Lesson 3: More costs of running a restaurant

Goals

- Create an equation to represent certain expenses for a restaurant, and interpret (orally and in writing) the solution.
- Determine whether a relationship is proportional and explain (orally) the reasoning.
- Determine whether a restaurant is making a profit using estimates of ongoing expenses, number of meals sold, average price per meal, and average cost per meal.

Lesson Narrative

This lesson is optional. In this lesson, students apply expressions and directed numbers to the context of balancing projected income and expenses for a restaurant. Students decide how to model these incomes and expenses in their calculations.

As with all lessons in this unit, all related topics have been addressed in prior units. This lesson provides an *optional* opportunity to go deeper and make connections between domains.

Addressing

- Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.
- Recognise and represent proportional relationships between quantities.
- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.
- Use measures of centre and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a year 8 science book are generally longer than the words in a chapter of a year 5 science book.

Instructional Routines

- Stronger and Clearer Each Time
- Discussion Supports
- Notice and Wonder

Student Learning Goals

Let's explore how much it costs to run a restaurant.

3.1 Are We Making Money?

Optional: 20 minutes

The purpose of this activity is for students to choose how they can apply maths concepts and strategies to a problem arising in a real-world context: predicting whether a restaurant will make a profit.

After students have estimated the monthly cost of their ongoing expenses on their list, poll the class on their total estimated monthly costs.

If students have completed the previous lesson about using spreadsheets to calculate the cost of ingredients for one serving of their recipe, they can use that as the basis for calculating the percentage of the markup. If students have not calculated the cost of ingredients in a previous lesson, then tell students that the *ingredients* for one serving of a meal typically cost about:

- £2 for a fast-food restaurant
- £5 for a casual sit-down restaurant
- £10 to £25 for a formal sit-down restaurant

Instructional Routines

- Stronger and Clearer Each Time

Launch

Point out that restaurants have many more expenses than just the cost of the food. Give students quiet work time followed by partner discussion.

Consider having students pause their work after the first question so that you can record their answers displayed for all to see, and all students will have access to this same information for the rest of the activity.

Speaking, Listening, Writing: Stronger and Clearer Each Time. Use this routine after students have assessed the profitability of restaurant A. Ask students to share their thinking with 2–3 consecutive partners. With each share, encourage listeners to push students for clarity in mathematical language (e.g., "How do you know the restaurant is making a profit?"). With each share, students' organisation of steps should get stronger and the explanations of reasoning should get clearer. This helps students to use mathematical language as they explain their profit analysis.
Design Principle(s): Optimise output (for explanation); Cultivate conversation

Student Task Statement

1. Restaurants have many more expenses than just the cost of the food.

    a. Make a list of other items you would have to spend money on if you were running a restaurant.

    b. Identify which expenses on your list depend on the number of meals ordered and which are independent of the number of meals ordered.

    c. Identify which of the expenses that are independent of the number of meals ordered only have to be paid once and which are ongoing.

    d. Estimate the monthly cost for each of the ongoing expenses on your list. Next, calculate the total of these monthly expenses.

2. Tell whether each restaurant is making a profit or losing money if they have to pay the amount you predicted in ongoing expenses per month. Organise your thinking so it can be followed by others.

    a. Restaurant A sells 6 000 meals in one month, at an average price of £17 per meal and an average cost of £4.60 per meal.

    b. Restaurant B sells 8 500 meals in one month, at an average price of £8 per meal and an average cost of £2.20 per meal.

    c. Restaurant C sells 4 800 meals in one month, at an average price of £29 per meal and an average cost of £6.90 per meal.

3.

    a. Predict how many meals your restaurant would sell in one month.

    b. How much money would you need to charge for each meal to be able to cover all the ongoing costs of running a restaurant?

4. What percentage of the cost of the ingredients is the markup on your meal?

Student Response

1. Answers vary. Sample response:

    - Expenses that depend on the number of meals sold: ingredients, disposable dishes or dish washing

    - One-time expenses: kitchen appliances, tables, chairs, decor, cash register, menus, uniforms, etc.

