Mathematics Education Innovation

Lesson 8: More about constant speed

Goals

- Calculate unit rates that represent speed or pace, use them to determine unknown distances or elapsed times, and explain (orally) the solution method.
- Interpret a verbal (written) description of a situation involving two objects moving at constant speeds, and create a diagram or table to represent the situation.

Learning Targets

- I can solve more complicated problems about constant speed situations.

Lesson Narrative

This lesson allows students to practise working with equivalent ratios, tables that represent them, and associated unit rates in the familiar context of speed, time, and distance. Students use unit rates (speed or pace) and ratios (of time and distance) to find unknown quantities (e.g., given distances and times, find a constant speed or pace; and given a speed or pace, solve problems about distance and time).

Addressing

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.
- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- Stronger and Clearer Each Time
- Three Reads
- Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Consider checking in advance whether there is a rail trail in your community, about which you could tell students during the Picnics on A Rail Trail activity.

Student Learning Goals

Let's investigate constant speed some more.

8.1 Back on the Treadmill Again

Warm Up: 10 minutes

Students have had experience determining speed given a ratio of time and distance. This task prompts students to use more than one strategy to solve speed-related problems (minutes passed and miles travelled) and practise reasoning in multiple ways, enabling them to see the connections across strategies.

There are several ways students can calculate how many miles Andre's dad could run in 30 minutes if travelling at a speed of 12 miles in 75 minutes. A few strategies:

- Using the speed. The ratio $12:75$ has an associated unit rate of $\frac{12}{75}$ or 0.16 miles per minute. To find the distance travelled in 30 minutes, multiply 0.16 miles per minute by 30. $(0.16) \times 30 = 4.8$, so he can run 4.8 miles in 30 minutes. Note that the rate per 1 associated with this unit rate is called **speed**.
- Using the pace. $\frac{75}{12}$ or 6.25 minutes per mile is also a unit rate for the ratio. To find the distance travelled in 30 minutes, divide 30 by 6.25. $30 \div 6.25 = 4.8$. Note that the rate per 1 associated with this unit rate is called **pace.**
- Using a scale factor: Noticing that in the "time" column of a table, 75 multiplied by $\frac{30}{75}$ is 30, and $\frac{30}{75} = 0.4$. The unknown number of miles is 4.8, because $12 \times (0.4) = 4.8$.

distance (miles)	time (minutes)
12	75
?	30

- Scaling up: Noticing that, going up in the "time" column, 75 is $30 \times (2.5)$. The unknown number of miles is then $12 \div 2.5 = 4.8$. As students work, notice different strategies being used so that they can be represented during discussion later.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Arrange students in groups of 2 and give them 3 minutes of quiet think time, followed by sharing with a partner and whole-class discussion. Ask students to be mindful of how they are thinking about the questions and be prepared to share their reasoning.

Student Task Statement

While training for a race, Andre's dad ran 12 miles in 75 minutes on a treadmill. If he runs at that rate:

1. How long would it take him to run 8 miles?
2. How far could he run in 30 minutes?

Student Response

1. It will take 50 minutes to run 8 miles. Possible strategies:

	distance (miles)	time (minutes)	
	12	75	
$\times \frac{1}{12}$	1	6.25	$\times \frac{1}{12}$
$\times 8$	8	50	$\times 8$

or

distance (miles)	time (minutes)
12	75
$\times 6.25$	
8	50
$\times 6.25$	

2. He can run 4.8 miles in 30 minutes. Possible strategies:

distance (miles)	time (minutes)
12	75
0.16	1
4.8	30

$\times \frac{1}{75}$ $\times \frac{1}{75}$

$\times 30$ $\times 30$

or

distance (miles)	time (minutes)
12	75
4.8	30

$\times 0.16$

$\times 0.16$

Activity Synthesis

Invite students to share with their partner: their solution ideas and their explanations of how unit rates and scaling can be helpful. Then, give each student a new partner to repeat the process. Ask students to practise using mathematical language to be as clear as possible when sharing with the class, when and if they are called upon.

Select students with different strategies to share with the class. Record their methods and display them for all to see. If any relevant strategies are missing, demonstrate them and add them to the display. Help students notice how unit rates and scaling can be helpful in solving similar problems.

Tell students when we find how the number of miles per minute or metres per second an object is moving, we are finding the **speed** of the object. When we find the number of minutes per mile or seconds per metre, we are finding the **pace** of the object.

