Lesson 15: Making and measuring boxes

Goals

- Apply operations with decimals to calculate the surface area of paper boxes.
- Describe (orally) sources of measurement error, and justify an appropriate level of precision for reporting the answer.
- Measure and compare (orally and in writing) the dimensions of paper boxes.

Learning Targets

- I can use the four operations on decimals to find surface areas and reason about real-world problems.

Lesson Narrative

In this optional culminating lesson of the unit, students construct open-top origami boxes by folding different-size square paper. Before folding, they make conjectures about how the paper size affects the length and area measurements of the boxes. Later, they test their conjectures by finding and analysing those measurements. While arithmetic operations on decimals are central to this work, students also build on their geometric work from earlier units. As they investigate the relationship between the side lengths of the origami paper and the edge lengths of the boxes, they also connect to their work on ratios.

This lesson is organised into two parts:

- Part 1: Measure, predict, and fold. Students carefully measure the sheets of square paper, predict the measurements of the boxes created from different-size sheets, and fold their paper into a box.
- Part 2: Measure, calculate, and compare. Students carefully measure the dimensions of the boxes, calculate their surface areas, and then compare the sizes of the boxes. They also reflect on the accuracy of their predictions from Part 1.

Depending on the instructional choices made, this lesson could take one or more class meetings. The time estimates are intentionally left blank because the amount of time needed might vary depending on factors such as:

- The size of the class.
- How familiar students are folding paper into shapes.
- How student work is ultimately shared with the class (not at all, informally, or with formal presentations).

Consider defining the scope of work further for students and setting a time limit for each part of the activity to focus students' work and optimise class time.

Addressing

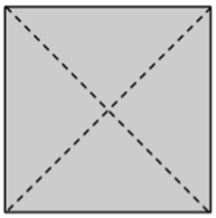
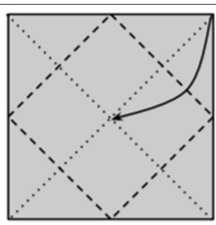
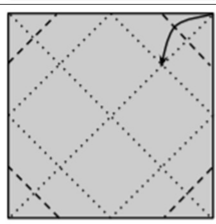
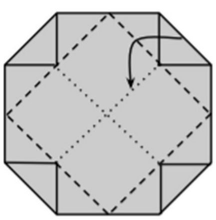
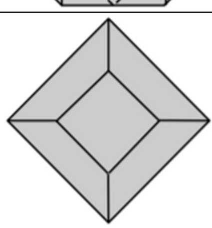
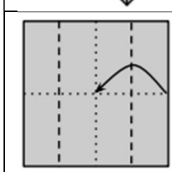
- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

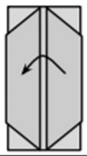
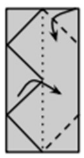
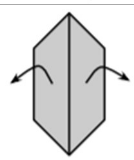
Instructional Routines

- Collect and Display
- Discussion Supports

Required Materials

Copies of blackline master

	1. Start with a square sheet of paper. Fold it in half along each diagonal (the dashed lines), make a crease, and then unfold.
	2. Fold one corner into the centre, make a crease, and unfold. Repeat with the remaining three corners.
	3. Fold the four corners to the crease line you made in the last step. Do not unfold!
	4. Re-fold along the creases you made in step 2.
	5. It should look like this now. Flip the whole thing over, and rotate it so that it is res.
	6. Fold the left edge and the right edge to the centre crease.

	7. Fold over just the top layer from right to left.
	8. Fold in the top right corner and the bottom right corner. Then, flip the two top flaps from left to right.
	9. Repeat the process on the left side. Fold in the top left corner and the bottom left corner.
	10. Fold the top flap of paper back from right to left.
	11. Pull the top layers away from each other, and allow your box to become three dimensional. Crease the edges to make it look like a box!

Origami paper
Rulers

Required Preparation

Choose at least three different sizes of origami paper for students to use. Common length and width sizes of square origami paper include 6 inch, 7 inch, 8 inch, 9 inch, and 9.75 inch. If origami paper is not available, cut squares of paper from available paper (thinner is better). Pre-make sample boxes of different sizes.

In order to help students fold their own origami boxes, both an embedded video and printed instructions are provided. The printed instructions are in the Folding Paper Boxes blackline master. If using the printed instructions, prepare 1 copy for every 2 students. These can be re-used with multiple classes.

Student Learning Goals

Let's use what we know about decimals to make and measure boxes.

