Lesson 9: Describing large and small numbers using powers of 10

Goals

- Describe (orally and in writing) large and small numbers as multiples of powers of 10.
- Interpret a diagram for base-ten units, and explain (orally) how the small squares, long rectangles, and large squares relate to each other.

Learning Targets

- Given a very large or small number, I can write an expression equal to it using a power of 10.

Lesson Narrative

This lesson serves as a prelude to standard form and builds on work students have done in previous years with numbers in base ten. Students use base-ten diagrams to represent different powers of 10 and review how multiplying and dividing by 10 affect the place value of each digit. They use their understanding of base-ten structure as they express very large and very small numbers using exponents.

Students also practise communicating—describing and writing—very large and small numbers in an activity, which requires attending to precision. This leads to a discussion about how powers of 10 can be used to more easily communicate such numbers.

Building On

- Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the value of each digit when a decimal number is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \left(\frac{1}{10}\right) + 9 \times \left(\frac{1}{100}\right) + 2 \times \left(\frac{1}{100}\right)$.

Addressing

- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

Building Towards

- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as

3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

- Perform operations with numbers expressed in standard form, including problems where both decimal and standard form are used. Use standard form and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimetres per year for seafloor spreading). Interpret standard form that has been generated by technology.

Instructional Routines

- Collect and Display
- Compare and Connect
- Think Pair Share

Required Materials

Pre-printed cards, cut from copies of the blackline master

Using Powers of 10 to Describe Large and Small Numbers Partner A Card You are Partner A. • Here are the answers to Partner B's questions: • Light waves travel through space at a speed of 3 hundred million metres per second. • The population of India is about 1 300 000 000 people. • The wavelength of a gamma ray is 0.0000000000048 metres. • The tardigrade (water bear) is 5 ten-thousandths of a metre long.	Using Powers of 10 to Describe Large and Small Numbers Partner B Card You are Partner B. Here are the answers to Partner A's questions: • Around the world, about 14 000 000 000 pencils are made each year. • The mass of a proton is 0.00000000000000000000000000167 kilograms. • The population of Russia is about 144 million people. • The diameter of a bacteria cell is about 2 ten-millionths of a metre.

Required Preparation

Print and cut up cards from the included blackline master. Prepare 1 set of cards for every 2 students.

Student Learning Goals

Let's find out how to use powers of 10 to write large or small numbers.

9.1 Thousand Million Billion Trillion

Warm Up: 5 minutes

In this warm-up, students connect thousand, million, billion, and trillion to their respective powers of ten—10^3, 10^6, 10^9, and 10^{12}. Understanding powers of 10 associated with these denominations will help students reason about quantities in real-world contexts such as the number of cells in a human body (trillions), world population (billions), etc.

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time and then 1 minute to compare their responses with their partner. Given the limited time, it may not be possible for students to create examples for each of the values in the second question. Tell students to try to at least find 1 or 2 examples and then to find others as time allows. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may think that, for example, $1\,000\,000 = 10^7$ because the number $1\,000\,000$ has 7 digits. Ask these students if it is true that $10 = 10^2$.

Some students may confuse the prefix "milli-" with the word "million." The word "million" literally means "a big thousand," and so both "million" and "mille" are related to the Latin "mille," meaning "thousand." While "milli-" is talking about a thousand parts (thousandths), "million" is talking about a thousand thousands.

Student Task Statement

1. Match each expression with its corresponding value and word.

expression
10^{-3}
10^6
10^9
10^{-2}
10^{12}
10^3
value
$1\,000\,000\,000\,000$
$\frac{1}{100}$
$1\,000$
$1\,000\,000\,000$
$1\,000\,000$

$\frac{1}{1\,000}$
word
billion
milli-
million
thousand
centi-
trillion

2. For each of the numbers, think of something in the world that is described by that number.

Student Response

expression	value	word
10^3	1 000	thousand
10^6	1 000 000	million
10^9	1 000 000 000	billion
10^{12}	1 000 000 000 000	trillion
10^{-2}	$\frac{1}{100}$	centi-
10^{-3}	$\frac{1}{1\,000}$	milli-

2. Answers vary. Sample response: 10^3 (thousand): number of students in a school. 10^6 (million): population of a state. 10^9 (billion): population of China. 10^{12} (trillion): number of stars in a large galaxy. 10^{-2} (hundredth): There are 100 centimetres in a metre. 10^{-3} (thousandth): There are 1 000 millilitres in a litre.

