Lesson 14: Finding solutions to inequalities in context

Goals

- Interpret inequalities that represent situations with a constraint.
- Solve an equation of the form $px + q = r$ to determine the boundary point for an inequality of the form $px + q > r$ or $px + q < r$.
- Use substitution or reasoning about the context to justify (orally and in writing) whether the values that make an inequality true are greater than or less than the boundary point.

Learning Targets

- I can describe the solutions to a inequality by solving a related equation and then reasoning about values that make the inequality true.
- I can write an inequality to represent a situation.

Lesson Narrative

In this lesson, students see more examples of inequalities in a context. This time, many inequalities involve negative coefficients. This illustrates the idea that solving an inequality is not as simple as solving the corresponding equation. After students find the boundary point, they must do some extra work to figure out the direction of inequality. This might involve reasoning about the context, substituting in values on either side of the boundary point, or reasoning about number lines. All of these techniques involve making the problem more concrete and visual and asking "Does this make sense?" In this lesson, the emphasis is on reasoning about the context.

It is important to understand that the goal is not to have students learn and practise an algorithm for solving inequalities like "whenever you multiply or divide by a negative, flip the inequality." Rather, we want students to understand that solving a related equation tells you the lower or upper bound of an inequality. To know whether values greater than or less than the boundary number make the inequality true, it's best to either think about the context or test some values that are above and below the boundary number. This way of reasoning about inequalities will serve students well long into their future studies, whereas students are very likely to forget a procedure memorised for a special case.

Addressing

- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Building Towards

- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Instructional Routines

- Co-Craft Questions
- Discussion Supports
- Think Pair Share

Required Preparation

Several activities suggest providing students with blank number lines to use for rough work. One way to accomplish this is to print a line with unlabelled, evenly-spaced tick marks, and place these into plastic pockets. Students can write on these with dry erase markers and wipe them off.

Student Learning Goals

Let's solve more complicated inequalities.

14.1 Solutions to Equations and Solutions to Inequalities

Warm Up: 10 minutes

This warm-up highlights the link between an inequality and its associated equation. This will be solidified throughout the lesson as students solve the associated equation and reason in context to determine the direction of inequality. Notice students who use the value -10 as a boundary as they test values to find solutions to the inequalities.

Launch

Give students 5 minutes of quiet work time followed by a whole-class discussion. Optionally, provide students with blank number lines for rough work.

Student Task Statement

1. Solve $-x = 10$
2. Find 2 solutions to $-x > 10$
3. Solve $2x = -20$
4. Find 2 solutions to $2x > -20$

Student Response

1. -10
2. Answers vary. Possible responses: -12, -28.7, -209. (Any value that is less than -10 works.)
3. -10

4. Answers vary. Possible responses: -9, 0, $82\frac{3}{4}$. (Any value that is greater than -10 works.)

Activity Synthesis

Display two number lines for all to see that each include -10 and some integer values to its left and right. Ask a few students to share their responses to the first two questions, recording their responses on one number line and gauging the class for agreement. Ask a few students to share their responses to the last two questions, recording their responses on the other number line and gauging the class for agreement.

Highlight the fact that $-x = 10$ and $2x = -20$ have the same solution (-10), but the inequalities $-x > 10$ and $2x > -20$ don't have the same solutions. Select students to share strategies they had for finding solutions. If not mentioned by students, discuss the fact that since -10 makes the sides equal, the neighbourhood of values around -10 is a good place to start looking for solutions.

14.2 Earning Money for Football Stuff

15 minutes

Previously in this unit, students wrote expressions and equations that are similar to the ones in this activity. Here, they are prompted in a scaffolded way to notice that they can express not just that an outcome can be *equal* to a value, but that an outcome can be *at least as much as* a value by using the new notation $\geq$.

Instructional Routines

- Co-Craft Questions
- Think Pair Share

Launch

Optionally, provide access to blank number lines to use for rough work.

