Lesson 2: Chance experiments

Goals

- Comprehend and use the terms “impossible,” “unlikely,” “equally likely as not,” “likely,” and “certain” (in spoken and written language) to describe the likelihood of an event.
- Interpret percentages, fractions, and decimals that represent the likelihood of events.
- Order a given set of events from least likely to most likely, and justify (orally) the reasoning.

Learning Targets

- I can describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.
- I can tell which event is more likely when the chances of different events are expressed as fractions, decimals, or percentages.

Lesson Narrative

In this lesson students investigate chance events. They use language like impossible, unlikely, equally likely as not, likely, or certain to describe a likelihood of a chance **event**. Students are making sense of situations and sorting them into these categories. In some cases, a value is assigned to the likelihood of an event using a fraction, decimal, or percentage chance. By comparing loose categories early and numerical quantities later, students are attending to precision when sorting the scenarios. Later, students will connect this language to more precise numerical values on their own.

Addressing

- Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Instructional Routines

- Collect and Display
- Compare and Connect
- Discussion Supports
- Take Turns
- Think Pair Share

Required Materials

Dice

cubes with sides numbered from 1 to 6

Pre-printed slips, cut from copies of the blackline master

Set 1 Likelihood The weather report says there is a 20% chance of rain tomorrow. The chance of rain tomorrow.	Set 1 Likelihood 10% of people are left handed. The chance that a randomly chosen person is left handed.
Set 1 Likelihood The offspring of two fruit flies in a science experiment have a 75% chance of having red eyes. The chance that the first fly to hatch has red eyes.	Set 1 Likelihood Half of the cards in a deck are red and half are black. Shuffle the cards and select the first card. The chance that the card is red.

Set 2 Likelihood 2 out of every 5 dentists recommend a certain brand of toothpaste. The chance that a random dentist recommends the toothpaste.	Set 2 Likelihood The chance that your opponent will play rock first in a game of paper, rock, scissors.
Set 2 Likelihood A pile contains 6 square pattern blocks and you choose one. The chance that the block you choose has 4 sides of the same length.	Set 2 Likelihood A fishbowl contains 5 balls where each one has an even number from 2 to 10 written on it and you choose one. The chance that you draw out a ball with the number 3 on it.
Set 2 Likelihood In general English usage, $\frac{4}{25}$ of words begin with the letter T. The chance that a randomly chosen word in a novel begins with the letter T.	Set 2 Likelihood The probability that a certain medical test gives the right result is 0.95. The chance that this medical test is correct for a random patient.

Required Preparation

Print and cut up slips from the Card Sort: Likelihood blackline master. One copy is needed for every 3 students. 2 standard dice are needed for a demonstration.

Student Learning Goals

Let's investigate chance.

2.1 Which is More Likely?

Warm Up: 5 minutes

The purpose of this warm-up is to engage students' intuition about likelihood of events. The following activities in this lesson continue to develop more formal ways of thinking about likelihood leading to the definition of probability in the next lesson.

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time and time to share their response with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

Students may think that it is required to pull out a specific shoe rather than any left shoe. Ask students to visualise the problem and determine how many right shoes are in the cupboard.

Student Task Statement

Which is more likely to happen?

- When reaching into a dark cupboard and pulling out one shoe from a pile of 20 pairs of shoes, you pull out a left shoe.
- When listening to a playlist—which has 5 songs on it—in shuffle mode, the first song on the playlist plays first.

Student Response

It is more likely that you will pull out a left shoe than the first song on the playlist will be the first song played in shuffle mode. Since the shoes come in pairs, it is equally likely that you would get a left or right shoe, so half of the time we would expect to get a right shoe. For the playlist, there are 5 different songs that could play first and only 1 of them is the first song on the list.

Activity Synthesis

The purpose of the discussion is to help students recognise their own intuition about the likelihood of an event even when prior outcomes are not available.

Have partners share responses with the class, and ask at least one student for each option for their reasoning. Give students time to discuss their reasoning until they come to an agreement.

It may be helpful to reiterate that the outcomes from these actions are not certain. It is certainly possible to do both things and draw out a right shoe and listen to the first song on the list first, but it is not very likely based on the situations.

