Strand 3: Number

Strand 3 further develops the proficiency learners have gained through their study of strand 3 at junior cycle. Learners continue to make meaning of the operations of addition, subtraction, multiplication and division of whole and rational numbers and extend this sense-making to complex numbers.

They extend their work on proof and become more proficient at using algebraic notation and the laws of arithmetic and induction to show that something is always true. They utilise a number of tools: a sophisticated understanding of proportionality, rules of logarithms, rules of indices and 2D representations of 3D solids to solve single and multi-step problems in numerous contexts.

As they engage with this strand and make connections across other strands, learners develop and reinforce their synthesis and problem-solving skills.

At each syllabus level students should be able to

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems	– recognise irrational numbers and appreciate that **R** ≠ **Q** – work with irrational numbers – revisit the operations of addition, multiplication, subtraction and division in the following domains: • **N** of natural numbers • **Z** of integers • **Q** of rational numbers • **R** of real numbers and represent these numbers on a number line – investigate the operations of addition, multiplication, subtraction and division with complex numbers **C** in rectangular form a+ib – illustrate complex numbers on an Argand diagram – interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate – develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place-value understanding – consolidate their understanding of factors, multiples, prime numbers in **N** – express numbers in terms of their prime factors – appreciate the order of operations, including brackets – express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form	– geometrically construct √2 and √3 – prove that √2 is not rational – calculate conjugates of sums and products of complex numbers – verify and justify formulae from number patterns – investigate geometric sequences and series – prove by induction • simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series • simple inequalities such as $n! > 2^n$, $2^n \geq n^2$ $(n \geq 4)$ $(1+x)^n \geq 1+nx$ $(x > -1)$ • factorisation results such as 3 is a factor of $4^n - 1$ – apply the rules for sums, products, quotients of limits – find by inspection the limits of sequences such as $\lim_{n\to\infty} \frac{n}{n+1}$; $\lim_{n\to\infty} r^n$, $\lvert r\rvert < 1$ – solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment – derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems (continued)	– appreciate that processes can generate sequences of numbers or objects – investigate patterns among these sequences – use patterns to continue the sequence – generalise and explain patterns and relationships in algebraic form – recognise whether a sequence is arithmetic, geometric or neither – find the sum to n terms of an arithmetic series	
3.2 Indices	– solve problems using the rules for indices (where $a, b \in \mathbf{R}$; $p, q \in \mathbf{Q}$; $a^p, a^q \in \mathbf{Q}$; $a, b \neq \mathbf{0}$): • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $a^0 = 1$ • $(a^p)^q = a^{pq}$ • $a^{\frac{1}{q}} = \sqrt[q]{a} \quad q \in \mathbf{Z},\ q \neq 0, a > 0$ • $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p \ p, q \in \mathbf{Z},\ q \neq 0, a > 0$ • $a^{-p} = \frac{1}{a^p}$ • $(ab)^p = a^p b^p$ • $(\frac{a}{b})^p = \frac{a^p}{b^p}$	– solve problems using the rules of logarithms • $\log_a(xy) = \log_a x + \log_a y$ • $\log_a(\frac{x}{y}) = \log_a x - \log_a y$ • $\log_a x^q = q \log_a x$ • $\log_a a = 1$ and $\log_a 1 = 0$ • $\log_a x = \frac{\log_b x}{\log_b a}$

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.3 Arithmetic	– check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result – accumulate error (by addition or subtraction only) – make and justify estimates and approximations of calculations; calculate percentage error and tolerance – calculate average rates of change (with respect to time) – solve problems that involve • calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price) • compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions) • costing: materials, labour and wastage • metric system; change of units; everyday imperial units (conversion factors provided for imperial units) – make estimates of measures in the physical world around them	– use *present value* when solving problems involving loan repayments and investments
3.4 Length, area and volume	– investigate the nets of prisms, cylinders and cones – solve problems involving the length of the perimeter and the area of plane figures: disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs, and figures made from combinations of these – solve problems involving surface area and volume of the following solid figures: rectangular block, cylinder, right cone, triangular-based prism (right angle, isosceles and equilateral), sphere, hemisphere, and solids made from combinations of these – use the trapezoidal rule to approximate area	