Activity Synthesis

Ask students to share the corresponding expressions, words, and values. Record and display their responses for all to see. If time is limited, consider displaying the completed table for all to see and discussing any questions or disagreements. Then, invite students to share their examples for the final question. After each student shares, ask the class whether they agree that the given example could be described by that value.

If students struggle to find something that could be described by each value, consider sharing some of the following examples:

- Thousand (10^3)
  - Number of students in a school
  - Population of an endangered species
  - Cost of a car that barely runs

    - Number of brain cells of a jellyfish
- Million (10^6)
    - Population of a state
    - Cost of the most expensive car in the world
    - Number of brain cells of a cockroach
- Billion (10^9)
    - Population of India
    - Population of China
    - Number of students in the world
    - Number of brain cells of a monkey
    - Number of trees in the U.S.
- Trillion (10^{12})
    - Amount of wealth produced by a developed country in a year in U.S. dollars
    - Number of stars in a large galaxy
    - Number of brain cells of 10 students

9.2 Base-ten Representations Matching

20 minutes

In this activity, students use their understanding of decimal place value and base-ten diagrams to practise working with the structure of standard form before it is formally introduced. They express numbers as sums of terms, each term being multiples of powers of 10. For example, 254 can be written as $2 \times 10^2 + 5 \times 10^1 + 4 \times 10^0$.

Notice students who choose different, yet correct, diagrams for the first and last problems. Ask them to share their reasoning in the discussion later.

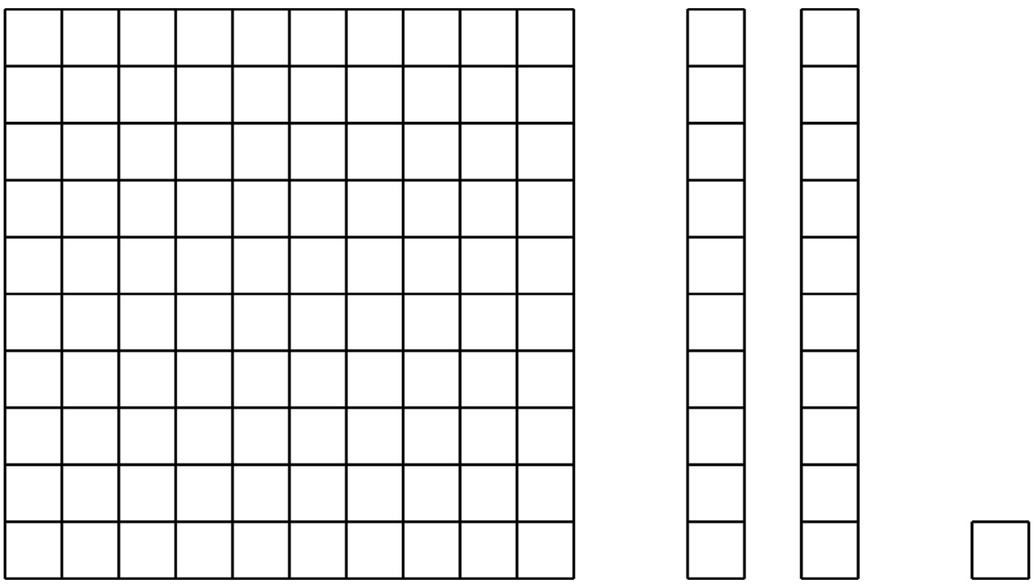
Instructional Routines

- Compare and Connect

Launch

Display the following diagram for all to see. Pause for quiet think time after asking each question about the diagram. Ask students to explain their thinking. Here are some questions to consider:

- "If each small square represents 1 tree, what does the whole diagram represent?" (121 trees)
- "If each small square represents 10 books, what does the whole diagram represent?" (1 210 books)
- "If each small square represents 1 000 000 stars, what does the whole diagram represent?" (121 000 000 stars)
- "If each small square represents 0.1 seconds, what does the whole diagram represent?" (12.1 seconds)
- "If each small square represents 10^3 people, what does the whole diagram represent?" (121 000 people)



