13.10 Ejercicios

- Utilizar multiplicadores de Lagrange para hallar el extremo indicado, suponer que x y y son positivos.

5. Minimizar $F(x,y) = x^2 + y^2$
Restricción: $x + 2y - 5 = 0$

$\nabla F = \lambda \nabla g$

$\langle 2x, 2y \rangle = \lambda \langle 1, 2 \rangle$

$2x i + 2y j = \lambda (i + 2j)$

$$\left.\begin{array}{l} 2x = \lambda \\ 2y = 2\lambda \end{array}\right\} \begin{array}{l} x = \lambda/2 \\ y = \lambda \end{array}$$

$x + 2y - 5 = 0$

$\dfrac{\lambda}{2} + 2\lambda = 5 \rightarrow \lambda = 2$

$x = 1$

$y = 2$

$F(1,2) = 1^2 + 2^2$

$= 1 + 4$

$= 5$

7) Maximizar $F(x,y) = 2x + 2xy + y$
Restricción : $2x + y = 100$

$\nabla F = \lambda \nabla g$
$\langle 2x, 2y, 2z \rangle = \lambda \langle 1, 1, 1 \rangle$
$(2+2y)i + (2x+1)j = 2\lambda i + \lambda j$

$$\left.\begin{array}{l} 2+2y = 2\lambda \rightarrow y = \lambda - 1 \\ 2x+1 = \lambda \rightarrow x = \dfrac{\lambda-1}{2} \end{array}\right\} y = 2x$$

$2x + y = 100$
$\frac{\cancel{4}x}{\cancel{4}} = \frac{100}{4}$
$\boxed{x = 25}$,

$2 + 2y = 2(2x+1)$
$2 + y = 4x + 2$
$2y = 4x$
$y = 2x \rightarrow y = 2(25)$
$y = 50$

$Pc\,(25, 50)$

$F(25,50) = 2(25) + 2(25)(50) + 50$
$= 50 + 2500 + 50$
$= 2600 > 0$

- Utilizar los multiplicadores de Lagrange para hallar los extremos indicados suponiendo que x, y ∧ z son positivo.

(11) Minimizar $F(x,y,z) = x^2+y^2+z^2$

Restricción : $x+y+z-9=0.$

$$x+y+z=9$$

$$\left.\begin{aligned} 2x &= \lambda \\ 2y &= \lambda \\ 2z &= \lambda \end{aligned}\right\} x = y = z$$

$$x+x+x-9=0$$

$$\frac{3x}{3}-\frac{9}{3}=0 \qquad P(3,3,3)$$

$$x-3=0 \rightarrow \boxed{x=3} \rightarrow \boxed{y=3} \rightarrow \boxed{z=3}$$

$$F(3,3,3) = 3^2+3^2+3^2 = 9+9+9 = \boxed{27}$$

$$F(3,3,3)=27$$

$$F(1) = 3(1)^2 = \boxed{3 < 27}$$

• Utilizar los multiplicadores de Lagrange para hallar todos los extremos de f indicados sujetos a dos restricción. Indicar x, y ∧ z son no negativos.

(17) Maximizar $f(x,y,z) = xyz$
Restricción $x+y+z=32$, $x-y+z=0$.

$$\nabla f = \lambda \nabla g + \mu \nabla h$$

$$yz\,i + xz\,j + xy\,k = \lambda(i+j+k) + \mu(i-j+k).$$

$$\left.\begin{array}{l} yz = \lambda + \mu \\ xz = \lambda + \mu \\ xy = \lambda + \mu \end{array}\right\} yz = xy \rightarrow x = z.$$

$$\left.\begin{array}{l} x+y+z = 32 \\ x-y+z = 0 \end{array}\right\} 2x + 2z = 32$$

$$x = z = 8$$

$$y = 16$$

$P(8,8,16)$

$F(8,8,16) = 8(8)(16) = 1024.$