2.2 How Likely Is It?

10 minutes

As preparation for talking about probability, students are asked to engage their intuition about the concept by loosely grouping scenarios into categories based on their likelihood by reasoning abstractly about situations in context. Some of the categories are meant to be loosely interpreted while others (such as "certain" and "impossible") have more precise meanings.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2.

Tell students that a “standard dice” is an object that has the numbers 1 through 6 printed on a cube so that each face shows a different number. This item will be referenced throughout the unit.

It may help students to understand the categories of likelihood with an example of opening a book to a random page:

- Impossible: Opening a 100 page book to page -300.
- Unlikely: Opening a 100 page book to exactly page 45.
- Equally likely as not: Opening a 100 page book to a page numbered less than 51.
- Likely: Opening a 100 page book to a page numbered greater than 10.
- Certain: Opening a 100 page book to a page numbered less than 1,000.

Allow students 5--7 minutes quiet work time followed by partner and whole-class discussion.

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select 5–6 of the situations to complete.
Supports accessibility for: Organisation; Attention; Social-emotional skills Conversing: Discussion Supports. As students work with a partner to group the scenarios into categories, use this routine to support small group discussion. Invite students to take turns, selecting an event and explaining to their partner which category they think it belongs to (i.e., impossible, unlikely, equally likely as not, likely, or certain). Encourage students to

challenge each other when they disagree. This will help students clarify their reasoning about multiplication of signed numbers.
Design Principle(s): Support sense-making; Optimise output (for explanation)

Student Task Statement

1. Label each event with one of these options:

   impossible, unlikely, equally likely as not, likely, certain

   a. You will win grand prize in a raffle if you purchased 2 out of the 100 tickets.

   b. You will wait less than 10 minutes before ordering at a fast food restaurant.

   c. You will get an even number when you roll a normal dice.

   d. A four-year-old child is over 6 feet tall.

   e. No one in your class will be late to class next week.

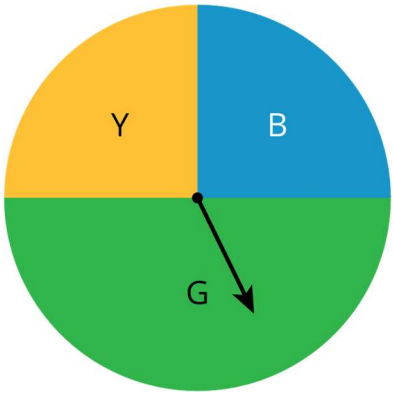
   f. The next baby born at a hospital will be a boy.

   g. It will snow at our school on July 1.

   h. The Sun will set today before 11:00 p.m.

   i. Spinning this spinner will result in green.

   j. Spinning this spinner will result in red.



2. Discuss your answers to the previous question with your partner. If you disagree, work to reach an agreement.

3. Invent another situation for each label, for a total of 5 more events.

Student Response

1.

   a. unlikely

b. likely

c. equally likely as not

d. impossible (unlikely is acceptable)

e. Answers vary.

f. equally likely as not

g. impossible (or possibly unlikely depending on the location of your school)

h. certain

i. equally likely as not

j. impossible

2. No answer needed.

3. Answers vary. Sample response:

- Impossible: Finding a rectangle that has 5 different sides.
- Unlikely: A peacock being born as an albino.
- Equally likely as not: A coin is flipped and it lands with the tails side showing.
- Likely: A box of crayons contains the colour red.
- Certain: A triangle has 3 sides.

Activity Synthesis

The purpose of this discussion is for students to see that the loose categories can be understood in a more formal way. Some students may begin to attach numbers to the likelihood of the events and that is a good way to begin the transition to thinking about probability.

There may be some discussion about the category "equally likely as not." Are events in this category required to be at exactly 50% or would an event with 55% or 48% likelihood be placed in this category as well? At this stage, the categories are meant to be loose, so it is not necessary that everyone agree on what goes in each category.

Questions for discussion

- "Were any of the scenarios listed difficult to categorise?"
- "Which categories are the most strict about what can go in them?" ("Certain" should represent scenarios that *must* happen with 100% certainty. "Impossible" should represent scenarios that *cannot* happen.)

- "What does it mean for an event to be certain?" (That it must happen.)
- "What does it mean for an event to be likely?" (Between about 50% and 100% chance.)