Give students 10 minutes to work on the task, followed by a brief whole-class discussion.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, display only the first expression and ask students what they notice before inviting them to select one or more diagrams that could represent it. Ask 1-2 students to explain their match before revealing the remaining expressions.
Supports accessibility for: Conceptual processing; Organisation

Anticipated Misconceptions

Some students may think that the small square must always represent one unit. Explain to these students that, as in the launch, the small square might represent 10 units, 0.1 units, or any other power of 10.

Some students may have trouble writing the value of expressions that involve powers of 10, especially if they involve negative exponents. As needed, suggest that they expand the expression into factors that are 10, and remind them that 10^{-1} corresponds to the tenths place, 10^{-2} corresponds to the hundredths place, etc.

Student Task Statement

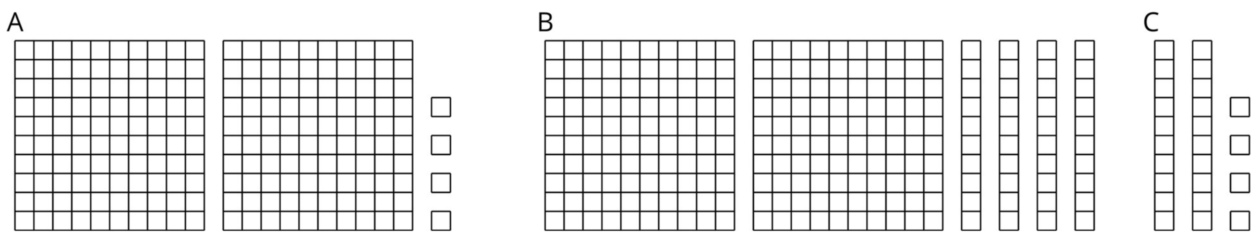
1. Match each expression to one or more diagrams that could represent it. For each match, explain what the value of a single small square would have to be.

   a. $2 \times 10^{-1} + 4 \times 10^{-2}$

   b. $2 \times 10^{-1} + 4 \times 10^{-3}$

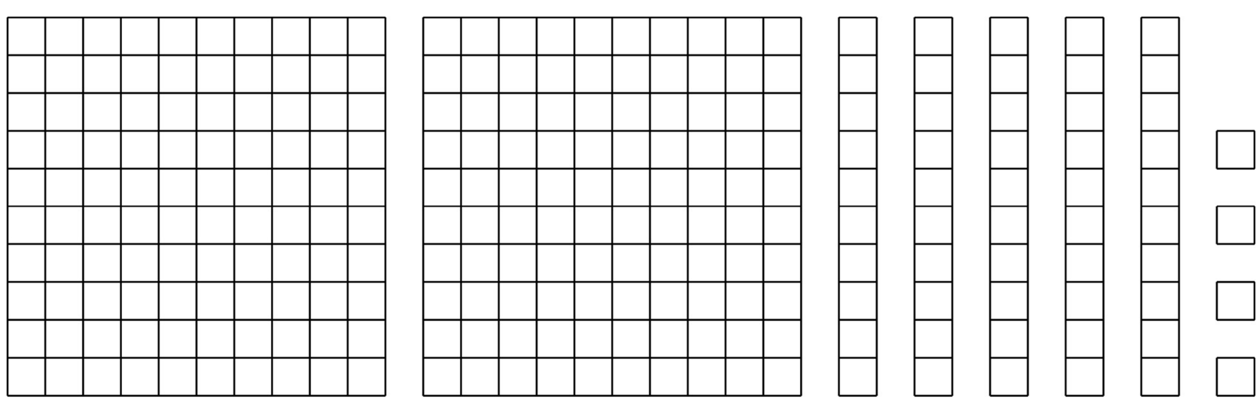
   c. $2 \times 10^{3} + 4 \times 10^{1}$

   d. $2 \times 10^{3} + 4 \times 10^{2}$



2.

   a. Write an expression to describe the base-ten diagram if each small square represents 10^{-4}. What is the value of this expression?



   b. How does changing the value of the small square change the value of the expression? Explain or show your thinking.

c. Select at least two different powers of 10 for the small square, and write the corresponding expressions to describe the base-ten diagram. What is the value of each of your expressions?

