The Context of this Research

Visualising abstract concepts is important for many teaching subjects, especially for mathematics. Manipulatives can be seen as a specific form of visualisation where students themselves are able to manipulate the representation of a concept in mathematics, for example, by holding, turning, rotating a 3D shape in their hands. Spatial reasoning is also an important process in geometry[8].

MANIPULATIVES AND GAMES IN EDUCATION

As already established, manipulatives are physical objects that can be used to demonstrate or help investigate concepts, such as those taught in the mathematics classroom. Research suggests that using manipulatives can have a positive impact on student mathematics literacy and on the understanding of mathematical concepts. Usually, these manipulatives are provided by teachers and are not created by students[9]. In this case though, in the learning process, the students do not merely passively absorb information during lessons or from textbooks, they develop their own knowledge based on what they have already learned and experienced according to the theory of constructivist learning[10]. A student's experience of the environment is usually physical and three-dimensional, especially during preschool. Therefore, it is possible that certain types of mathematical manipulatives may be used by children before learning to read and, to some extent, these manipulatives can be created by students in a constructivist manner. Creating objects such as manipulatives requires a variety of skills and competencies, especially those in the field of mathematics.

Playing games and solving puzzles can foster strategic and critical thinking, while making them can foster creativity and assist in inquiry-based learning. When planning activities that involve the use of games, it is important to consider all these aspects in order to get the most out of the game[11]. Experiments in this direction were recently carried out in a private school in Canada, where students not only played, but also digitally remodelled existing physical games[12]. As the final outcome, a group of students used a 3D printer to physically create their digital models. The students examined existing materials, and tested different strategies by playing, or understanding the code behind them. They also had to adapt them and create (or recreate) their own models.

In addition to understanding the rules, which is the first learning aspect of playing, other competencies can be developed from the use of games as a teaching and learning tool. Social skills such as resilience, self-expression,

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8 Mix, K. S., & Battista, M. T. (2018). Visualizing mathematics: The role of spatial reasoning in mathematical thought. Springer.

9 Fenyvesi, K., Budinski, N., Kaukolinna, M., Lakos, D., & Lavicza, Z. (2020). Playful Development of Mathematical Thinking Skills in Primary and Secondary School with the Logifaces STEAM Education Toolkit, LUMAT Research Symposium, p40.

10 G. E. Hein, (1991). Constructivist learning theory. Institute for Inquiry. Available at:/http://www. exploratorium. edu/ifi/resources/constructivistlearning. htmlS.

11 Olson, J. C. (2007). Developing students' mathematical reasoning through games. Teaching Children Mathematics, 13(9), 464-471. https://doi.org/10.5951/tcm.13.9.0464

12 D. Lieban, M. Barreto, S. Reichenberger and Z. Lavicza. (2018). Developing Mathematical and Technological Competencies of Students Through Remodeling Games and Puzzles. In Bridges 2018 proceedings. http://archive.bridgesmathart.org/2018/bridges2018-379.pdf.

and collaboration can be improved when playing. A number of these competencies were identified by a group of psychologists, musicians and teachers/educators in a workshop in Brazil. Moreover, as already mentioned, they found that mathematical and technological skills can also be developed and improved. From this experience, the Brazilian study also found that the use of games as a teaching tool can be adapted as much as desired in differentiated learning/teaching, which shows the great value of using games in education. Currently we are considering extending our teaching research to different learning environments and classroom settings. One idea is to use various, more accessible technologies, such as smartphones and tablets, to complement the activities. Through this, we believe that a greater number of students may be reached and engaged in the learning process we describe in this paper.

Mathematical discussions influence the development of games on different levels. As a particular example explored in Lieban et al.[13], in the course of a study carried out by the GeoGebra project, another programme aiming at the development of mathematical skills using games, a student decided to change the geometrical basis of the game. Equilateral triangle prisms were used instead of squares, which became a great challenge: finding the centre of the triangle was necessary in the course of the game. Some trials with the tools in the GeoGebra platform guided the student to the precise solution. Nevertheless, when using 3D modelling software, Tinkercad, the student found only the grid as a support, not being able to use the same tools and strategies used in GeoGebra. As a consequence, the shape of the pieces was changed again, the square prism being chosen as the piece this time. This is a good example of the adaptability necessary in the interaction with mathematical games and how students are able to find solutions through their mathematical skills.