8.2 Picnics on the Rail Trail

30 minutes

This task asks students to answer questions in the context of constant speed. If you wish to have students engage in more aspects of mathematical modelling, have students keep their books or devices closed and only display the stem that establishes the scenario. Ask students what they notice and wonder, and select questions to answer that are established through class discussion.

In this activity, students reason about the distances between the two friends, elapsed time, and speed. Students reason both quantitatively and abstractly; students can estimate some of the solutions or check that they make sense in the given context (e.g., an earlier meeting time of the two friends would mean that one or both of them are travelling faster). As students work in groups, monitor the strategies they use to solve the last three problems, such as detailed diagrams of the path with marked-off distances, different ways of using tables, and so on.

Instructional Routines

- Group Presentations
- Three Reads
- Notice and Wonder

Launch

Share some information about the system of Rail Trails in the United States, as some students may be unfamiliar with non-motorised trails. Explain to students that, since they are built on old railway lines, these trails have very little gain or loss in height—making it reasonable to maintain a constant speed while walking, running, or cycling.

If you (optionally) decide to take a less structured approach and compel students to engage in more aspects of mathematical modelling, have students keep their books or devices closed and only display the stem that establishes the scenario. Ask students, "What do you notice? What do you wonder?" Record the things they notice and wonder for all to see. Select questions that students posed for the class to explore that are similar to the questions in the task.

Arrange students in groups of 3–4. Give students a few minutes of quiet think time to complete the first two questions, and then time to discuss their responses with a partner. Encourage students to look at their partner's approach and choices (e.g., how they work out the values in the table or sets up the table, how calculations are done, etc.). Ask students to pause for a brief whole-class discussion afterwards.

To make sure students are on the right track, display at least one student solution for each of the first two problems for all to see before they move on to complete the activity. Give students 8–10 minutes to work and discuss in groups, and tell them that each group will be assigned a problem to explain to the class. Provide each group with tools for creating a visual display.

Representation: Internalise Comprehension. Represent the same information through different modalities by using drawings or diagrams. If students are unsure where to begin, suggest that they draw a picture or diagram that represents the situation.
Supports accessibility for: Conceptual processing; Visual-spatial processing Reading: Three

Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. Use the first read to orient students to the situation by asking students to describe the situation without using numbers (e.g., Two friends live alongside different parts of a trail, one day they walk towards each other for a picnic). Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., The distance between then is 24 miles to start, Kiran walks at a speed of 3 miles per hour, etc.). For the third read, reveal the first three questions and provide students with independent work time. Once students are ready, brainstorm possible solution strategies to respond to Kiran's suggestion: "If I walk 3 miles per hour toward you, and you walk 3.4 miles per hour toward me, it's the same as if you stay put and I jog 6.4 miles per hour." This will help students connect the language in the word problem about important quantities and rates with the reasoning needed to solve the problem.
Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

Encourage students who are struggling to make sense of the mathematics to make a picture of the path and mark off distances after certain time periods.

Look for students misinterpreting expressions of time. For example, 2.5 hours after 8 a.m. is 10:30 a.m., not 10:50 a.m.

Students who are unsure about how to calculate distance apart in the table may benefit from creating a table with 4 columns: time in hours, how far Kiran has travelled, how far Clare has travelled, and the distance between them.

Student Task Statement

Kiran and Clare live 24 miles away from each other along a rail trail. One Saturday, the two friends started walking toward each other along the trail at 8:00 a.m. with a plan to have a picnic when they meet.

Kiran walks at a **speed** of 3 miles per hour while Clare walks 3.4 miles per hour.

1. After one hour, how far apart will they be?
2. Make a table showing how far apart the two friends are after 0 hours, 1 hour, 2 hours, and 3 hours.
3. At what time will the two friends meet and have their picnic?
4. Kiran says “If I walk 3 miles per hour toward you, and you walk 3.4 miles per hour toward me, it’s the same as if you stay put and I jog 6.4 miles per hour.” What do you think Kiran means by this? Is he correct?
5. Several months later, they both set out at 8:00 a.m. again, this time with Kiran jogging and Clare still walking at 3.4 miles per hour. This time, they meet at 10:30 a.m. How fast was Kiran jogging?