Student Response

1.

a. C if a small square is 10^{-2} or B if a small square is 10^{-3}

b. A if a small square is 10^{-3}

c. A if a small square is 10^{1}

d. C if a small square is 10^{2} or B if a small square is 10^{1}

2.

a. $4 \times 10^{-4} + 5 \times 10^{-3} + 2 \times 10^{-2}$ which is 0.0254.

b. Answers vary. Sample response: Changing the value of the small square changes the powers of 10. The long rectangle always has an exponent that is 1 higher than for the small square, and the large square always has an exponent that is 2 higher than for the long rectangle.

c. Answers vary. Sample response: If the small square has a value of 10^{6}, then the expression is $4 \times 10^{6} + 5 \times 10^{7} + 2 \times 10^{8}$, which is 254 000 000. If the small square has a value of 10^{-1}, then the expression is $4 \times 10^{-1} + 5 \times 10^{0} + 2 \times 10^{1}$, which is 25.4.

Activity Synthesis

Select students to share their responses to the first and last problems. Bring attention to the fact that diagrams B or C could be used depending on the choice of the value of the small square.

The main goal of the discussion is to make sure students see the connection between decimal place value to sums of terms that are multiples of powers of 10. To highlight this connection explicitly, consider discussing the following questions:

- "How are the diagrams related to our base-ten numbers and place value system?" (In base-ten numbers, each place value is ten times larger than the one to its right; so every 1 unit of a place value can be composed of 10 units of the next place value to its right. The diagrams work the same way: each shape representing a base-ten unit can be composed of 10 that represent another unit that is one tenth of its value.)
- "How are the diagrams related to numbers written using powers of 10?" (We can think of each place value as a power of ten. So a ten would be 10^{1}, a hundred would be 10^{2}, a tenth would be 10^{-1}, and so on.)

- "If each large square represents 10^2, what do 2 large squares and 4 long rectangles represent?" (A long rectangle is a tenth of the large square, so we know the long rectangle represents 10^1. This means 2 large squares and 4 long rectangles represent $2 \times 10^2 + 4 \times 10^1$.)
- "Why is it possible for one base-ten diagram to represent many different numbers?" (Because of the structure of our place value system—where every group of 10 of a base-ten unit composes 1 of the next higher unit—is consistent across all place values.)

Speaking, Listening: Compare and Connect. As students prepare to share their responses to the first and last problems, look for those using different strategies for matching base-ten representations. During the discussion, ask students to share what worked well in a particular approach. Listen for and amplify any comments that describe each representation (i.e., place value, base-ten unit, powers of 10). Then encourage students to make the connection between decimal place value to sums of terms that are multiples of powers of 10. This will foster students' meta-awareness and support constructive conversations as they compare strategies for describing large and small quantities.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

9.3 Using Powers of 10 to Describe Large and Small Numbers

15 minutes

This activity motivates students to find easier ways to communicate about very large and very small numbers, using powers of 10 and working toward using standard form. Students take turns reading aloud and writing down quantities that involve long strings of digits, noticing the challenges of expressing such numbers.

As students work, monitor the different ways students communicate the number of zeros precisely to their partners. Some might use standard vocabulary (billion, ten-thousandth, etc.), or some may communicate the number of zeros after the decimal point or after the significant digits. Select students using different strategies to share later.

Instructional Routines

- Collect and Display

Launch

Arrange students in groups of 2. Distribute a pair of cards (one for Partner A and one for Partner B) from the blackline master to each group. Ask partners not to show their card to each other.

Tell students that one partner should read an incomplete statement in the materials and the other partner should read aloud the missing information on the card. The goal is for each partner to write down the missing quantity correctly. Partners should take turns reading and writing until all four statements for each person are completed.

Consider explaining (either up front or as needed during work time) that students who have the numbers can describe or name them in any way that they think convey the quantities fully. Likewise, those writing the numbers can write in any way that captures the quantities accurately.

