# Lesson 10: Different options for solving one equation

## Goals

- Critique (orally and in writing) a given solution method for an equation of the form $p(x+q)=r$.
- Evaluate (orally) the usefulness of different approaches for solving a given equation of the form $p(x+q)=r$.
- Recognise that there are two common approaches for solving an equation of the form $p(x+q)=r$, i.e., expanding using the distributive property or dividing each side by p.

## Learning Targets

- For an equation like $3(x+2)=15$, I can solve it in two different ways: by first dividing each side by 3, or by first rewriting $3(x+2)$ using the distributive property.
- For equations with more than one way to solve, I can choose the easier way depending on the numbers in the equation.

## Lesson Narrative

The purpose of this lesson is to practise solving equations of the form $p(x+q)=r$, and to notice that one of the two ways of solving may be more efficient depending on the numbers in the equation.

### Addressing

- Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Building Towards

- Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Instructional Routines

- Algebra Talk
- Discussion Supports
- Think Pair Share

## Student Learning Goals

Let's think about which way is easier when we solve equations with parentheses.

# 10.1 Algebra Talk: Solve Each Equation

**Warm Up: 5 minutes**

The purpose of this Algebra Talk is to promote seeing structure in equations of the form $p(x+q)=r$. The goal is for students to see $(x-3)$ as a chunk of the equation. For each equation, the equation is true if $(x-3)$ is 10. These understandings help students develop fluency. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

### Instructional Routines

- Algebra Talk
- Discussion Supports

### Launch

Display one equation at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

*Representation: Internalise Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.
*Supports accessibility for: Memory; Organisation*

### Student Task Statement

$100(x-3) = 1\ 000$

$500(x-3) = 5\ 000$

$0.03(x-3) = 0.3$

$0.72(x+2) = 7.2$

### Student Response

- 13
- 13
- 13
- 8

### Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

It may help to display an equation like $100(x-3)=1\,000$ but cover the $(x-3)$ with your hand or with an eraser. "100 times something is 1 000. What is the something?"

*Speaking: Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ."
Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
*Design Principle(s): Optimise output (for explanation)*

## 10.2 Analysing Solution Methods

**15 minutes**

From previous work in the unit, students should already understand that distributing first is a valid solution method, though this activity reinforces that understanding. The purpose of this activity is to make explicit a common pitfall (in Noah's method). Monitor for students with different, valid reasons for agreeing or disagreeing. For example, disagree with Noah because . . .

- his answer $\frac{19}{2}$ doesn't make the original equation true.
- a bar model shows that adding 9 to each side does not result in a diagram that can be represented with $2x=19$.
- when you add 9 to the left side, you are adding it to $2x-18$ by the distributive property, which doesn't result in $2x$.

### Instructional Routines

- Discussion Supports
- Think Pair Share

## Launch

Arrange students in groups of 2. Give 5–10 minutes quiet work time and time to share their reasoning with their partner, followed by a whole-class discussion.

Explain to students that their job is to analyse three solution methods for errors. They should share with their partner whether they agree or disagree with each method, and explain why.

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "I agree/disagree with _____ because…" or "Instead of _____, he/she should have..." *Supports accessibility for: Language; Social-emotional skills*

## Anticipated Misconceptions

If students aren't sure how to begin analysing Noah's method, ask them to explain what it means for a number to be a solution of an equation. Alternatively, suggest that they draw a bar model of $2(x-9)=10$.

## Student Task Statement

Three students each attempted to solve the equation $2(x-9)=10$, but got different solutions. Here are their methods. Do you agree with any of their methods, and why?