Arrange students in groups of 2. Allow 10 minutes of quiet work time and partner discussion followed by a whole-class discussion. Depending on the needs of your class, you may decide to ask students to pause after the first question for the whole-class discussion before tackling the second question.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Look for students who correctly write an expression but struggle in writing an equation with the correct inequality sign. Consider pausing for a brief whole-class discussion inviting students to share strategies for determining the correct sign.
Supports accessibility for: Memory; Organisation Conversing: Co-craft Questions. Reveal only the context for Andre's summer job, without revealing the questions that follow. Ask students to create mathematical questions about this situation. Give students 1–2 minutes to write down mathematical questions that could be asked about the situation. Invite

students to share their questions with the class, before revealing the rest of the activity. Listen for amplify questions that contain the phrase "at least."
Design Principle(s): Maximise meta-awareness; Support sense-making

Student Task Statement

1. Andre has a summer job selling magazine subscriptions. He earns £25 per week plus £3 for every subscription he sells. Andre hopes to make at least enough money this week to buy a new pair of football boots.

    a. Let n represent the number of magazine subscriptions Andre sells this week. Write an expression for the amount of money he makes this week.

    b. The least expensive pair of boots Andre wants cost £68. Write and solve an equation to find out how many magazine subscriptions Andre needs to sell to buy the boots.

    c. If Andre sold 16 magazine subscriptions this week, would he reach his goal? Explain your reasoning.

    d. What are some other numbers of magazine subscriptions Andre could have sold and still reached his goal?

    e. Write an *inequality* expressing that Andre wants to make at least £68.

    f. Write an inequality to describe the number of subscriptions Andre must sell to reach his goal.

2. Diego has budgeted £35 from his summer job earnings to buy shorts and socks for football. He needs 5 pairs of socks and a pair of shorts. The socks cost different amounts in different stores. The shorts he wants cost £19.95.

    a. Let x represent the price of one pair of socks. Write an expression for the total cost of the socks and shorts.

    b. Write and solve an equation that says that Diego spent exactly £35 on the socks and shorts.

    c. List some other possible prices for the socks that would still allow Diego to stay within his budget.

    d. Write an inequality to represent the amount Diego can spend on a single pair of socks.

Student Response

1.

    a. $3n + 25$

    b. $3n + 25 = 68, n = 14\frac{1}{3}$

c. Yes. $16 > 14\frac{1}{3}$. He made £73, which is more than enough to buy the boots.

d. Answers vary. Sample responses: 15, 17, 100. (Any whole number greater than 14 would make sense.)

e. $3n + 25 \geq 68$

f. $n \geq 14\frac{1}{3}$

2.

a. $5x + 19.95$

b. $5x + 19.95 = 35$, $x = 3.01$. In this situation, Diego paid £3.01 for each pair of socks.

c. Answers will vary. Any price under £3.01 is an acceptable response.

d. $x \leq 3.01$.

Activity Synthesis

Here is what we want students to understand as a result of this activity:
In order to find the solution to an inequality like $3n + 25 \geq 68$, we can solve an equation to find the point where $3n + 25 = 68$. This is the point that separates numbers that are solutions to the inequality from numbers that are not solutions. To find whether the solution to the inequality is $n \geq 14\frac{1}{3}$ or $n \leq 14\frac{1}{3}$, we can substitute some values of n that are greater than $14\frac{1}{3}$ and some that are less than $14\frac{1}{3}$ to check. Alternatively, we can think about the context: If Andre wants to make more money, he needs to sell more magazines, not fewer. If Diego wants to spend less than £35, he needs to spend less for socks, not more. Ask students:

- How does solving the equation help us solve an inequality? What does the solution tell us about solutions to the inequality?
- What are some ways we can determine whether the solution to an inequality should use less than or greater than?
- How can we check whether a value is a solution to the inequality?
- Could Andre sell exactly $14\frac{1}{3}$ subscriptions?
- Can Diego pay exactly £3.01 for each pair of socks?
- How can we tell if there are restrictions on the solutions of the inequality, such as only positive numbers or only whole numbers?

14.3 Granola Bars and Savings

15 minutes

The purpose of this activity is for students to interact with contexts in which the direction of inequality is the opposite of what they might expect if they try to solve like they would with an equation. For example, in the second problem, the original inequality is $9(7 - x) \leq 36$, but the solution to the inequality is $x \geq 3$.

