Grades 9-12 (S)

Duration: 15 min

Tools: one Logifaces Set / 2-3 pairs or 4-6 students

Individual / Pair work

Keywords: Cosine, Angle of planes

536 - Different Slopes

MATHS / TRIGONOMETRY

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

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DESCRIPTION

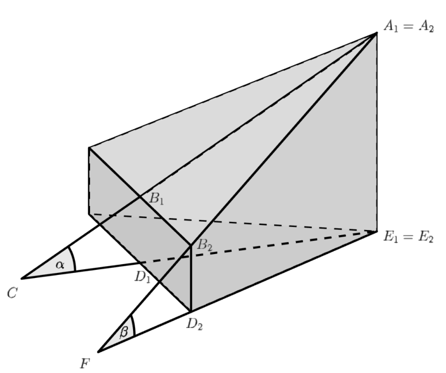
In the 9 pcs or 16 pcs Set, students choose those blocks that have two vertical edges with the same height and one with different height. These are blocks 112, 113, 122, 133, 223 and 233.

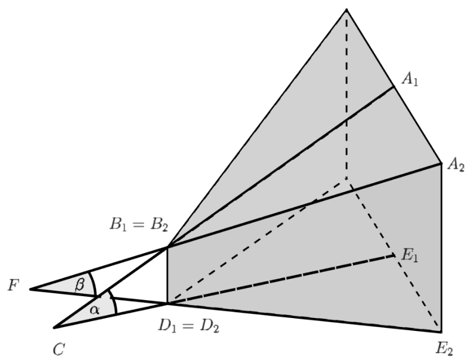
They draw the block and mark the following two angles on the diagram: let α be the angle of the inclination of the planes of the triangular faces, and let β be the angle of the two legs of the trapezium face.

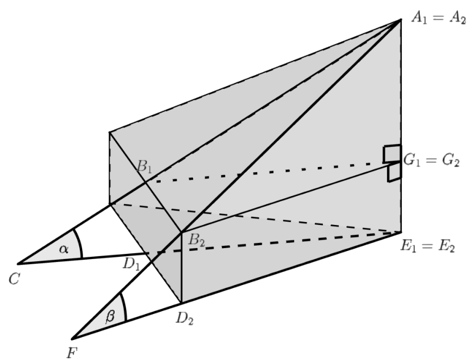
They then calculate the sizes of the angles α and β.

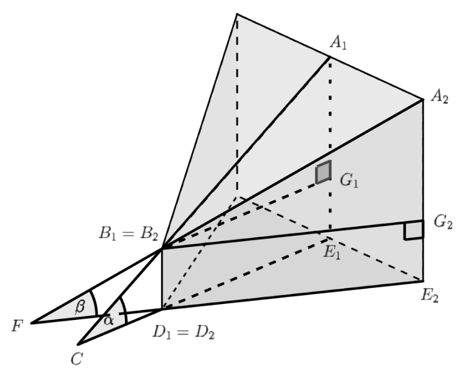
SOLUTIONS / EXAMPLES

The pictures below show the drawings of the two types of blocks with the notations used in the calculations.









To calculate the angles, let G_1 and G_2 be points on the line A_1E_1 and A_2E_2, respectively, such that B_1G_1 and B_2G_2 are parallel to the plane of the base triangle.

The triangles $A_1B_1G_1$ and $A_2B_2G_2$ are right-angled triangles with angles $\angle A_1B_1G_1 = \alpha$ and $\angle A_2B_2G_2 = \beta$, thus by definition, $cos(\alpha) = \frac{|G_1B_1|}{|A_1B_1|}$ and $cos(\beta) = \frac{|G_2B_2|}{|A_2B_2|}$ (where the notation $|AB|$ is used for the length of the AB line segment).

As seen in the diagram B_1G_1 is the altitude of the base triangle (a_b) and A_1B_1 is the altitude of the top triangle (a_t). B_2G_2 is the length of the base triangle's edge (a) and A_2B_2 is the top triangle's not horizontal edge (b).
So $cos(\alpha)$ is the ratio of the altitudes of the base (a_b) and top triangles (a_t), and $cos(\beta)$ is the ratio of the length of the base triangle's edge (a) and the top triangle's not horizontal edge (b): $cos(\alpha) = \frac{a_b}{a_t}$ and $cos(\beta) = \frac{a}{b}$.
Based on this formula, the angles can be calculated:

Block	a_b	a_t	α	a	b	β
112	$2\sqrt{3}$	$\sqrt{13}$	$\alpha \approx 16°$	4	$\sqrt{17}$	$\beta \approx 14°$
113	$2\sqrt{3}$	4	$\alpha = 30°$	4	$\sqrt{20}$	$\beta \approx 27°$
122	$2\sqrt{3}$	$\sqrt{13}$	$\alpha \approx 16°$	4	$\sqrt{17}$	$\beta \approx 14°$
133	$2\sqrt{3}$	4	$\alpha = 30°$	4	$\sqrt{20}$	$\beta \approx 27°$
223	$2\sqrt{3}$	$\sqrt{13}$	$\alpha \approx 16°$	4	$\sqrt{17}$	$\beta \approx 14°$
233	$2\sqrt{3}$	$\sqrt{13}$	$\alpha \approx 16°$	4	$\sqrt{17}$	$\beta \approx 14°$

PRIOR KNOWLEDGE

Calculation of altitude of triangles, Trigonometric ratios (specifically cosine), Measurements of angles

RECOMMENDATIONS / COMMENTS

Exercises 537 - Ratio of Heights and 538 - Ratio of Areas are recommended to calculate the angle α in a different approach.

Exercise 539 - Angle of Planes is recommended to calculate the angle between the planes of the base and top triangles in blocks 123 and 132.