EXAMPLES OF EDUCATIONAL GAMES

There is a wide range of educational games, and we selected two examples in the field of mathematics to provide context for the Logifaces game: how it follows the tradition of manipulatives to a certain extent while also offering something uniquely new. In the following we briefly explain the functions of Cuisenaire rods and PUSE. These games were selected because they are based on geometrical forms that are linked to mathematics and can also be used as manipulatives and can also be used interdisciplinary way for exercises in other subjects.

One of these originates from 1931 and the other was developed more recently, in 2010. Both combine mathematics and other subjects using a playful approach following the Montessori spirit and contain objects that can be assembled in configurations and forms, which can be created either as the students desire or following certain rules. These aspects can also be found in the Logifaces game, which combines mathematics and other subjects through assembling geometrical forms.

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13 D. Lieban and Z. Lavicza. (2019). Dissecting a Cube as a Teaching Strategy for Enhancing Students' Spatial Reasoning: Combining Physical and Digital Resources. In Bridges 2019 proceedings. https://archive.bridgesmathart.org/2019/bridges2019-319.pdf.

Cuisenaire rods

Cuisenaire rods are a kind of mathematical game and manipulatives. As can be observed in Figure 1, they consist of a set of ten rods with different colours and lengths. Each of the rods represents the numbers from one to ten with the smallest cube unit serving as the measure of the increase in height between each rod. This mathematics learning aid was invented by the Belgian primary school teacher Georges Cuisenaire and is used today as an interactive tool to discover mathematics and mathematical concepts[14].

Figure 1:
A set of Cuisenaire rods as used in school today. Image source: https://www.abcschoolsupplies.ie/product/wooden-cuisenaire-rods-introductory-set-of-74

Students that are challenged in understanding logical operations profit in particular when using Cuisenaire rods. The idea was born when Cuisenaire observed the difference in student enjoyment when teaching his two subjects, music and mathematics. He therefore tried to create a tool combining these subjects and spark the enjoyment students experience in music classes in mathematics classes too and used the rods as an aid. Although having been developed years earlier in 1931, Cuisenaire rods did not become broadly popular until the 1950s, when Caleb Gattegno, a British mathematician and specialist in maths education, popularised them and further developed their use. Since then they have been used in more than 100 countries and 10,000 schools all over the world. The areas of utilisation are extensive, for example they can be used to develop algebraic thinking, and understanding of sequences, patterns and counting[15]. The rods can be used to visualise addition, subtraction, multiplication and division, and even proportions, ratios and fractions, thereby developing the mathematic thinking in students. Another possible use for the Cuisenaire rods is in language teaching, more specifically in the so-called Silent Way or in connection with verb tenses[16].

The Poly-Universe in School Education (PUSE) methodology

The second example of the use of manipulatives in the context of maths teaching is the PUSE methodology. The acronym stands for Poly-Universe in School Education and is the name of an Erasmus+ project based on the Poly-Universe game[17], which was created by János Saxon-Szász. This approach also combines geometric shapes and art, however this time the art is visual rather than acoustic. PUSE includes triangles, squares with a missing corner and circles with straight edges.

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14 K. Delaney, (2001). Cuisenaire rods: 40 years on. Australian Primary Mathematics Classroom, 6(2), 26-31.

15 M. Ollerton, H. Williams and S. Gregg. Cuisenaire: from Early Years to Adult (2018). From https://www.atm.org.uk/Shop/Cuisenaire---from-Early-Years-to-Adult/Cuisenaire---from-Early-Years-to-Adult-Book-and-Download/ACT103pk

16 C. Gattegno, (2010). The common sense of teaching foreign languages. Educational Solutions World.

17 E. Stettner, & G. Emese, (2016). Teaching Combinatorics with "Poly-Universe". In Proceedings of Bridges 2016: Mathematics, Music, Art, Architecture, Education, Culture (pp. 553-556). Tessellations Publishing.