Give students 10 minutes to work. Leave a few minutes for a whole-class discussion.

Action and Expression: Internalise Executive Functions. Begin with a whole-class think aloud to demonstrate how students should work together.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Your teacher will give you a card that tells you whether you are Partner A or B and gives you the information that is missing from your partner's statements. Do not show your card to your partner.

Read each statement assigned to you, ask your partner for the missing information, and write the number your partner tells you.

Partner A's statements:

1. Around the world, about ____________________ pencils are made each year.
2. The mass of a proton is ____________________ kilograms.
3. The population of Russia is about ____________________ people.
4. The diameter of a bacteria cell is about ____________________ metre.

Partner B's statements:

1. Light waves travel through space at a speed of ____________________ metres per second.
2. The population of India is about ____________________ people.
3. The wavelength of a gamma ray is ____________________ metres.
4. The tardigrade (water bear) is _____________ metres long.

Student Response

Partner A's statements:

1. Around the world, about 14 000 000 000 pencils are made each year.
2. The mass of a proton is 0.00000000000000000000000000167 kilogram.
3. The population of Russia is about 144 000 000 people.
4. The diameter of a bacteria cell is about 0.0000002 metre.

Partner B's statements:

1. Light waves travel through space at a speed of 300 000 000 metres per second.

2. The population of India is about 1 300 000 000 people.

3. The wavelength of a gamma ray is 0.0000000000048 metre.

4. The tardigrade (water bear) is 0.0005 metre long.

Are You Ready for More?

A "googol" is a name for a really big number: a 1 followed by 100 zeros.

1. If you square a googol, how many zeros will the answer have? Show your reasoning.

2. If you raise a googol to the googol power, how many zeros will the answer have? Show your reasoning.

Student Response

Writing a googol as 10^{100} makes it easier to solve this problem.

1. 200 zeros, because $(10^{100})^2 = 10^{200}$.

2. $10^{10^{102}}$ zeros, because $(10^{100})^{10^{100}} = 10^{100\times10^{100}} = 10^{10^2\times10^{100}} = 10^{10^{102}}$.

Activity Synthesis

Ask previously identified students to share how they described their partner's numbers or recorded those given to them. The purpose of the discussion is for students to hear different strategies for communicating very small and very large numbers. For example, 150 000 000 000 can be described as "one hundred fifty billion," as "15 followed by 10 zeros," as 150×10^9, or as $(1.5) \times 10^{11}$.

For negative powers, discuss the idea that multiplying by 10^{-1} means multiplying by $\frac{1}{10}$, which in turn increases the number of decimal places. For example, consider 48×10^{-13}. We know that 1×10^{-13} is 1 multiplied by $\frac{1}{10}$, 13 times, which is 0.0000000000001. So 48×10^{-13} is $48 \times (0.0000000000001)$, which is 0.0000000000048. Display the following table for all to see and briefly explain to students how to write each number using powers of 10.

quantities	using powers of 10
150 000 000 000 metres	150×10^9 metres
300 000 000 metres per second	300×10^6 metres per second
0.0000000000048 metres	48×10^{-13} metres
0.00000000000000000000000000167 kilogram	167×10^{-29} kilogram

Conversing, Representing: Collect and Display. While pairs are working, circulate and listen to students talk about very large and very small numbers. Write down common or important phrases you hear the ways students communicate the number of zeros to their partners. Listen for students who use standard vocabulary (billion, ten-thousandth, etc.), provide the number of zeros after the decimal point or after the significant digits, or refer to powers of 10. Display their representations together with the students' language onto a visual display. This will help students use mathematical language when communicating about very small and very large numbers during their paired and whole-group discussions.
Design Principle(s): Maximise meta-awareness; Support sense-making

Lesson Synthesis

The focus of the discussion is the structure of our place value system and the rationale and usefulness of describing large and small numbers in different ways. Consider asking some of the following questions:

