Lesson 4: Money and debts

Goals

- Apply addition of directed numbers to calculate an account balance after a deposit or withdrawal, and explain (orally and using other representations) the solution method.
- Explain (orally and in writing) how directed numbers can be used to represent situations involving money, including deposits or withdrawals, and assets or debts.
- Write an equation with an unknown addend to represent a situation where the amount of change is unknown.

Learning Targets

- I understand what positive and negative numbers mean in a situation involving money.

Lesson Narrative

In this lesson, students are introduced to using negative numbers in the context of money to represent debts or debits.

It is common to use money contexts to represent directed numbers. One point that often gets overlooked is that it is a *convention* that we do this, rather than a necessity. Any situation in which we use a negative number to represent a debt (for example), we could equally well just use a positive number and distinguish it by calling it a debt. The reason we use directed numbers in this context is that it allows us to represent a whole class of problems with the same expression. For example, if a person has £50 in the bank and writes a £20 cheque, we can represent the balance as $50 - 20$. If they had written an £80 cheque, we can still write the balance as $50 - 80$, as long as we have adopted the convention that negative numbers represent what the person owes the bank (and assuming the bank allows overdrafts). Using a mathematical structure (the directed numbers) to represent a context (a current account balance) is an example of modelling with mathematics.

Building On

- Apply and extend previous understanding of numbers to the system of rational numbers.

Addressing

- Apply and extend previous understanding of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Building Towards

- Apply and extend previous understanding of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Think Pair Share

Required Preparation

This lesson presents opportunities to practise performing operations on directed values, but the emphasis is really on noticing that money can be represented with positive and negative values. If the computation requirements might get in the way of that understanding, consider providing access to calculators.

Student Learning Goals

Let's apply what we know about directed numbers to money.

4.1 Concert Tickets

Warm Up: 10 minutes

There are many ways to think about debt, and the way the lender views it differs from the way from the borrower does. This warm-up introduces the idea that we can represent a debt with a directed number. From the perspective of the person who owes money, the debt is usually viewed as a negative number. From the perspective of the bank, it may be viewed as a positive number.

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Ask students,

"Priya is buying concert tickets for her and her friends with the money she earns at her part-time job. This month, she has earned £135. Can she buy three £50 tickets for a concert?"

Ask students to discuss with a partner for 1 minute.

Explain that sometimes we can borrow money from a bank to buy things we cannot afford at the time, and then pay the money back to the bank in the future. Give students time for partner discussion followed by whole-class discussion.

Student Task Statement

Priya wants to buy three tickets for a concert. She has earned £135 and each ticket costs £50. She borrows the rest of the money she needs from a bank and buys the tickets.

1. How can you represent the amount of money that Priya has after buying the tickets?
2. How much more money will Priya need to earn to pay back the money she borrowed from the bank?
3. How much money will she have after she pays back the money she borrowed from the bank?

Student Response

Answers vary. Sample response:

1. Since Priya owes the bank money, this could be represented by a negative number, -£15.
2. Priya needs to earn £15 more. Sample explanations:
   - Because $150 - 135 = 15$.
   - Because $-15 + 15 = 0$.
3. She will have £0 after she pays the money back.

Activity Synthesis

The most important thing for students to understand is that when representing a debt with a negative number, the additive inverse tells how much money is needed to pay off the debt. If Priya has -£15, then she needs £15 more to raise her balance back to £0.

If no students suggest that we can represent Priya's money using a negative number, introduce the idea.

4.2 Cafeteria Food Debt

10 minutes

In this activity, students solve problems about debts that can be represented with addition and subtraction equations. Some problems ask students to calculate the balance after the transaction and some questions ask students to calculate the amount of the transaction, given the starting and ending balances. Students draw number lines to represent each problem.

The series of questions involves a running balance after deposits and withdrawals are made. Students may represent this by drawing a new diagram for each question, or by adding on to the same diagram. Either approach will work.