Noah's method:

| | | |
|---|---|---|
| $2(x-9)$ | $=10$ | |
| $2(x-9)+9$ | $=10+9$ | add 9 to each side |
| $2x$ | $=19$ | |
| $2x \div 2$ | $=19 \div 2$ | divide each side by 2 |
| $x$ | $=\frac{19}{2}$ | |

Elena's method:

| | | |
|---|---|---|
| $2(x-9)$ | $=10$ | |
| $2x-18$ | $=10$ | apply the distributive property |
| $2x-18-18$ | $=10-18$ | subtract 18 from each side |
| $2x$ | $=-8$ | |
| $2x \div 2$ | $=-8 \div 2$ | divide each side by 2 |
| $x$ | $=-4$ | |

Andre's method:

$$\begin{aligned} 2(x-9) &= 10 && \\ 2x-18 &= 10 && \text{apply the distributive property} \\ 2x-18+18 &= 10+18 && \text{add 18 to each side} \\ 2x &= 28 && \\ 2x \div 2 &= 28 \div 2 && \text{divide each side by 2} \\ x &= 14 && \end{aligned}$$

### Student Response

Answers vary. Sample responses:

1. I disagree with Noah's method, because $2(x-9)+9$ is not $2x$. Noah should multiply out the brackets by the 2 before adding a number to each side.
2. I disagree with Elena's method, because $2x-18-18$ is $2x-36$, not $2x$. Instead of subtracting 18, it would be better to add 18.
3. I agree with Andre's method, because all of his moves are valid, and 14 makes the original equation true when substituted for $x$.

### Activity Synthesis

Invite students to share as many unique reasons they agree or disagree with each method as time allows. See the Activity Narrative for anticipated approaches. Pay particular attention to Noah's method, since this represents a common error.

*Speaking: Discussion Supports.* To help students produce statements that explain why they agree (or disagree) with the different solution methods, provide sentence frames such as "I notice ___ so I …," "That could be true because…" and "This method works because…" This will provide students with a structure to communicate their reasoning while promoting access to both content and language.
*Design Principle(s): Optimise output (for explanation)*

## 10.3 Solution Pathways

**15 minutes**

The purpose of this activity is to practice solving equations of the form $p(x+q)=r$, recognising that there are two valid approaches, and making judgments about which one is more sensible for a given equation.

### Instructional Routines

- Discussion Supports

## Launch

Display this equation and a balance diagram to match: $3(x+2)=21$. Tell students, “Any time you want to solve an equation in this form, you have a choice to make about how to proceed. You can either divide each side by 3 or you can multiply out the brackets.” Demonstrate each solution method side by side, while appealing to reasoning about the balance diagram.

Keep students in the same groups. 5–10 minutes of quiet or partner work time followed by a whole-class discussion.

*Representation: Internalise Comprehension*. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
*Supports accessibility for: Memory; Conceptual processing*

## Student Task Statement

For each equation, try to solve the equation using each method (dividing each side first, or applying the distributive property first by multiplying out the brackets). Some equations are easier to solve by one method than the other. When that is the case, stop doing the harder method and write down the reason you stopped.

1. $2\,000(x-0.03)=6\,000$

2. $2(x+1.25)=3.5$

3. $\frac{1}{4}(4+x)=\frac{4}{3}$

4. $-10(x-1.7)=-3$

5. $5.4=0.3(x+8)$

## Student Response

1. 3.03
2. 0.5
3. $\frac{4}{3}$ or equivalent
4. 2
5. 10

### Activity Synthesis

Reveal the solution to each equation and give students a few minutes to resolve any discrepancies with their partner.

Display the list of equations in the task, and ask students to help you label them with which solution method was easier, either "divide first" or "multiply out the brackets first." Discuss any disagreements and the reasons one method is easier than the other. (There is really no right or wrong answer here. Some people might prefer moves that eliminate fractions and decimals as early as possible. Some might want to minimise the number of computations.)

*Speaking: Discussion Supports.* Use this routine to help students produce mathematical language to communicate about which method is more efficient. Give groups of students 3–4 minutes to discuss which method would be easiest for each problem. Next, select groups to share how their solution methods minimise the number of computations needed, or address eliminating fractions and decimals. Consider providing sentence frames such as: "Dividing first was easier because ________, " or "Multiplying out the brackets first was easier because_____." Call on students to restate and/or revoice their peers' descriptions using mathematical language.
*Design Principle(s): Optimise output (for justification)*

## Lesson Synthesis

Possible questions for discussion:

- "What are the two main ways we can approach solving equations like the ones we saw today?" (divide first or multiply out the brackets first)
- "What kinds of things do we look for to decide which approach is better?" (powers of ten, operations that result in whole numbers, moves that will eliminate fractions or decimals)
- "How can we check if our answer is a solution to the original equation?" (Substitute our answer for the variable and see if it makes the equation true.)