    - Other ongoing expenses that don't depend on the number of meals sold:

expense	estimated cost

	(pounds per month)
rent	2 000
maintenance	400
phone	100
utilities	500
insurance	300
payroll services	100
salaries	8 000
permits	150
cleaning/laundry	500
total fixed expenses	12 050

2. Answers vary depending on the estimated monthly operating costs.

    a. Restaurant A makes a profit if their total fixed expenses are less than £74 400 per month, because $(17 + \text{-}4.60) \times 6\,000 = 74\,400$.

    b. Restaurant B makes a profit if their total fixed expenses are less than £49 300 per month, because $(8 + \text{-}2.20) \times 8\,500 = 49\,300$.

    c. Restaurant C makes a profit if their total fixed expenses are less than £106 080 per month, because $(29 + \text{-}6.90) \times 4\,800 = 106\,080$.

3. Answers vary. Sample response:

    a. About 6 000 meals per month

    b. At least £7.00, because the inequality $6\,000(x + \text{-}5) > 12\,050$ can represent the average menu price x for which the restaurant will make a profit.

4. Answers vary. Sample response: At least 40%, because $7 \div 5 = 1.4$, which means the menu price needs to be at least 140% of the cost of the ingredients to have enough money to cover the restaurant's other operating costs.

Activity Synthesis

Poll the class on whether they think each of the restaurants A, B, and C made money for the month. Select students to share their reasoning.

Note: The way the problem is written, there is not just one correct answer to the question. Whether or not each restaurant made money depends on how much the students estimated for the total monthly expenses (excluding food).

if students estimated the total ongoing expenses (excluding food) to	restaurants that would have made money	restaurants that would have lost money

be:		
below £49 300	A, B, and C	none
between £49 300 and £74 400	A and C	B
between £74 400 and £106 080	C	A and B
above £106 080	none	A, B, and C

Next, ask students to trade with a partner, and check their work for calculating the percentage of the mark up.

Ask students to discuss:

- "How does the amount you plan to charge for each meal compare to your partner's amount?"
- "How does your markup percentage compare to your partner's percentage?"

Consider telling students that many restaurant owners use 300% as an estimate of a good percentage for the mark up on their meals to be able to make a profit. That means the price of the meal is 4 times what the cost of the ingredients were.

3.2 Disposable or Reusable?

Optional: 20 minutes

The purpose of this activity is for students to write and solve equations as a strategy to compare the projected costs of using reusable versus disposable plates and forks. First, students examine dot plots representing the average number of customers served per day at a sample of restaurants to make a prediction about how many customers they might serve per day. Then, students see that the cost of buying disposable plates and forks can be modelled with a proportional relationship, while the cost of buying and washing reusable plates and forks can be modelled with an equation in the form $px + q = r$.

Instructional Routines

- Discussion Supports
- Notice and Wonder

Launch

Display the dot plots about the number of customers served per day. Invite students to share what they notice and wonder.

Some things students may notice:

- Many of the fast food restaurants serve more customers per day than the full service restaurants.
- There is a lot of overlap between the two distributions, from 300 to 600 customers.

Some things students may wonder:

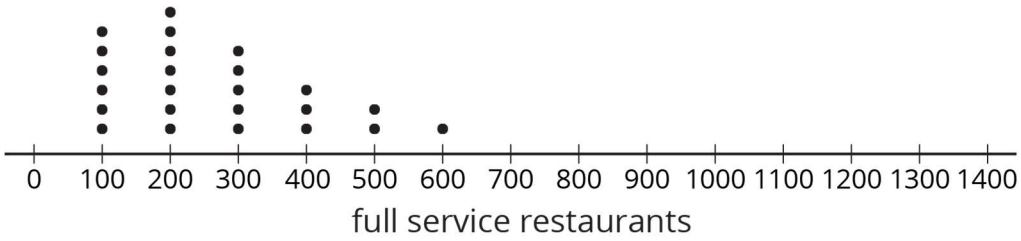
- Were the restaurants included in the samples selected at random?
- Is there a meaningful difference between the average number of customers served at these two types of restaurants?
- About how many customers would my restaurant serve per day?

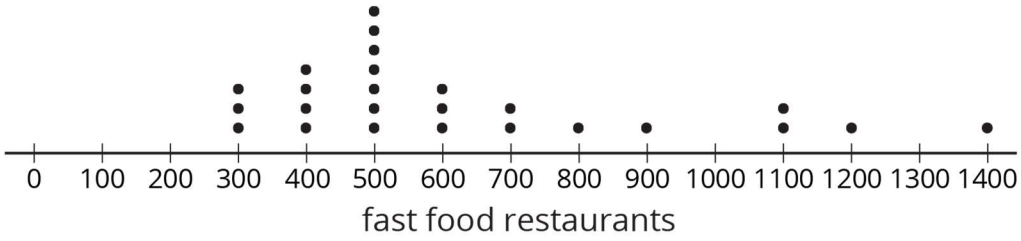
Give students quiet work time followed by whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. After students have solved the first 2-3 problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organisation; Attention

Student Task Statement

A sample of full service restaurants and a sample of fast food restaurants were surveyed about the average number of customers they serve per day.