The focus in this activity is on writing a new equation to represent each situation, and on creating a diagram to represent the situation. If students struggle, encourage them to think about what they have already learned about adding and subtracting integers, and assure them that they can use that understanding to reason about money. If students might struggle with the computations, consider providing access to a calculator to take the focus off of computation.

Instructional Routines

- Discussion Supports

Launch

Remind students that a deposit is money paid into an account. If students do not read carefully, they may not realise that they are expected to write an equation and create a diagram for each question, and only record a numerical answer. Ensure they understand what they are expected to do before they begin working.

Give students quiet work time followed by whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts for students who benefit from support with organisational skills in problem solving. Give students time to work on the first 1-2 transactions before checking in. Invite 1-2 students to think aloud and share how they came up with an equation and how they represented the transaction on a number line. Record their thinking on a display chart and keep the work visible as students continue to work.
Supports accessibility for: Organisation; Attention

Anticipated Misconceptions

Some students may struggle to write an equation for each problem. Prompt them to identify what amount is unknown in each situation.

Student Task Statement

At the beginning of the month Kiran had £24 in his school cafeteria account. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then, represent each transaction on a number line. What is the unknown quantity in each case?

1. In the first week he spent £16 on lunches. How much was in his account then?
2. Then he deposited some more money and his account balance was £28. How much did he deposit?
3. Then he spent £34 on lunches the next week. How much was in his account then?
4. Then he deposited enough money to pay off his debt to the cafeteria. How much did he deposit?

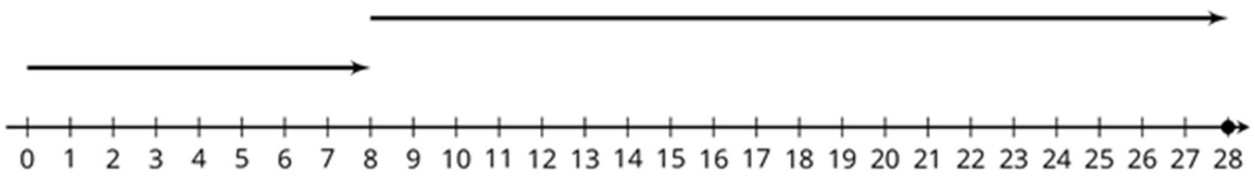
5. Explain why it makes sense to use a negative number to represent Kiran's account balance when he owes money.

Student Response

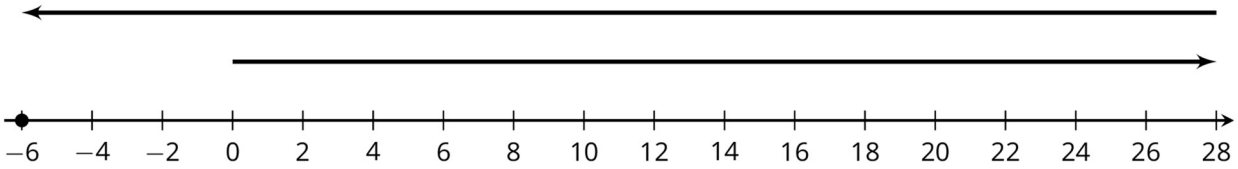
Answers vary. Sample responses:

1. $24 + (-16) = n; n = 8$

2. $8 + m = 28; m = 20$



3. $28 + (-34) = p; p = -6$



4. $(-6) + q = 0; q = 6$

5. It makes sense that his balance is negative because he must pay back a positive amount of money to get to zero and stop owing money.

Activity Synthesis

The most important thing for students to understand is that all the rules they have learned for adding and subtracting directed numbers still work when applied to the context of negative amounts of money.