- "How do base-ten diagrams help us make sense of (or explain) the exponents in powers of 10?" (When using diagrams, grouping 10 of the next smaller unit means multiplying by 10. When dealing with powers of 10, multiplying by 10 increases the exponent by 1. Likewise, decomposing a base-ten unit into 10 of the next smaller unit means multiplying by $\frac{1}{10}$, so the exponent in the power of 10 goes down by 1.)
- "How does using powers of 10 make it easier to communicate about very large or very small numbers?" (We can write in a smaller space. It's also faster to read and easier to understand the size of a number and to compare numbers. Using powers of 10 helps us avoid errors of missing zeros or extra zeros.)
- "What are some different ways to describe a large number like 123 billion?" ($123 \times (1\,000\,000\,000$ or 123×10^9.)
- "What are some different ways to describe a small number like 0.0000000789?" (789 ten-billionths, $789 \times \frac{1}{10\,000\,000\,000}$, or 789×10^{-10}.)

9.4 Better with Powers of 10

Cool Down: 5 minutes

Student Task Statement

1. Write 0.000000123 as a multiple of a power of 10.
2. Write 123 000 000 as a multiple of a power of 10.

Student Response

1. Answers vary since students don't yet know the standard of standard form. Sample response: $(1.23) \times 10^{-7}$.
2. Answers vary. Sample response: $(1.23) \times 10^{8}$.

Student Lesson Summary

Sometimes powers of 10 are helpful for expressing quantities, especially very large or very small quantities. For example, the United States Mint has made over

500 000 000 000

pennies. In order to understand this number, we have to count all the zeros. Since there are 11 of them, this means there are 500 billion pennies. Using powers of 10, we can write this as: 500×10^9 (five hundred times a billion), or even as: 5×10^{11} The advantage to using powers of 10 to write a large number is that they help us see right away how large the number is by looking at the exponent.

The same is true for small quantities. For example, a single atom of carbon weighs about

0.0000000000000000000000199

grams. We can write this using powers of 10 as 199×10^{-25} or, equivalently, $(1.99) \times 10^{-23}$ Not only do powers of 10 make it easier to write this number, but they also help avoid errors since it would be very easy to write an extra zero or leave one out when writing out the decimal because there are so many to keep track of!

Lesson 9 Practice Problems

1. **Problem 1 Statement**

   Match each number to its name.

   a. 1 000 000

   b. 0.01

   c. 1 000 000 000

   d. 0.000001

   e. 0.001

   f. 10 000

   – One hundredth

   – One thousandth

   – One millionth

   – Ten thousand

   – One million

   – One billion

Solution

a. one million

b. one hundredth

c. one billion

d. one millionth

e. one thousandth

f. ten thousand

2. **Problem 2 Statement**

Write each expression as a multiple of a power of 10:

a. 42 300

b. 2 000

c. 9 200 000

d. Four thousand

e. 80 million

f. 32 billion

Solution

a. Answers vary. Sample responses: 423×10^2, 4.23×10^4

b. Answers vary. Sample response: 2×10^3

c. Answers vary. Sample responses: 92×10^5, 9.2×10^6

d. Answers vary. Sample response: 4×10^3

e. Answers vary. Sample response: 8×10^7

f. Answers vary. Sample responses: 32×10^9, 3.2×10^{10}

3. **Problem 3 Statement**

Each statement contains a quantity. Rewrite each quantity using a power of 10.

a. There are about 37 trillion cells in an average human body.

b. The Milky Way contains about 300 billion stars.

c. A sharp knife is 23 millionths of a metre thick at its tip.

d. The wall of a certain cell in the human body is 4 nanometres thick. (A nanometre is one billionth of a metre.)

Solution

a. 37×10^{12} (or equivalent)

b. 300×10^{9} (or equivalent)

c. 23×10^{-6} (or equivalent)

d. 4×10^{-9} (or equivalent)

4. **Problem 4 Statement**

A fully inflated basketball has a radius of 12 cm. Your basketball is only inflated halfway. How many more cubic centimetres of air does your ball need to fully inflate? Express your answer in terms of π. Then estimate how many cubic centimetres this is by using 3.14 to approximate π.

Solution

$1,152\pi$ cubic cm, 3,617.28 cubic cm

5. **Problem 5 Statement**

Solve each of these equations. Explain or show your reasoning.