2.3 Take a Chance

10 minutes (there is a digital version of this activity)

In this lesson, students begin to move towards a more quantitative understanding of likelihood by observing a game that has two rounds with different requirements for winning in each round. The game is also played multiple times to help students understand that the actual number of times an **outcome** occurs may differ from expectations based on likelihood at first, but should narrow towards the expectation in the long-run. By repeating the process many times, students are recognising a structure beginning to form with the results. Activities in later lessons will more formally show this structure forming from the repeated processes.

Instructional Routines

- Collect and Display

Launch

Arrange students in groups of 2. Following the demonstration game, allow 5 minutes of partner work followed by a whole-class discussion.

Select two students to play this game of chance that consists of 2 rounds. Give 1 standard dice to each student.

Round 1: One student chooses three numbers that will count as a win for them. The other student wins if any of the other numbers come up. Roll the dice.

Round 2: Whoever lost the first round gets to choose 4 numbers that will count for a win for them while their partner gets the remaining 2 numbers. Roll the dice.

Have the students play this game of chance 10 times by playing the two rounds each time. Display the results of each round for all to see. For example:

	person choosing number	winner	chooser wins?
game 1, round 1	Lin	Noah	
game 1, round 2	Lin	Lin	X
game 2, round 1	Noah	Noah	X
game 2, round 2	Lin	Lin	X

Tell students that the questions ask about whether the person choosing the number wins the game or not. They should not concentrate on who won more often, but how often the number chooser wins the round. In the example table, the column with the Xs is the focus.

The digital version is slightly different, with an applet generating random numbers from 1 to 6. This applet is based on the work of pirsquared in GeoGebra.

Conversing, Reading: Collect and Display. As students discuss the questions with a partner, write down the words and phrases students use to describe the likelihood of either player winning each round of the game. As students review the language collected in the visual display, encourage students to revise and improve how ideas are communicated. For example, a phrase such as: "The person who chooses 4 numbers has more ways to win the round" can be improved with the phrase, "The person who chooses 4 numbers has 4 out of 6 ways to win the round." This will help students assign numbers to the likelihood of the person choosing the winning number for each round. This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

Your teacher will have 2 students play a short game.

1. When the first person chose 3 numbers, did they usually win?
2. When the person chose 4 numbers, did you expect them to win? Explain your reasoning.

Student Response

Answers vary. Sample response:

1. The person who chose 3 numbers won more than lost, but not by a lot.
2. I expected the person who chose 4 numbers to usually win since they had 4 ways to win and the other person only had 2.

Are You Ready for More?

On a game show, there are 3 closed doors. One door has a prize behind it. The contestant chooses one of the doors. The host of the game show, who knows where the prize is located, opens one of the *other* doors which does not have the prize. The contestant can choose to stay with their first choice or switch to the remaining closed door.

1. Do you think it matters if the contestant switches doors or stays?
2. Practice playing the game with your partner and record your results. Whoever is the host starts each round by secretly deciding which door has the prize.
    a. Play 20 rounds where the contestant always stays with their first choice.
    b. Play 20 more rounds where the contestant always switches doors.

3. Did the results from playing the game change your answer to the first question? Explain.

Student Response

Answers vary. Sample response:

1. It appears like it doesn't matter: there are two doors left, one has the prize, and one doesn't.
2. No response required.
3. Switching should win more often than not switching, but other results are possible by chance.

Activity Synthesis

Define a **chance experiment**. A chance experiment is the process of making an observation of something when there is uncertainty about which of two or more possible outcomes will occur. Each time a standard dice is thrown, it is a chance experiment. This is because we cannot be certain of which number will be the outcome of this experiment.

Some questions for discussion:

- "Who did you expect to win each time: the person choosing the number or the other person? Explain your reasoning."
- "In one round of the game it was more likely that the person choosing the numbers would win. We'll be talking a lot in this unit about how likely or probable an event is to happen and even assigning numbers to the likelihood."
- "What if we had to assign numbers to the 'likelihood' of the person choosing the numbers winning for each round? For each part of the game, what percentage or fraction would you assign to the likelihood of the person who chose the numbers winning?" (For the first round of the game, there is a 50% or $\frac{1}{2}$ chance for each person to win. For the second round of the game, the person choosing the numbers should have a 67% or $\frac{4}{6}$ or $\frac{2}{3}$ chance to win.)
- "What percentage or fraction would you assign to waiting for less than 10 minutes before your order is taken at the fast food restaurant from the previous task?" (Answers vary. Values between 80% and 100% are expected. It is not necessary to agree on a particular value at this point.)
- "How could we get more evidence to support these answers?" (Collect data from fast food restaurants to find a fraction of customers who order their food within 10 minutes.)