Review each of the following types of computations and discuss how they apply to the school cafeteria situations:

- Adding numbers with the same sign
- Adding numbers with opposite signs
- Adding opposites makes 0
- Subtracting as addition with a missing addend
- Subtracting as adding the additive inverse

Speaking, Listening: Discussion Supports. Demonstrate the use of mathematical language to describe thinking. For example, "If Kiran begins the month with £24, I know that he starts with a positive amount. I know he spends £16, and spending money is subtraction or negative. To create an equation, I need both values." To provide an opportunity for both listening and speaking, ask students, "Who can restate my reasoning in a different way?" This helps invite more student participation and meta-awareness of language and reasoning.
Design Principle(s): Optimise output (for explanation)

4.3 Bank Statement

10 minutes

In this activity, students see that withdrawals, in addition to debts, can also be represented using negative numbers. Students continue using addition and subtraction to solve problems about debt. While solving the last problem, students may begin wondering about multiplying and dividing directed numbers, which will be addressed in the next several lessons.

You may wish to ask students to pause after the first question for discussion. The decision about which numbers to represent with positive versus negative values hinges on whether you are thinking from the perspective of the person or the perspective of the account. Point out that the final balance is represented with a negative number to show that the person owes the bank money (this should be brought out in the launch). Therefore, from the perspective of the account, deposits are positive values and withdrawals are negative values. It would be possible to proceed either way, but it will facilitate discussion later if everyone uses the same convention as a result of work on the first question.

As students work, monitor for students who are expressing their reasoning as addition and subtraction equations or expressions.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Co-Craft Questions

Launch

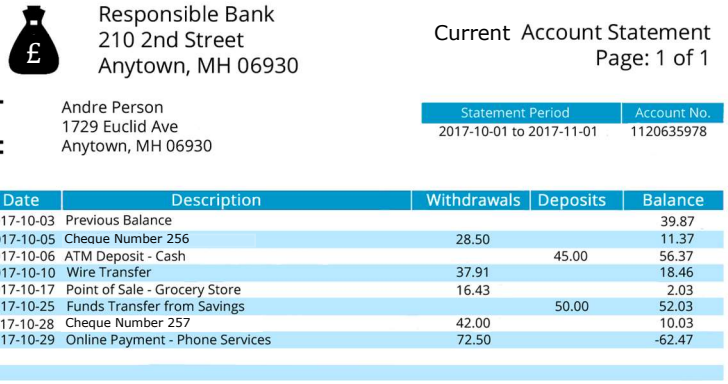
Display the bank statement image for all to see, without the questions. Ask students to think of two things they notice and two things they wonder. Give students 1 minute of quiet think time. Select a few students to share. Make sure students understand the meaning of *deposit* and *withdrawal*.

Give students quiet work time followed by whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity. *Supports accessibility for: Memory; Conceptual processing Reading, Writing: Co-craft Questions.* To support students in making sense of the bank statement, display the image of the bank statement without the remaining text of the problem. Clarify the meaning of the terms in the bank statement (e.g., deposit, withdrawal, balance) in this context. Note that the word "balance" has an everyday meaning ("I fell down when I lost my balance"). Then ask pairs of students to write down possible mathematical questions that might be asked about the situation. Invite pairs to share with the whole class. Highlight questions that relate to calculating a balance based on a given deposit or withdrawal. This will allow students to use mathematical language related to a bank statement as they reason about the accounting situation.
Design Principle(s): Cultivate conversation; Support sense-making

Student Task Statement

Here is a bank statement.



Responsible Bank
210 2nd Street
Anytown, MH 06930

Current Account Statement
Page: 1 of 1

Andre Person
1729 Euclid Ave
Anytown, MH 06930

Statement Period	Account No.
2017-10-01 to 2017-11-01	1120635978

Date	Description	Withdrawals	Deposits	Balance
2017-10-03	Previous Balance			39.87
2017-10-05	Cheque Number 256	28.50		11.37
2017-10-06	ATM Deposit - Cash		45.00	56.37
2017-10-10	Wire Transfer	37.91		18.46
2017-10-17	Point of Sale - Grocery Store	16.43		2.03
2017-10-25	Funds Transfer from Savings		50.00	52.03
2017-10-28	Cheque Number 257	42.00		10.03
2017-10-29	Online Payment - Phone Services	72.50		-62.47

1. If we put withdrawals and deposits in the same column, how can they be represented?
2. Andre withdraws £40 to buy a music player. What is his new balance?
3. If Andre deposits £100 in this account, will he still be in debt? How do you know?