$2(3-2c)=30$

$31=5(b-2)$

$3x-2=7-6x$

Solution

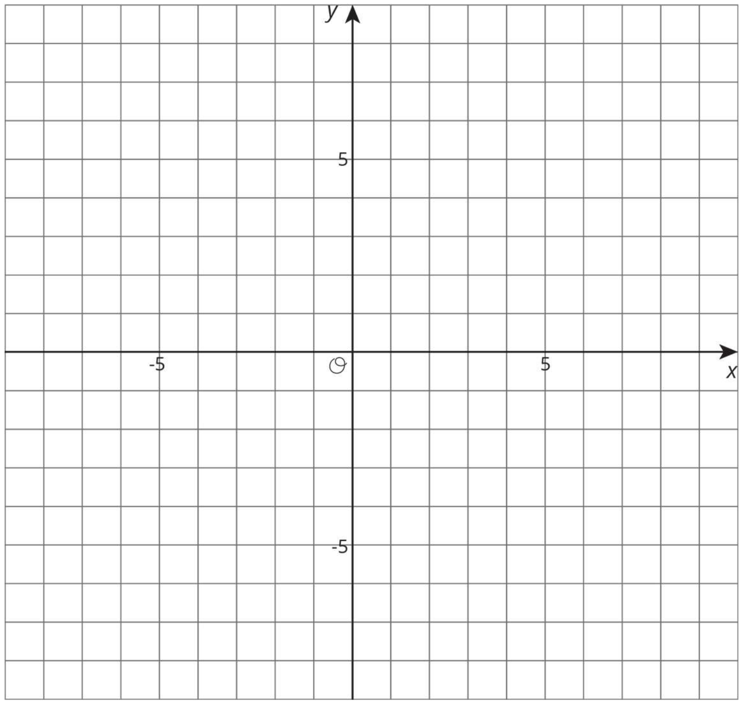
a. $c=$ -6. Responses vary. Sample response: Divide each side by 2, then subtract 3 from each side, then divide each side by -2.

b. $b=\frac{41}{5}$. Responses vary. Sample response: Distribute 5 on the right side, add 10 to each side, then divide each side by 5.

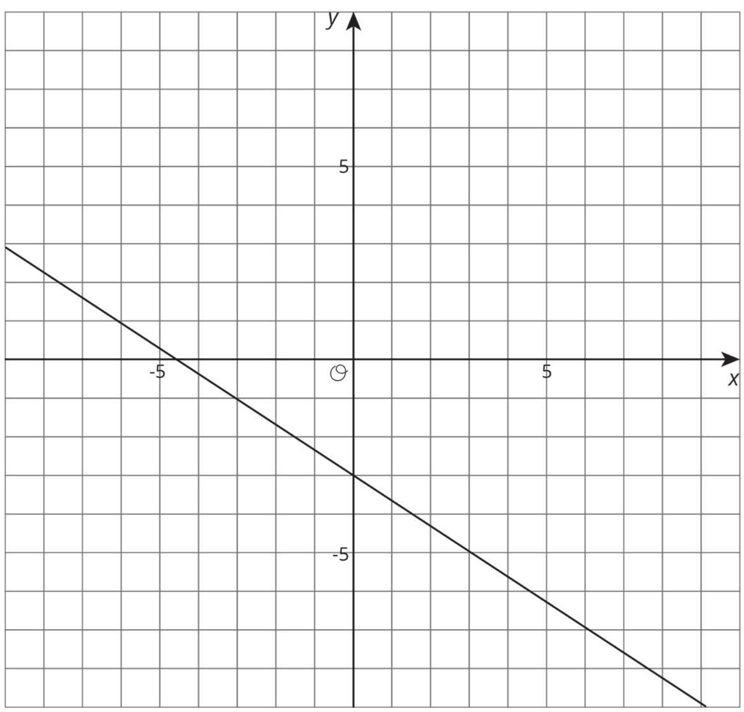
c. $x=1$. Responses vary. Sample response: Add 2 to each side, then add $6x$ to each side, then divide each side by 9.

6. **Problem 6 Statement**

Graph the line going through (-6,1) with a gradient of $\frac{-2}{3}$ and write its equation.



Solution



$y = \frac{-2}{3}x - 3$