2.4 Card Sort: Likelihood

10 minutes

The last activity in this lesson moves one more step closer to quantifying likelihood of scenarios by ordering them individually rather than into groups. Some of the scenarios have a numerical probability expressed as a percentage, some in decimal form, some as a fraction, and some do not have a numerical probability given. This gives students the opportunity to work with probabilities expressed as percentages, decimals, and fractions.

You will need the blackline master for this activity.

Instructional Routines

- Compare and Connect
- Take Turns

Launch

Arrange students in groups of 3. Distribute one copy of the blackline master for each group.

Tell students to take turns ordering the cards by beginning with one card, then adding in additional cards one at a time before, after, or between cards as needed.

Give students 5 minutes for group work followed by a whole-class discussion.

Anticipated Misconceptions

Students may have trouble understanding the Rock, Paper, Scissors context. Tell these students that a player randomly chooses one of the three items to play in each round. If students still struggle, tell them that each of the three items are expected to be played with equal likelihood.

Student Task Statement

1. Your teacher will give you some cards that describe events. Order the events from least likely to most likely.
2. After ordering the first set of cards, pause here so your teacher can review your work. Then, your teacher will give you a second set of cards.
3. Add the new set of cards to the first set so that all of the cards are ordered from least likely to most likely.

Student Response

Set 1: Left handed, Rain, Cards, Flies.

Sets 1 and 2: Numbered balls, Left handed, Letter T, Rain, Rock, Toothpaste, Cards, Flies, Medical test, Pattern blocks.

Activity Synthesis

The purpose of the discussion is for students to talk about the methods they used to sort the cards and compare likelihood of different situations.

Some questions for discussion:

- "How were the numerical values of the likelihoods written?" (Some were written as percentages, some as fractions, and some as decimal values.)
- "How did you compare them when there was a mix of percentages, fractions, and decimals?"
- "Some of the cards did not have a percentage, fraction, or decimal. How did you determine where those cards would go in the order?"

Speaking, Listening: Compare and Connect. Ask students to prepare a visual display of their sorted cards. Students should consider how to display their cards so that another student can interpret the reasoning behind the order they selected. Some students may wish to add notes or details to their displays to help communicate their thinking. Invite students to investigate each other's work, ask students to compare the order between displays. Listen for and amplify any comments about the use of percentages, fractions, and decimals to compare the likelihood of different situations. Then encourage students to make connections between the various ways to quantify the likelihood of a situation. Listen for and amplify language students use to explain that the percentage, fraction, and decimal used to quantify the likelihood of the same scenario are all equivalent. This will foster students' meta-awareness and support constructive conversations as they compare and connect quantities that represent the likelihood of a situation.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

Lesson Synthesis

Ask students "what is a chance event?"

In the same groups of 2, have partners come up with examples of each of these types of events:

- impossible
- unlikely
- equally likely as not
- likely
- certain

Ask partners to share responses with the class.

2.5 According To

Cool Down: 5 minutes

Students rank scenarios based on likelihood expressed in different numerical forms.

Student Task Statement

Here are some scenarios:

- According to market research, a business has a 75% chance of making money in the first 3 years.
- According to lab testing, $\frac{5}{6}$ of a certain kind of experimental light bulb will work after 3 years.
- According to experts, the likelihood of a car needing major repairs in the first 3 years is 0.7.

1. Write the scenarios in order of likelihood from least to greatest after three years: the business makes money, the light bulb still works, and the car needs major repairs.
2. Name another chance experiment that has the same likelihood as one of the scenarios.

Student Response

1. The car needs major repairs, the business makes money, the light bulb still works.
2. Answers vary. Sample responses:
   - Flipping two coins and at least one not landing as heads has a 75% chance of happening.
   - Rolling a standard dice and it lands with any number other than 1 face up.
   - That a number greater than 3 is selected when selecting a number between 1 and 10 randomly.