Student Response

1. After one hour Kiran has walked 3 miles and Clare has walked 3.4 miles, reducing their total distance apart to $24 - 3 - 3.4 = 17.6$ miles.
2. Here is the table:

elapsed time (hours)	distance apart (miles)
0	24
1	17.6
2	11.2
3	4.8

3. Since the distance between them is decreasing by 6.4 miles each hour, they will meet after 3.75 hours, which will be at 11:45am.
4. What Kiran means is that each hour the total distance between them is decreasing by 6.4 miles, so the amount of time it will take them to meet is the same as if one person stays put and the other travels at 6.4 miles per hour. However, the location of their meeting would change to Clare's house instead of somewhere in between the houses on the Rail Trail.
5. In 2.5 hours Clare travelled $(3.4) \times (2.5) = 8.5$ miles. $24 - 8.5 = 15.5$. This means Kiran jogged the remaining 15.5 miles in 2.5 hours. 15.5 miles in 2.5 hours means Kiran jogged 6.2 miles per hour. Alternative solution: If they meet on the trail after 2.5 hours (8 am to 10:30 am), then their combined speed is 9.6 miles per hour, since $\frac{24}{2.5} = 9.6$. From the problem, Clare is walking 3.4 miles per hour, so Kiran must be jogging 6.2 miles per hour, since $9.6 - 3.4 = 6.2$.

Are You Ready for More?

1. On his trip to meet Clare, Kiran brought his dog with him. At the same time Kiran and Clare started walking, the dog started running 6 miles per hour. When it got to Clare it turned around and ran back to Kiran. When it got to Kiran, it turned around and ran back to Clare, and continued running in this fashion until Kiran and Clare met. How far did the dog run?

2. The next Saturday, the two friends leave at the same time again, and Kiran jogs twice as fast as Clare walks. Where on the rail trail do Kiran and Clare meet?

Student Response

1. We know that they meet after 3.75 hours. So the dog was running 6 miles per hour for 3.75 hours. Therefore, the dog ran 22.5 miles.

2. They meet 8 miles from Clare's starting point. It may appear this problem doesn't have enough information, but since Kiran will always travel twice as far as Clare, he must travel 16 miles and Clare must travel 8 miles.

Activity Synthesis

Invite selected groups to present the solutions to their assigned problems. If possible, start from the most common strategies and move to the least common. Highlight effective uses of unit rates, equivalent ratios, scaling, and table representations in students' work.

8.3 Swimming and Cycling

Optional: 10 minutes

This optional activity is more opportunity to practise working with rates, in a new situation that involves constant speed of multiple people moving at the same time. This problem has less scaffolding than the previous activity. There are many different unit rates students may choose to calculate while solving this problem. Specifying the units and explaining the context for a rate gives students an opportunity to attend to precision.

Monitor for students that use different strategies to solve the problem:

- Creating a drawing or diagram that represents the situation
- Finding how far each person travels in the same amount of time (such as: In 24 minutes, Jada cycles 4 miles while her cousin swims 1 mile.)
- Finding how long it takes each person to travel the same distance (such as: To go 2 miles, it takes Jada 12 minutes and her cousin 48 minutes.)
- Calculating the pace of each individual
- Calculating the speed of each individual
- Calculating the combined speed of how fast they are moving away from each other

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports

Launch

Give students a few minutes of quiet think time to complete the two questions, then pause for a brief whole-class discussion afterwards.

Writing, Speaking: Discussion Supports. Provide sentence frames to students to support their explanations of their process for determining who was moving faster and by how much. For example, "First, I _____. Then, I _____." or "I know that _____ is moving faster because _____."
Design Principle(s): Support sense-making, Optimise output (for explanation)

Anticipated Misconceptions

Some students may confuse the meaning of the speed and pace, thinking that 24 minutes per mile is faster than 6 minutes per mile. Make sure they have labelled the units on their rates, and prompt them to consider what the words "per mile" tell us about the situation.

Student Task Statement

Jada cycles 2 miles in 12 minutes. Jada's cousin swims 1 mile in 24 minutes.

1. Who is moving faster? How much faster?
2. One day Jada and her cousin line up on the end of a swimming pier on the edge of a lake. At the same time, they start swimming and cycling in opposite directions.
    a. How far apart will they be after 15 minutes?
    b. How long will it take them to be 5 miles apart?

Student Response

1. Jada cycles faster than her cousin swims, by 7.5 miles per hour (or equivalent).
2.
    a. $3\frac{1}{8}$ miles. Possible strategy: Jada cycles 1 mile in 6 minutes. There are $2\frac{1}{2}$ groups of 6 minutes in 15 minutes. This means Jada cycles 2.5 miles in 15 minutes. Jada's cousin swims $\frac{5}{8}$ mile in 15 minutes, because $15 \div 24 = \frac{15}{24}$, which is equivalent to $\frac{5}{8}$. The total distance between them is $2\frac{4}{8} + \frac{5}{8}$, or $3\frac{1}{8}$ miles.
    b. 24 minutes. Possible strategy: Jada and her cousin are moving away from each other at a rate of 12.5 miles per hour. $5 \div 12.5 = 0.4$, so it will take them 0.4 hours to be 5 miles apart. This is equivalent to 24 minutes, because $0.4 \times 60 = 24$.