1. How does the average number of customers served per day at a full service restaurant generally compare to the number served at a fast food restaurant? Explain your reasoning.
2. About how many customers do you think your restaurant will serve per day? Explain your reasoning.

3. Here are prices for plates and forks:

	plates	forks
disposable	165 paper plates for £12.50	600 plastic forks for £10
reusable	12 ceramic plates for £28.80	24 metal forks for £30

   a. Using your predicted number of customers per day from the previous question, write an equation for the total cost, d, of using disposable plates and forks for every customer for n days.

   b. Is d proportional to n? Explain your reasoning.

   c. Use your equation to predict the cost of using disposable plates and forks for 1 year. Explain any assumptions you make with this calculation.

   d. How much would it cost to buy enough reusable plates and forks for your predicted number of customers per day?

   e. If it costs £10.75 a day to wash the reusable plates and forks, write an expression that represents the total cost, r, of buying and washing reusable plates and forks after n days.

   f. Is r proportional to n? Explain your reasoning.

   g. How many days can you use the reusable plates and forks for the same cost that you calculated for using disposable plates and forks for 1 year?

Student Response

1. Fast food restaurants generally have more customers per day than full-service restaurants, but there is a lot of overlap. The difference in means is about 366 customers, which small compared with the ranges.

2. Answers vary. Sample response: 240 customers per day because 240 is close to the centre of the distribution for full-service restaurants, and I think my restaurant will be a typical full-service restaurant.

3. Answers vary. Sample response for 240 customers per day:

   a. $d = 22.18n$

   b. The equation represents a proportional relationship because it is in the form $y = kx$, and the constant of proportionality is 22.18. However, this was using an average of 240 customers per day, and in real life the restaurant serves a different number of people each day, so it could be close to proportional, but not exactly.

c. Assuming the restaurant is open 365 days in the year and serves an average of 240 customers per day, it would cost £8 095.70 to use disposable plates and forks for every customer. This also assumes that each customer uses exactly one plate and one fork.

d. £876 to buy 240 reusable plates and 240 reusable forks

e. $10.75d + 876 = r$

f. No, this equation does not represent a proportional relationship. It cannot be rewritten in the form $y = kx$ and if graphed, it would go through the point (0, 876) instead of (0, 0).

g. If $10.75d + 876 = 8\,095.70$, then $d = 671.6$, which is about 1 year and 10 months.

Activity Synthesis

Poll the class on how many days they can you use the reusable plates and forks for the same cost as using disposable plates and forks for 1 year. Select students to share what this tells us about the situation. (If their answer is greater than the number of days they planned for their restaurant to be open during the year, then this means that buying and washing reusable plates and forks is cheaper than using disposable plates and forks.)

Select students to share their reasoning about whether there is a proportional relationship between the cost of using disposable or reusable and the number of days.

Students might share the following ideas:

- The relationship for the cost of using disposable looks like it is proportional because we wrote it in the form $y = kx$.
  - The relationship for the cost of using disposable would be close, but not exactly proportional, because the equation is assuming an average number of customers per day and in real life the restaurant serves a different number of people each day.
- The relationship for the cost of using reusable is not proportional because:
  - there is a start-up cost of buying the reusable plates and forks.
  - the equation cannot be written in the form $y = kx$. There has to be a term that is added that represents the start-up costs.
  - if we graphed the relationship, it would not go through the origin, but would cross the y-axis at a point that represents the start-up costs.

Speaking: Discussion Supports. As students share their reasoning about whether there is a proportional relationship between the cost of using disposable or reusable and the number of days, provide sentence frames to support their justifications, such as, "I know ___ because

. . . .” Press for details in students’ explanations by asking students to elaborate on an idea (e.g., “Why did you...?” and “Can you give an example of . . . ?”). This will help students communicate clearly and use more precise language.
Design Principle(s): Optimise output (for justification)