15.1 Folding Paper Boxes

Optional:

In this activity, students work with decimals by building paper boxes, taking measurements of the paper and the boxes, and calculating surface areas. Although the units are specified in the problem, students need to measure very carefully in order to give an estimate to the nearest millimetre. Next, students compare the side lengths of several paper squares and estimate what the measurements of the boxes will be once the squares have been folded.

Closely monitor student progress on the first question to make sure that students are careful in their measurements. On the second question, make sure they think carefully about what level of precision to use in reporting the relationship between two measurements. For examples:

- If one sheet of square paper is very close to twice the length of another, reporting the answer as 2 is reasonable given the possible error in measurement.
- If the relationship is very close to a fraction (e.g. $\frac{3}{2}$ for the 6 inch and 9 inch squares) students might report the number as a fraction.
- If students report the quotient as a decimal, three digits (ones, tenths, hundredths) is appropriate because the measurements in the first problem have 3 digits.

Encourage students to make strong creases when folding their paper at the end of the activity. Suggest that they use the side of a thumbnail or a ruler to flatten the crease after making the initial fold. Students will use these boxes in the next activity.

Instructional Routines

- Collect and Display

Launch

Choose at least three different sizes of square paper for students to use. To see the mathematical structure more clearly, the smallest should be 6 inches by 6 inches and the largest should be 12 inches by 12 inches. For the other size squares, common length and width sizes of origami paper include 7 inch, 8 inch, 9 inch, and 9.75 inch. Pre-make boxes of different sizes from square paper with lengths 6 inches, 8 inches, and 12 inches.

Arrange students in groups of 3–4. Provide each group with at least three different sizes of paper. Since the folding process is involved and students are asked to measure the squares they use to make the boxes, make some extra squares available for each group. Students will need metric rulers or tape measures, marked in millimetres.

Give students 10 minutes for the problems. When students are finished, demonstrate how to fold a paper square into a box and then have students fold their paper squares. This can be done with an accompanying video or with instructions such as those included in the blackline master. In either case, consider going through the process with students step by step and practicing beforehand to make sure that it goes smoothly.

Note that these particular instructions make a box with a square base; the following activity, which prompts students to record the length and width of the box's base, is based on this premise. If a different origami construction is used, the instructions and possibly the task statements will need to be adjusted.

Video 'Origami Box' available here: https://player.vimeo.com/video/304138309.

Representing, Conversing: Collect and Display. As students work on the problems, use this routine to capture the ways they communicate about the measurements, including the precision at which they were taken. Additionally, capture the ways students discuss the relationships between the dimensions of the sheets of paper and predicted dimensions of the boxes. Listen for terms such as quotient, divide, multiply, and rounding. Notate student language and relevant sketches/diagrams to reference while students are working and then discuss in the synthesis. This will help students communicate about the accuracy of the measurements and the size comparisons between boxes.
Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

Check that students are obtaining accurate measurements to the nearest tenth of a centimetre. Are they placing the ruler at the edge of a side? Are they starting at zero?

Student Task Statement

Your teacher will demonstrate how to make an open-top box by folding a sheet of paper. Your group will receive 3 or more sheets of square paper. Each person in your group will make 1 box. Before you begin folding:

1. Record the side lengths of your papers, from the smallest to the largest.
   - Paper for Box 1: ____________ cm
   - Paper for Box 2: _____________ cm
   - Paper for Box 3: ______________ cm
2. Compare the side lengths of the square sheets of paper. Be prepared to explain how you know.
   a. The side length of the paper for Box 2 is _______ times the side length of the paper for Box 1.
   b. The side length of the paper for Box 3 is _______ times the side length of the paper for Box 1.
3. Make some predictions about the measurements of the three boxes your group will make:
   - The surface area of Box 3 will be _______ as large as that of Box 1.
   - Box 2 will be _______ times as tall as Box 1.

– Box 3 will be _______ times as tall as Box 1.

Now you are ready to fold your paper into a box!

Student Response

Answers vary. Sample responses (based on paper sizes of 6 inches by 6 inches, 8 inches by 8 inches, and 12 inches by 12 inches):

1. Paper for Box 1: 15.2 cm, paper for Box 2: 20.3 cm, paper for Box 3: 30.5 cm.

2. The paper for Box 2 is about 1.3 times as long as the paper for Box 1, found by dividing 20.3 by 15.2. Alternatively, using the inch measurements, the paper for Box 2 is $\frac{4}{3}$ times as long as the paper for Box 1. The paper for Box 3 is about 2 times as long as the paper for Box 1, found by dividing 30.5 by 15.2 (or by dividing 12 by 6).