Figure 2: *different PUSE setsFigure*

Figure 3: *different combinations of PUSE elements. Image source: http://poly-universe.com/mathematics-set-up-and-combination-possibilities-of-the-poly-universe-game-family/*

Figure 4: *child playing with a PUSE set. Image source: http://poly-universe.com/#services*

The inventor of the Poly-Universe game has been involved in the PUSE Project since 2017. The essence of the game is to form an artistic and mathematical system by filling the two-dimensional plane with geometric forms, as shown in Figures 3 and 4. While creating a desire for discovery it also provides the experience of continuous success and enables the player to create their own task. Poly-Universe can help develop are art sensitivity, complex logical thinking, mathematical and combination skills, attention and concentration, visual memory and even social skills, such as openness and team spirit. The PUSE project describes the utilisation possibilities as follows: "The complexity emerging out of Poly-universe's simplicity makes it more than a game, more than art, more than mathematics: these elements come all together – creating synergy in education"[18].

Aspects of the above examples found in Logifaces

Both Cuisenaire rods and the PUSE methodology combine mathematical objects with arts using an interdisciplinary approach. The examples can also be used for teaching other subjects, including languages,

18 E. Stettner, & G. Emese, (2016). Teaching Combinatorics with "Poly-Universe". In Proceedings of Bridges 2016: Mathematics, Music, Art, Architecture, Education, Culture (pp. 553-556). Tessellations Publishing.

engineering and science. These games can therefore be seen as STEAM games that use geometric forms as their basis, and constitute manipulatives. These geometric forms can be held in a student's hands, observed and combined to form different shapes and are generic, which is useful for the visualisation of concepts in other STEAM subjects. Both games were created with consideration to artistic creativity and motivational aspects and take a playful approach in the teaching of subjects commonly perceived as harder than art. They have certain shapes and a certain thickness but are in general used as two-dimensional visualisations and use colour coding as an additional dimension with meaning.

Logifaces blocks are similar: the blocks can also be combined into various shapes that are either suggested by the game or decided by the students and, as it is also based on geometric forms, they can be used as manipulatives. The interdisciplinary background is another similarity; the Logifaces game is the result of an artistic design idea and has even received an award in this category. Consequently the Logifaces game has similar artistic and visualisation capabilities, being able to teach STEAM subjects in an integrative and interactive way and foster longer attention spans, combinatory skills, creativity and social skills such as communication. Just as in the previously mentioned examples, the developer of the game, Daniel Lakos, has a strong link to the arts with his Logifaces game receiving the Hungarian Design Award. While Cuisenaire has links to both mathematics and music, and his rods can be used to represent sounds, Saxon is a painter and created Polyuniverse with a high visual content, Daniel Lakos is an architect and created a game that has a three dimensional component. The Logifaces approach may have high motivation-creating qualities, like the other two examples, and can be used in educational settings for students with maths learning disabilities.

However, there is a distinct difference between the examples above and the Logifaces approach: the examples described are used in two dimensions and use colour coding as an additional data dimension, whereas Logifaces does not need colour coding as it consists of three dimensional blocks. This opens up numerous possibilities: Firstly, not only children with special cognitive challenges, such as dyscalculia, can profit from this game but also children with special physical disabilities, such as the visually impaired, are able to profit more from this approach. The additional data dimension can also be perceived by other senses, such as touch, which adds to the blocks quality of being manipulatives.

Secondly, the game is more abstract than the other examples, but has the same number of data dimensions, which provide additional possibilities for use with STEAM subjects and, if needed, concepts that are more abstract can be communicated. Also, there is more freedom in the shapes that can be created because colour does not play a role, so all the blocks can be used. This can help create complex structures such as the representation of, for example, optics in physics. The colour dimension can still be added to code words and concepts, for example, if an additional data dimension is needed.

