as true or false. Explain or show your reasoning.

1. $10^{-5} = -10^5$
2. $(10^2)^{-3} = (10^{-2})^3$
3. $\frac{10^3}{10^{14}} = 10^{-11}$

Student Response

1. False, because $10^{-5} = \frac{1}{100\,000}$, whereas $-10^5 = -100\,000$.
2. True, because both $(10^2)^{-3}$ and $(10^{-2})^3$ are equal to 10^{-6}.
3. True, because $\frac{10^3}{10^{14}} = 10^{3-14} = 10^{-11}$.

Student Lesson Summary

When we multiply a positive power of 10 by $\frac{1}{10}$, the exponent *decreases* by 1: $10^8 \times \frac{1}{10} = 10^7$ This is true for *any* positive power of 10. We can reason in a similar way that multiplying by 2 factors that are $\frac{1}{10}$ *decreases* the exponent by 2: $\left(\frac{1}{10}\right)^2 \times 10^8 = 10^6$

That means we can extend the rules to use negative exponents if we make $10^{-2} = \left(\frac{1}{10}\right)^2$. Just as 10^2 is two factors that are 10, we have that 10^{-2} is two factors that are $\frac{1}{10}$. More generally, the exponent rules we have developed are true for *any* integers n and m if we make $10^{-n} = \left(\frac{1}{10}\right)^n = \frac{1}{10^n}$

Here is an example of extending the rule $\frac{10^n}{10^m} = 10^{n-m}$ to use negative exponents: $\frac{10^3}{10^5} = 10^{3-5} = 10^{-2}$ To see why, notice that $\frac{10^3}{10^5} = \frac{10^3}{10^3 \times 10^2} = \frac{10^3}{10^3} \times \frac{1}{10^2} = \frac{1}{10^2}$which is equal to 10^{-2}.

Here is an example of extending the rule $(10^m)^n = 10^{m \times n}$ to use negative exponents: $(10^{-2})^3 = 10^{(-2)(3)} = 10^{-6}$To see why, notice that $10^{-2} = \frac{1}{10} \times \frac{1}{10}$. This means that $(10^{-2})^3 = \left(\frac{1}{10} \times \frac{1}{10}\right)^3 = \left(\frac{1}{10} \times \frac{1}{10}\right) \times \left(\frac{1}{10} \times \frac{1}{10}\right) \times \left(\frac{1}{10} \times \frac{1}{10}\right) = \frac{1}{10^6} = 10^{-6}$

Lesson 5 Practice Problems

1. **Problem 1 Statement**

   Write with a single exponent: (ex: $\frac{1}{10} \times \frac{1}{10} = 10^{-2}$)

   a. $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

   b. $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

   c. $\left(\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}\right)^2$

   d. $\left(\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}\right)^3$

   e. $(10 \times 10 \times 10)^{-2}$

Solution

a. 10^{-3}

b. 10^{-7}

c. 10^{-8}

d. 10^{-9}

e. 10^{-6}

2. **Problem 2 Statement**

Write each expression as a single power of 10.

a. $10^{-3} \times 10^{-2}$

b. $10^{4} \times 10^{-1}$

c. $\frac{10^5}{10^7}$

d. $(10^{-4})^5$

e. $10^{-3} \times 10^{2}$

f. $\frac{10^{-9}}{10^5}$

Solution

a. 10^{-5}

b. 10^{3}

c. 10^{-2}

d. 10^{-20}

e. 10^{-1}

f. 10^{-14}

3. **Problem 3 Statement**

Select **all** of the following that are equivalent to $\frac{1}{10\,000}$:

a. $(10\,000)^{-1}$

b. $-(10\,000)$

c. $(100)^{-2}$

d. $(10)^{-4}$

e. $(-10)^{2}$

Solution ["A", "C", "D"]

4. **Problem 4 Statement**

Match each equation to the situation it describes. Explain what the constant of proportionality means in each equation.

Equations:

a. $y = 3x$

b. $\frac{1}{2}x = y$

c. $y = 3.5x$

d. $y = \frac{5}{2}x$

Situations:

- A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 square feet of dirt.
- I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
- A store has a “4 for £10” sale on hats.
- For every 48 cookies I bake, my students get 24.

Solution

Explanations vary. Sample responses:

a. Water and salt. For each cup of salt, there are 3 cups of water.

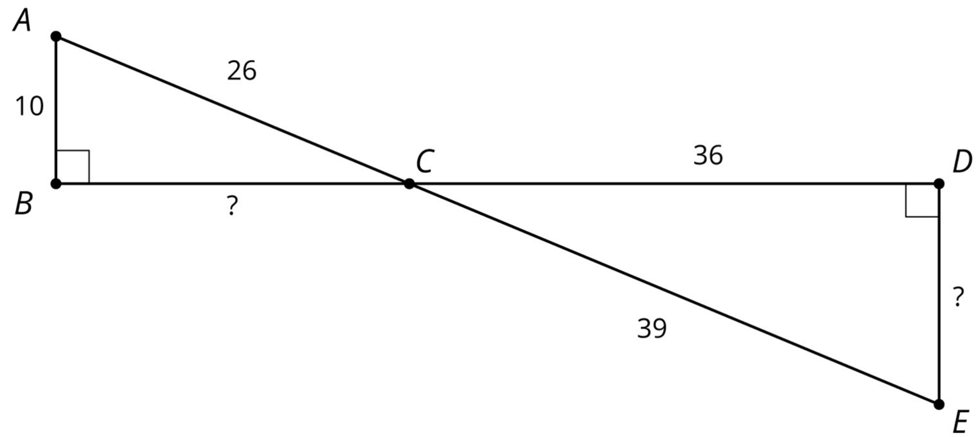
b. Cookies. For every cookie I bake, my students get half.

c. Dump truck. The dump truck hauls 3.5 square feet of dirt in each load.

d. Sale on hats. Each hat costs £2.50.

5. **Problem 5 Statement**

a. Explain why triangle ABC is similar to EDC.



b. Find the missing side lengths.

Solution

a. Explanations vary. Sample explanation: Both triangles contain a right angle, and angles ACB and ECD are vertical angles. The triangles are similar because two pairs of corresponding angles are congruent.

b. Side BC measures 24, and side DE measures 15. (The scale factor is $\frac{39}{26}$ or 1.5.)