3. The surface area of Box 3 will be about four times the surface area of Box 1. Box 2 will be about 1.3 (or one and a third) times the height of Box 1. Box 3 will be 2 times as tall as Box 1.

Activity Synthesis

(Note: the discussion questions below assume square paper of side lengths 6 inches, 8 inches, and 12 inches were used).

The goal of this discussion is for students to think critically about the accuracy of their measurements and predictions. Consider asking some of these discussion questions. Sample responses are shown in parentheses, but expect students' answers to vary.

- What did you find for the length and width of the smallest square? (15.2 cm. Also expect some answers of 15.3 cm and possibly a wider range of values.)
- What was challenging about measuring the length of the squares? (The millimetres are so small that it was hard to tell which millimetre it was closest to. It was hard to measure *straight* across.)
- How confident are you about the accuracy of your measurements (For the first square, very confident about the 15 in 15.2 cm, but not confident about the 0.2.)
- How many times as long as Paper 1 was Paper 3? (About 2, very close to 2, or a decimal number that is close to 2.)
- What are some advantages and disadvantages of reporting the quotient of the side length of Paper 3 and that of Paper 1 as 2? (Advantage: it describes the general relationship clearly, and 2 is an easy number to grasp. Disadvantage: it was not exactly twice as long.)
- What are some advantages and disadvantages of reporting the quotient of the side length of Paper 3 and that of Paper 1 as 2.007 (or another value that is very close to 2

and is proposed by students)? (Advantage: the number is more accurate than 2. Disadvantage: it is too accurate. The measurements were done by hand and were not precise enough to judge the quotient to the nearest ten-thousandth.)

- How much taller do you think Box 3 will be compared to Box 1? (Twice, because the side length of the paper making Box 3 is twice as long and twice as wide as the that of paper making Box 1.)

15.2 Sizing Up Paper Boxes

Optional:

Using the boxes that they built in the previous activity, students now measure and compare the length, height, and surface area of the boxes. This work requires fluency in operations with decimal numbers and care in measurement. In measuring the dimensions of the box, there are multiple layers of imprecision that can be expected.

- The length, width, and height will not be an exact number of millimetres and so students round to the nearest millimetre. In some cases, this may essentially be a guess between two different values.
- The box is made by folding paper and this process is not exact. The box is therefore not exactly a rectangular prism with a square base, and measurements of the length, width, and height vary depending on which part of the box is measured.
- When finding the surface area of their box, students will add and multiply their measurements. In performing operations, any errors in the measurements propagate, making it challenging to trace where they originated.

Instructional Routines

- Discussion Supports

Launch

Keep students in the same groups. Provide access to rulers. Give groups 8–10 minutes to complete the first table collaboratively. There are three sets of measurements and a surface area calculation for each box. Each student can fill out the row for the box that they made. If a group has more than three paper sizes, adjust the tables in the activity accordingly. Ask students to pause for a class discussion after they have completed the table.

Select a couple of groups to share their measurements and surface area calculations. Display their responses for all to see. Ask other groups if their responses are close, and if not, ask what their answers are. Come to a general agreement about the approximate measurements and areas.

Then, ask students to complete the remaining questions.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisation and

problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organisation; Attention

Anticipated Misconceptions

Students may neglect to attend to units of measurement when calculating area. Encourage them to attend to the units being used.

Student Task Statement

Now that you have made your boxes, you will measure them and check your predictions about how their heights and surface areas compare.

a. Measure the length and height of each box to the nearest tenth of a centimetre. Record the measurements in the table.

	side length of paper (cm)	length of box (cm)	height of box (cm)	surface area (sq cm)
Box 1				
Box 2				
Box 3				

b. Calculate the surface area of each box. Show your reasoning and decide on an appropriate level of precision for describing the surface area (Is it the nearest 10 square centimetres, nearest square centimetre, or something else?). Record your answers in the table.

1. To see how many times as large one measurement is when compared to another, we can calculate their quotient. Divide each measurement of Box 2 by the corresponding measurement for Box 1 to complete the following statements.

   a. The length of Box 2 is _______ times the length of Box 1.

   b. The height of Box 2 is _______ times the height of Box 1.

   c. The surface area of Box 2 is _______ times the surface area of Box 1.

2. Find out how the dimensions of Box 3 compare to those of Box 1 by calculating quotients of their lengths, heights, and surface areas. Show your reasoning.

   a. The length of Box 3 is _______ times the length of Box 1.

   b. The height of Box 3 is _______ times the height of Box 1.

   c. The surface area of Box 3 is _______ times the surface area of Box 1.