Thirdly, the three-dimensional shape itself may be of value in helping to foster visuospatial skills, identify orientation within geometric shapes and other important skills connected to three-dimensional thinking. The other two examples do not involve these skills in any way. Visuospatial skills are the ability to imagine the rotation of objects and the transition between 2D and 3D, and are connected to virtual 3D manipulation, which is required in engineering[19] [20]. The manipulation of 2D and 3D objects in two-dimensional representation requires understanding of drawing conventions and the ability to correctly reading a drawing[21]. Pittalis and Christou (2010) identify

- 3D geometry thinking
- he ability to manipulate a 3D model
- the ability to recognise and manipulate nets and 3D objects and compare them
- and the ability to calculate the volumes of solids as challenges for students
- creating and manipulating 3D objects in parallel and perpendicular directions

as great challenges for both young students and adults alike. Other researchers have observed that students lack visuospatial skills, especially when manipulating prisms in perpendicular and other directions[22]. These skills are not supported by the two examples discussed above, but do play an important role in science subjects and engineering. In the course of our investigation, we examined the topics of motivation, creativity, interdisciplinarity, three-dimensional thinking and support for students challenged in subjects connected to logic and computational thinking.

THE ERASMUS+ PROJECT

The main objective of the Logifaces methodology project is to prepare a new visual mathematics and art teaching system and adapt it to and disseminate it in EU-wide school education. The project is based on the Logifaces game, which is a spatial geometric, logic and sensorimotor skill development game. The target group of the project are primary and secondary school students, aged 6 to 18. Schools and institutes operating in the different education systems of various countries were invited to develop and test STEAM exercises using the game, involving students in both elementary and secondary school.

The introduction of the Logifaces teaching methodology aims to have a positive effect on school education because it involves the use of a game as a tool that switches dimensions, demonstrates spatial geometry, and develops sensorimotor and art skills. It creates fun and motivation in early childhood, provides support for children

19 Duffy, G., Sorby, S. A., & Bowe, B. (2016). Visualizing electric circuits: The role of spatial visualization skills in electrical engineering. 70th EDGD midyear conference, American society for engineering education, 113– 117

20 Sorby, S. A., Metz, S., & Ribe, R. (2017). Implementing Training for Spatial Visualization Skills: Research and Best Practices for Engaging Future Engineers. Proceedings of the 2017 Conference for Industry and Education Collaboration, American Society for Engineering Education, 1–7.

21 Pittalis, M., & Christou, C. (2010). Types of reasoning in 3D geometry thinking and their relation with spatial ability. Educational Studies in Mathematics, 75(2), 191–212. https://doi.org/10.1007/s10649-010-9251-8

22 Ma, H. L., Wu, D., Chen, J. W., & Hsieh, K. J. (2009). Mithelmore's development stages of the right rectangular prisms of elementary school students in Taiwan. Proceedings of the 33rd conference of the international group for the psychology of mathematics education, 4, 57–64

challenged by mathematics, interdisciplinary teaching exercises connected to mathematics and art, and support for developing cultural competence. Details of the additional skills that may be developed by this project :

- Flexible thinking - developing problem-solving skills by working on different solutions
- Creativity - Using imagination to finding unique solutions for open ended tasks
- Spatial and visuospatial abilities - Understanding the link between different objects and mental rotation of abstract objects - being able to imagine rotations in 2D and 3D
- Communication - Verbal skills and communication by planning and constructing projects in groups while using technical and artistic language
- Imagination - Most of the shapes built with Logifaces do not look quite like the real thing. Imagination has to fill in all the gaps. Imagination is a critical aspect of creative problem solving and abstract thinking
- Tactile senses - Logifaces is based on polygonal modelling, which helps students get a sense of virtual complexity at an early age by connecting the virtual and the physical
- Team building - Logifaces can be used freely or with the aim of building a predefined shape. Leadership and project discussion develop naturally
- Artistic inspiration - Exploring sculpture and design. The relief-like surfaces blend ancient aesthetics with contemporary design. Logifaces blocks have inspired artists, photographers, and even fashion designers
- Calculus - Exploration with simple blocks helps develop subconscious skills in mathematics and logic. Children practice geometry and build an understanding of numbers and how units interrelate