Activity Synthesis

The key takeaway of this discussion is the idea that students can find and use different unit rates to solve the problem, so it is important to specify what a particular unit rate measures. Invite students who used different strategies to share how they solved the problem. Sequence the strategies from most common to least common. If any student used a drawing or diagram to represent the situation, consider having them share first. Some possible strategies to highlight include:

- Creating a double, triple, or quadruple number line diagram showing the elapsed time and distances travelled
- Finding how far they will travel in the same amount of time
- Calculating their individual paces in minutes per mile (6 minutes per mile for Jada, 24 minutes per mile for her cousin)
- Calculating their individual speeds in miles per minute ($\frac{1}{6}$ mile per minute for Jada, $\frac{1}{24}$ mile per minute for her cousin)
- Calculating their individual speeds in miles per hour (10 miles per hour for Jada, 2.5 miles per hour for her cousin)
- Calculating the combined speed of how fast they are moving away from each other ($\frac{5}{24}$ mile per minute or 12.5 miles per hour)

Some unit rates can be more helpful than others, depending on the question we are trying to answer. Consider asking discussion questions like these:

- “Which unit rate was most helpful for answering how far apart they will be after 15 minutes?” (their speed, either in miles per minute or miles per hour)
- “Which unit rate was most helpful for answering how long it will take them to be 5 miles apart?” (their pace)
- “How did the fact that they were travelling away from each other affect the problem?” (The total distance between them at any point was the sum of the distance each person had travelled. The rate at which they were moving away from each other was the sum of their individual rates of travel.)

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.
Supports accessibility for: Language; Social-emotional skills; Attention

Lesson Synthesis

In this lesson we dealt with people travelling certain distances in a certain amount of time (at a constant speed). Let’s think about how Jada was travelling 2 miles in 12 minutes.

- "What are some ways to communicate her speed?" (A common way is to say she travels $\frac{1}{6}$ of a mile per minute.)
- "How is it calculated?" (Divide 2 by 12.)
- "How would we calculate the other unit rate in this situation?" ($12 \div 2$)
- "What does it mean?" (It takes her 6 minutes to travel 1 mile. This is her pace.)
- "What are your favourite tools for making sense of and solving constant speed problems?" (Possible responses: double number lines, tables of equivalent ratios, dividing and multiplying.)

8.4 Penguin Speed

Cool Down: 5 minutes

Student Task Statement

A penguin walks 10 feet in 6 seconds. At this speed:

1. How far does the penguin walk in 45 seconds?
2. How long does it take the penguin to walk 45 feet?

Explain or show your reasoning.

Student Response

1. 75 feet. Possible strategy: The penguin's speed is $10 \div 6$, or $\frac{5}{3}$ feet per second. In 45 seconds, the penguin walks $45 \times \frac{5}{3}$, or 75 feet.
2. 27 seconds. Possible strategy: The penguin's pace is $6 \div 10$, or 0.6 seconds per foot. To walk 45 feet, it takes the penguin 45×0.6, or 27 seconds.

Student Lesson Summary

When two objects are each moving at a constant speed and their distance-to-time ratios are equivalent, we say that they are moving at the *same speed*. If their time-distance ratios are not equivalent, they are not moving at the same speed.

We describe **speed** in units of distance per unit of time, like *miles per hour* or *metres per second*.

- A snail that crawls 5 centimetres in 2 minutes is travelling at a rate of 2.5 centimetres per minute.
- A toddler that walks 9 feet in 6 seconds is travelling at a rate of 1.5 feet per second.

- A cyclist who cycles 20 kilometres in 2 hours is travelling at a rate of 10 kilometres per hour.

We can also use **pace** to describe distance and time. We measure pace in units such as *hours per mile* or *seconds per metre*.

- A snail that crawls 5 centimetres in 2 minutes has a pace of 0.4 minutes per centimetre.
- A toddler walking 9 feet in 6 seconds has a pace of $\frac{2}{3}$ seconds per foot.
- A cyclist who cycles 20 kilometres in 2 hours has a pace of 0.1 hours per kilometre.

Speed and pace are reciprocals. Both can be used to compare whether one object is moving faster or slower than another object.