3. Record your results in the table.

	side length of paper	length of box	height of box	surface area
Box 2 compared to Box 1				
Box 3 compared to Box 1				

4. Earlier, in the first activity, you made predictions about how the heights and surface areas of the two larger boxes would compare to those of the smallest box. Discuss with your group:

   – How accurate were your predictions? Were they close to the results you found by performing calculations?

   – Let's say you had another piece of square paper to make Box 4. If the side length of this paper is 4 times the side length of the paper for Box 1, predict how the length, height, and surface area of Box 4 would compare to those of Box 1. How did you make your prediction?

Student Response

a. Answers vary. Sample response: The table below is filled out based on 6-inch, 8-inch, and 12-inch papers. Refer to the Activity Synthesis for examples of possible errors in these measurements.

	side length of paper (cm)	length of box (cm)	height of box (cm)	surface area (sq cm)
Box 1	15.2	5.4	2.7	87.5
Box 2	20.8	7.2	3.6	156
Box 3	30.5	10.8	5.4	350

b. Answers vary. See table for sample results.

1. Answers vary. See table in response to question 4.

2. Answers vary. See table in response to question 4.

3. Sample response: The table is filled out using three significant digits in each case.

	side length of paper	length of box	height of box	surface area
Box 2 compared to Box 1	1.34	1.3	1.3	1.78
Box 3 compared to Box 1	2.01	2	2	4

4. Answers vary. Sample response:

- My estimates were accurate. I thought the length and height of Box 3 would be twice those of Box 1 and that the length and height of Box 2 would be about one and a third times those of Box 1. I thought the surface area of Box 3 would be 4 times the surface area of Box 1. These answers are close to my results of my calculations. The discrepancies probably come from error in measurement.
- I predicted the length and height of this box to be 4 times as long as those of Box 1, and the surface area of Box 4 to be 16 times as large.

Activity Synthesis

(Note: the discussion questions below assume sheets of square paper of side lengths 6 inches, 8 inches, and 12 inches were used).

For each size of paper used, ask students to report their measurements for the volume of the box and record for all to see. Ask:

- "Is there variation in the measurements? Which ones?" (Yes, possibly for the dimensions of the squares; almost certainly for the dimensions of the boxes.)
- "Why do you think the measurements were not all the same?" (It was difficult to line up the edges of the paper with the ruler. It was also challenging to measure to the nearest millimetre.)
- "Were there extra difficulties measuring the boxes, as opposed to measuring the squares of paper?" (Yes, e.g., the sides of the box were not completely flat; the sides of the box were not identical; the height of the box was different at different places.)
- "How accurate do you think your measurements were?" (In each case, within 1 or 2 millimetres.)
- "How does this influence the way you record a result? For example, for the surface area, did anyone record their answer to hundredths of a square centimetre? Why or why not?" (Some students might have done this, but it is overly precise and does not take into account the imprecision in measurements mentioned above.)
- "In the table for the third question, did you record units for your measurements? Is it important to record units?" (It is important that the measurements of the differences between boxes must be in the *same* units.)

Also discuss with the class their predictions and measurements for how the heights and surface area of the boxes compare to one another. Ask:

- "How much larger did you think the surface area of Box 3 would be than the surface area of Box 1?" (4 times because it takes 4 of the smaller sheet of paper to make one of the larger ones. Some students may also say 2 times, because the height, length, and width are about 2 times as long as those of Box 1.)

- "Did your measurements match your prediction for the surface areas? Why might that be?" (Students who did not think the surface area of Box 3 would be 4 times as large as the surface area of Box 1 can discuss their old thinking and new. Because area is found by taking a product of length measurements and *all* measurements (length, width, height) of Box 3 are twice the corresponding measurements of Box 1, Box 3 will have 4 times as much surface area as Box 1.)

Listening, Speaking, Conversing: Discussion Supports. Before the whole-class discussion, give students time to converse with their group about their predictions for how the heights and surface area of the boxes compare to one another. Provide this sentence frame for students to use: "My measurements matched/did not match my predictions because __. " Invite group members to press for details in their peers' explanations and challenging ideas by asking, "Why did you make that prediction?", "Could you explain that using a different example?", or "What evidence do you have to support your reasoning?" This will help students solidify their reasoning, and improve their explanations before the whole-class discussion.
Design Principle(s): Cultivate conversation

Lesson Synthesis

The discussions of student work at the end of each activity provides opportunities to summarise takeaways from this lesson. Students can use this optional lesson to practice fluency with calculations involving decimals. Highlight instances where students have to make an estimate in order to proceed, figure out how to determine significant digits, or apply their understanding of units when measuring and recording information.