Student Lesson Summary

A **chance experiment** is something that happens where the outcome is unknown. For example, if we flip a coin, we don't know if the result will be a head or a tail. An **outcome** of a chance experiment is something that can happen when you do a chance experiment. For example, when you flip a coin, one possible outcome is that you will get a head. An **event** is a set of one or more outcomes.

We can describe events using these phrases:

- Impossible
- Unlikely

- Equally likely as not
- Likely
- Certain

For example, if you flip a coin:

- It is *impossible* that the coin will turn into a bottle of ketchup.
- It is *unlikely* the coin will land on its edge.
- It is *equally likely as not* that you will get a tail.
- It is *likely* that you will get a head or a tail.
- It is *certain* that the coin will land somewhere.

The *probability* of an event is a measure of the likelihood that an event will occur. We will learn more about probabilities in the lessons to come.

Glossary

- chance experiment
- event
- outcome

Lesson 2 Practice Problems

Problem 1 Statement

The likelihood that Han makes a free throw in basketball is 60%. The likelihood that he makes a 3-point shot is 0.345. Which event is more likely, Han making a free throw or making a 3-point shot? Explain your reasoning.

Solution

It is more likely that Han makes a free throw. Since 0.345 is less than 0.5, making a 3-pointer is an unlikely event. Since 60% is greater than 50%, making a free throw is a more likely event.

Problem 2 Statement

Different events have the following likelihoods. Sort them from least to greatest:

60%

8 out of 10

0.37

20%

$\frac{5}{6}$

Solution

20%, 0.37, 60%, 8 out of 10, $\frac{5}{6}$

Problem 3 Statement

There are 25 prime numbers between 1 and 100. There are 46 prime numbers between 1 and 200. Which situation is more likely? Explain your reasoning.

- A computer produces a random number between 1 and 100 that is prime.
- A computer produces a random number between 1 and 200 that is prime.

Solution

A computer produces a random number between 1 and 100 that is prime. There is a 25% chance of getting a prime number from the first 100 numbers and only a 23% chance from the first 200 numbers.

Problem 4 Statement

It takes $4\frac{3}{8}$ cups of cheese, $\frac{7}{8}$ cups of olives, and $2\frac{5}{8}$ cups of sausage to make a signature pizza. How much of each ingredient is needed to make 10 pizzas? Explain or show your reasoning.

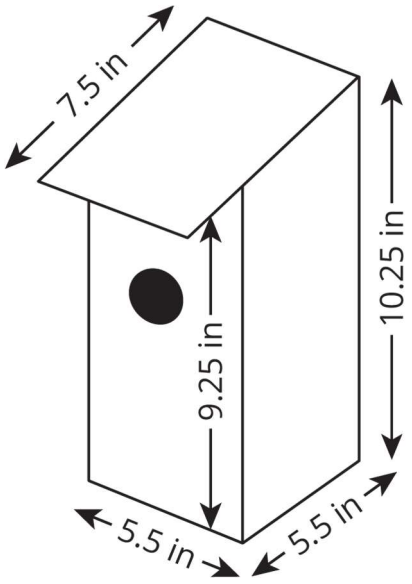
Solution

number of pizzas	cups of cheese	cups of olives	cups of sausage
1	$\frac{35}{8}$	$\frac{7}{8}$	$\frac{21}{8}$
10	$\frac{350}{8}$ or equivalent	$\frac{70}{8}$ or equivalent	$\frac{210}{8}$ or equivalent

With decimals, the answers are 43.75 cups of cheese, 8.75 cups of olives, and 26.25 cups of sausage.

Problem 5 Statement

Here is a diagram of a birdhouse Elena is planning to build. (It is a simplified diagram, since in reality, the sides will have a thickness.) About how many square inches of wood does she need to build this birdhouse?



Solution

286 square inches.

Problem 6 Statement

Select **all** the situations where knowing the surface area of an object would be more useful than knowing its volume.

a. Placing an order for tiles to replace the roof of a house.

b. Estimating how long it will take to clean the windows of a greenhouse.

c. Deciding whether leftover soup will fit in a container.

d. Estimating how long it will take to fill a swimming pool with a garden hose.

e. Calculating how much paper is needed to manufacture chocolate bar wrappers.

f. Buying fabric to sew a sofa cover.

g. Deciding whether one muffin pan is enough to bake a muffin recipe.

Solution ["A", "B", "E", "F"]