# 10.4 Solve Two Equations

**Cool Down: 5 minutes**

### Student Task Statement

Solve each equation. Show or explain your method.

1. $8.88 = 4.44(x - 7)$
2. $5\left(y + \frac{2}{5}\right) = -13$

### Student Response

1. $x = 9$

2. $y$ = -3

## Student Lesson Summary

Equations can be solved in many ways. In this lesson, we focused on equations with a specific structure, and two specific ways to solve them.

Suppose we are trying to solve the equation $\frac{4}{5}(x+27)=16$. Two useful approaches are:

- divide each side by $\frac{4}{5}$
- apply the distributive property by multiplying out the brackets

In order to decide which approach is better, we can look at the numbers and think about which would be easier to compute. We notice that $\frac{4}{5} \times 27$ will be hard, because 27 isn't divisible by 5. But $16 \div \frac{4}{5}$ gives us $16 \times \frac{5}{4}$, and 16 is divisible by 4. Dividing each side by $\frac{4}{5}$ gives:

$$\begin{aligned} \frac{4}{5}(x+27) &= 16 \\ \frac{5}{4} \times \frac{4}{5}(x+27) &= 16 \times \frac{5}{4} \\ x+27 &= 20 \\ x &= \text{-}7 \end{aligned}$$

Sometimes the calculations are simpler if we first use the distributive property. Let's look at the equation $100(x+0.06)=21$. If we first divide each side by 100, we get $\frac{21}{100}$ or 0.21 on the right side of the equation. But if we use the distributive property first, we get an equation that only contains whole numbers.

$$\begin{aligned} 100(x+0.06) &= 21 \\ 100x+6 &= 21 \\ 100x &= 15 \\ x &= \frac{15}{100} \end{aligned}$$

## Lesson 10 Practice Problems

1. **Problem 1 Statement**

   Andre wants to buy a backpack. The normal price of the backpack is £40. He notices that a store that sells the backpack is having a 30% off sale. What is the sale price of the backpack?

   **Solution**

   £28

2. **Problem 2 Statement**

On the first maths exam, 16 students received an A grade.  On the second maths exam, 12 students received an A grade. What percentage decrease is that?

**Solution**

25% $(4 \div 16 = 0.25)$

3. **Problem 3 Statement**

Solve each equation.

a. $2(x - 3) = 14$

b. $-5(x - 1) = 40$

c. $12(x + 10) = 24$

d. $\frac{1}{6}(x + 6) = 11$

e. $\frac{5}{7}(x - 9) = 25$

**Solution**

a. 10

b. -7

c. -8

d. 60

e. 44

4. **Problem 4 Statement**

Select **all** expressions that represent a correct solution to the equation $6(x + 4) = 20$.

a. $(20 - 4) \div 6$

b. $\frac{1}{6}(20 - 4)$

c. $20 - 6 - 4$

d. $20 \div 6 - 4$

e. $\frac{1}{6}(20 - 24)$

f. $(20 - 24) \div 6$

**Solution** ["D", "E", "F"]

5. **Problem 5 Statement**

Lin and Noah are solving the equation $7(x + 2) = 91$.

Lin starts by using the distributive property to multiply out the brackets. Noah starts by dividing each side by 7.

a. Show what Lin's and Noah's full solution methods might look like.

b. What is the same and what is different about their methods?

**Solution**

Answers vary. Sample response:

- Lin's solution method: $7x + 14 = 91$, $7x = 77$, $x = 11$
- Noah's solution method: $x + 2 = 13$, $x = 11$

a. Both methods involve dividing by 7, but Noah does the division first, while Lin does the division last. Also, Lin's method involves subtracting 14, while Noah's method involves subtracting 2. Both solutions are correct and valid. Noah's solution could be considered more efficient for this example, because it takes fewer steps and has equally complicated arithmetic work.