- An object with the higher speed is *faster* than one with a lower speed because the former travels a greater distance in the same amount of time.
- An object with the greater pace is *slower* than one with a smaller pace because the former takes more time to travel the same distance.

Because speed is a *rate per 1 unit of time* for ratios that relate distance and time, we can multiply the amount of time travelled by the speed to find the distance travelled.

time (minutes)	distance (centimetres)
2	5
1	2.5
4	$4 \times (2.5)$

Glossary

- pace
- speed

Lesson 8 Practice Problems

Problem 1 Statement

A kangaroo hops 2 kilometres in 3 minutes. At this rate:

a. How long does it take the kangaroo to travel 5 kilometres?

a. How far does the kangaroo travel in 2 minutes?

Solution

a. 7.5 minutes (or equivalent)

b. $\frac{4}{3}$ kilometres (or equivalent)

Problem 2 Statement

Mai runs around a 400-metre track at a constant speed of 250 metres per minute. How many minutes does it take Mai to complete 4 laps of the track? Explain or show your reasoning.

Solution

$\frac{32}{5}$ minutes (or equivalent). Possible responses:

–

distance (metres)	time (minutes)
250	1
500	2
400	1.6
1 600	6.4

– If each lap is 400 metres, then Mai runs 1 600 metres in 4 laps. Since every 250 metres takes her 1 minute to run, it would take her 1 600 ÷ 250 or 6.4 minutes to run 1 600 metres.

Problem 3 Statement

At 10:00 a.m., Han and Tyler both started running toward each other from opposite ends of a 10-mile path along a river. Han runs at a pace of 12 minutes per mile. Tyler runs at a pace of 15 minutes per mile.

a. How far does Han run after a half hour? After an hour?

b. Do Han and Tyler meet on the path within 1 hour? Explain or show your reasoning.

Solution

a. Han runs $2\frac{1}{2}$ miles in a half hour and 5 miles in an hour. This table can be used to determine the distances.

time (minutes)	distance (miles)
12	1
1	$\frac{1}{12}$
30	$2\frac{1}{2}$
60	5

b. No. Tyler travels 1 mile every 15 minutes, so he travels 4 miles in 60 minutes. Because Han travels 5 miles and Tyler travels 4 miles, and they are 10 miles apart, they are one mile apart after 1 hour.

Problem 4 Statement

Two skateboarders start a race at the same time. Skateboarder A travels at a steady rate of 15 feet per second. Skateboarder B travels at a steady rate of 22 feet per second. After 4 minutes, how much farther will skateboarder B have travelled? Explain your reasoning.

Solution

Skateboarder B will have travelled 1 680 feet farther. Possible reasoning: There are 240 seconds in 4 minutes, because $4 \times 60 = 240$. Skateboarder A travels 240 times 15, or 3 600 feet in 4 minutes. Skateboarder B travels 240 times 22, or 5 280 feet in 4 minutes, because $5\,280 - 3\,600 = 1\,680$.

Problem 5 Statement

There are 4 tablespoons in $\frac{1}{4}$ cup. There are 2 cups in 1 pint. How many tablespoons are there in 1 pint? If you get stuck, consider drawing a double number line or making a table.

Solution

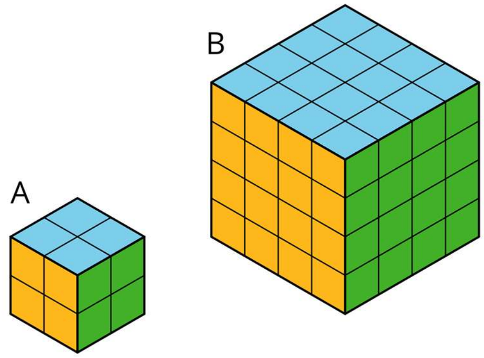
32 tablespoons

Problem 6 Statement

Two larger cubes are made out of unit cubes. Cube A is 2 by 2 by 2. Cube B is 4 by 4 by 4. The side length of cube B is twice that of cube A.

a. Is the surface area of cube B also twice that of cube A? Explain or show your reasoning.

b. Is the volume of cube B also twice that of cube A? Explain or show your reasoning.



Solution

a. No. Sample reasoning: The surface area of cube A is $6 \times (2 \times 2)$ or 24 square units. The surface area of cube B is $6 \times (4 \times 4)$ or 96 square units. The surface area of B is 4 times that of A.

b. No. Sample reasoning: The volume of cube B is 64 cubic units because $4^3 = 64$. The volume of cube A is 8 cubic units because $2^3 = 8$. 64 is not twice as much as 8.