Student Response

1. We could use negative numbers to represent the withdrawals.
2. Andre's new balance will be -£102.47, because $-62.47 + (-40) = -102.47$.

3. Yes, he will still be in debt. Sample explanations:
   - Because $-102.47 + 100 = -2.47$
   - Because $|-102.47| > |100|$

Are You Ready for More?

The *national debt* of a country is the total amount of money the government of that country owes. Imagine everyone in the United Kingdom was asked to help pay off the national debt. How much would each person have to pay?

Student Response

Answers vary as the population and national debt of the United Kingdom changes. If the population of the United Kingdom is about 68 million and the national debt is about £2.2 trillion, each person would have to pay about £32 000.

Activity Synthesis

The most important thing for students to understand is that the rules for adding and subtracting directed numbers can help them solve problems about debts.

Select students to share their solutions. Ask students to indicate whether they agree, disagree, or have any clarifying questions.

Lesson Synthesis

Main learning points:

- We can use positive numbers to represent payments into a bank account (deposits) and negative numbers to represent money taken out of an account (withdrawals).
- We can also use a negative balance to represent debt (owing money).
- We can use the additive inverse to quickly find how much money is needed to reach a balance of zero.

Discussion questions:

- What words do we use to mean "money added into" or "money taken out of" an account?
- How can we represent owing money?
- Why does it make sense to use negative numbers to represent debt?
- How can we tell how much money is needed to pay off a debt?

4.4 Buying a Bike

Cool Down: 5 minutes

Student Task Statement

1. Clare has £150 in her bank account. She buys a bike for £200. What is Clare's account balance now?

2. If Clare earns £75 the next week from delivering newspapers and deposits it in her account, what will her account balance be then?

Student Response

1. Clare's balance is -£50.

2. Clare's new balance is £25.

Student Lesson Summary

Banks use positive numbers to represent money that gets put into an account and negative numbers to represent money that gets taken out of an account. When you put money into an account, it is called a **deposit**. When you take money out of an account, it is called a **withdrawal**.

People also use negative numbers to represent debt. If you take out more money from your account than you put in, then you owe the bank money, and your account balance will be a negative number to represent that debt. For example, if you have £200 in your bank account, and then you write a cheque for £300, you will owe the bank £100 and your account balance will be -£100.

starting balance	deposits and withdrawals	new balance
0	50	0 + 50
50	150	50 + 150
200	-300	200 + (-300)
-100		

In general, you can find a new account balance by adding the value of the deposit or withdrawal to it. You can also tell quickly how much money is needed to repay a debt using the fact that to get to zero from a negative value you need to add its opposite.

Glossary

- deposit
- withdrawal

Lesson 4 Practice Problems

1. **Problem 1 Statement**

   The table shows five transactions and the resulting account balance in a bank account, except some numbers are missing. Fill in the missing numbers.

	transaction amount	account balance
transaction 1	200	200
transaction 2	-147	53
transaction 3	90	
transaction 4	-229	
transaction 5		0

Solution

	transaction amount	account balance
transaction 1	200	200
transaction 2	-147	53
transaction 3	90	143
transaction 4	-229	-86
transaction 5	86	0

2. **Problem 2 Statement**

   a. Clare has £54 in her bank account. A store credits her account with a £10 refund. How much does she now have in the bank?

   b. Mai's bank account is overdrawn by £60, which means her balance is -£60. She gets £85 for her birthday and deposits it into her account. How much does she now have in the bank?

   c. Tyler is overdrawn at the bank by £180. He gets £70 for his birthday and deposits it. What is his account balance now?

   d. Andre has £37 in his bank account and writes a cheque for £87. After the cheque has been cashed, what will the bank balance show?