Play-based learning in the early years is fundamental as we absorb communication and language skills in both early numeracy and literacy learning. This naturally leads to the acquisition of social skills, such as turn-taking and sharing, and to the desire to discover more. Play activities that are simple, magic and fun are vital. Later, during the years students spend in primary education play-based learning is important for acquiring numeracy and literacy skills, but above all for developing the desire to keep learning. In secondary school positive stimulation to learn and a vision of where this is all leading are primary. Hands-on, open-ended activities help stimulate constant reflection and inquiry-based thinking, and we believe Logifaces is a suitable tool for achieving these goals.

Logifaces aims to help teachers introduce and practice a variety of mathematical, scientific, artistic and other concepts. It was created by Hungarian architect and designer Daniel Lakos, who is also an assistant professor at Moholy-Nagy University, Budapest (MOME). A group of teachers, researchers and designers from Austria, Finland, Hungary, and Serbia joined forces to investigate and develop examples of education activities in the project entitled The Logifaces Methodology: analogue game for digital minds[23]. Many activities revolve around using the game blocks as manipulatives, such as the proposed workshop involving the creation of 3D models of the blocks

23 The coordinator of the Logifaces project is Planbureau Kft located in Budapest, Hungary. Other partners in the project are the Akademische Gymnasium in Vienna, Austria, the Experience Workshop in Jyväskylä, Finland, Johannes Kepler Universität Linz (JKU), Austria, Lauder Javne Zsidó Közösségi Óvoda, Általános Iskola, Középiskola és Zenei Alapfokú Művészeti Iskola in Hungary, and Osnovna i srednja skola sa domom ucenika Petro Kuzmjak in Serbia.

themselves. Using the blocks as a point of inspiration and motivation to engage students in mathematics and multidisciplinary learning projects is in line with the skill development recommended by the European Council. The geometry of the blocks begs their use as manipulatives in various STEAM areas revolving around spatial, visual, and computational thinking.

Exercises are created and collected based on using the game as a teaching tool[24]. The Logifaces project is aimed at creating a catalogue of exercises in STEAM subject education using the Logifaces game and at investigating the effects of such exercises. For this it is crucial to use the game in a teaching context to find different utilizations. Another core element within the goals of the Logifaces method is enhancing students' spatial reasoning and three-dimensional thinking. Lieban (2018) gives the example of cube-puzzles, arguably a kind of manipulative, to improve spatial reasoning in students by addressing the problem in two as well as in three dimensions[25]. Lieban (2018) discusses how knowing one representation of a problem can help students visualise and comprehend other representations. Not only may switching between dimensions be trained by the game, but more mathematical, artistic, and STEAM-related subjects can be supported.

Mathematical concepts involving the jumps between the planar and the spatial, spatial geometry, combinatorics, algebra, figurating numbers and tessellation can be introduced and connected to artistic thinking, such as analysing the light-shadow effect on different angles and slopes. Also graph theory can be explained connecting to, for example, computer related skills, like network science and polygonal modelling.

Art is taught through encouraging thinking about design, relief structures, light and shadow effect, and basic figurative forms. Social skills can also be taught, such as problem solving, cooperation when learning with peers, and non-verbal communication. In addition, the game may help inclusivity through collaborative activities with visually impaired children.

The exercises created will be available online at https://www.geogebra.org/m/pghjyunt and a paper version will also be available.

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24 Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. Educational Studies in mathematics, 47(2), 175-197.

25 D. Lieban, M. Barreto, S. Reichenberger and Z. Lavicza. (2018). Developing Mathematical and Technological Competencies of Students Through Remodeling Games and Puzzles. In Bridges 2018 proceedings. http://archive.bridgesmathart.org/2018/bridges2018-379.pdf.