Solution

a. £64 because $54 + 10 = 64$

b. £25 because $-60 + 85 = 25$

c. -£110 because $-180 + 70 = -110$

d. -£50 because $37 - 87 = -50$

3. **Problem 3 Statement**

Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select **all** that apply.

a. $g - 0.05$

b. $g - 0.05g$

c. $0.95g$

d. $0.05g$

e. $(1 - 0.05)g$

Solution ["B", "C", "E"]

4. **Problem 4 Statement**

Decide whether or not each equation represents a proportional relationship.

a. Volume measured in cups (c) vs. the same volume measured in ounces (z): $c = \frac{1}{8}z$

b. Area of a square (A) vs. the side length of the square (s): $A = s^2$

c. Perimeter of an equilateral triangle (P) vs. the side length of the triangle (s): $3s = P$

d. Length (L) vs. width (w) for a rectangle whose area is 60 square units: $L = \frac{60}{w}$

Solution

a. yes

b. no

c. yes

d. no

5. **Problem 5 Statement**

Add.

a. $5\frac{3}{4} + (-\frac{1}{4})$

b. $-\frac{2}{3} + \frac{1}{6}$

c. $-\frac{8}{5} + (-\frac{3}{4})$

Solution

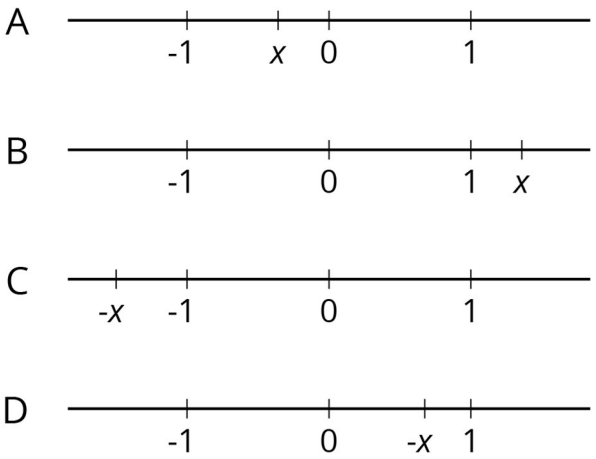
a. $5\frac{1}{2}$

b. $\frac{-3}{6}$ or $\frac{-1}{2}$

c. $\frac{-47}{20}$

6. **Problem 6 Statement**

In each diagram, x represents a different value.



For each diagram,

a. What is something that is *definitely* true about the value of x?

b. What is something that *could be* true about the value of x?

Solution

Answers vary. Sample responses:

Diagram A:

a. The value of x is definitely negative. The value of x is definitely greater than $\frac{-1}{2}$ and less than 0, since x is closer to 0 than it is to -1.

b. The value of x could be $\frac{-1}{3}$ or -0.4; these values are negative and between 0 and $\frac{-1}{2}$.

Diagram B:

a. The value of x is definitely positive. The value of x is definitely between 1 and 2, since x is a little greater than 1.

b. The value of x could be 1.4 or $1\frac{1}{3}$; these values are positive and between 1 and 1.5.

Diagram C:

a. The value of x is definitely positive, because $-x$ is negative (for example, $-(-1.5) = 1.5$). The value of x is definitely between 1 and 2, since the distance of $-x$ from 0 is a little greater than 1.

b. The value of x could be $1\frac{1}{2}$ or 1.4; these values are halfway (or a little less than halfway) between 1 and 2.

Diagram D:

a. The value of x is definitely negative. The value of x is definitely less than $\frac{-1}{2}$ and greater than -1, since $-x$ is farther away from 0 than it is from 1.

b. The value of x could be -0.7 or $\frac{-2}{3}$; these values are between 0 and -1 but closer to -1.