Some students might solve the associated equation and then test values of x to determine the direction of inequality. That method will be introduced in more generality in the next lesson. This activity emphasises thinking about the context in deciding the direction of inequality.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Keep students in the same groups. Give 5–10 minutes of quiet work time and partner discussion followed by a whole-class discussion.

Student Task Statement

1. Kiran has £100 saved in a bank account. (The account doesn't earn interest.) He asked Clare to help him figure out how much he could take out each month if he needs to have at least £25 in the account a year from now.

    a. Clare wrote the inequality $-12x + 100 \geq 25$, where x represents the amount Kiran takes out each month. What does $-12x$ represent?

    b. Find some values of x that would work for Kiran.

    c. We could express *all* the values that would work using either $x \leq \_\_$ or $x \geq \_\_$. Which one should we use?

    d. Write the answer to Kiran's question using mathematical notation.

2. A teacher wants to buy 9 boxes of granola bars for a school trip. Each box usually costs £7, but many grocery stores are having a sale on granola bars this week. Different stores are selling boxes of granola bars at different discounts.

    a. If x represents the amount in pounds of the discount, then the amount the teacher will pay can be expressed as $9(7 - x)$. In this expression, what does the quantity $7 - x$ represent?

    b. The teacher has £36 to spend on the granola bars. The equation $9(7 - x) = 36$ represents a situation where she spends all £36. Solve this equation.

c. What does the solution mean in this situation?

d. The teacher does not have to spend all £36. Write an inequality relating 36 and $9(7 - x)$ representing this situation.

e. The solution to this inequality must either look like $x \geq 3$ or $x \leq 3$. Which do you think it is? Explain your reasoning.

Student Response

1.

a. The difference in Kiran's account balance after one year (because there are 12 months in a year).

b. Answers vary. Sample responses: 1, 2, 6. (Any value less than or equal to 6.25 will work.)

c. $x \leq \_\_$. Kiran must draw less than a certain amount each month in order to end up with £25 in the account at the end of the year.

d. $x \leq 6.25$. Possible strategy: Solve the equation $-12x + 100 = 25$ and then use the reasoning in the previous problem part (or test values of x) to decide between $\geq$ and $\leq$.

2.

a. The price of 1 box after the discount.

b. $x = 3$. Possible strategy: divide each side by 9 resulting in $7 - x = 4$, then notice that $7 - 3$ is 4.

c. If the discount is £3, then the teacher will pay exactly £36 for the granola bars.

d. $9(7 - x) \leq 36$ or $36 \geq 9(7 - x)$

e. $x \geq 3$, because a discount higher than £3 per box will mean that the teacher will pay a lower price for granola bars.

Are You Ready for More?

Jada and Diego baked a large batch of cookies.

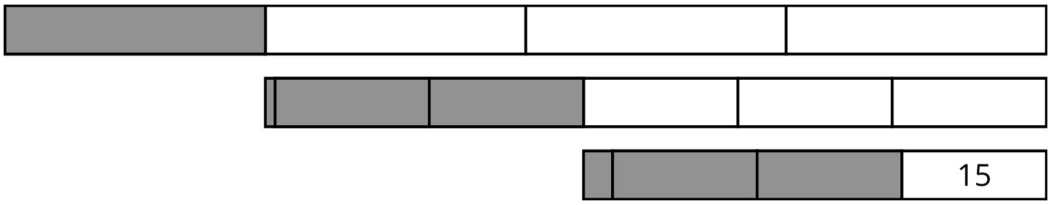
- They selected $\frac{1}{4}$ of the cookies to give to their teachers.
- Next, they threw away one burnt cookie.
- They delivered $\frac{2}{5}$ of the remaining cookies to a local nursing home.
- Next, they gave 3 cookies to some neighbourhood kids.

- They wrapped up $\frac{2}{3}$ of the remaining cookies to save for their friends.

After all this, they had 15 cookies left. How many cookies did they bake?

Student Response

108 cookies. Possible strategy: Draw a diagram to represent the situation:



Next, work backwards:

$15 \times 3 = 45, 45 + 3 = 48$

$48 \div 3 = 16, 16 \times 5 = 80, 80 + 1 = 81$

$81 \div 3 = 27, 27 \times 4 = 108$

An equation that represents the number of cookies remaining would be $\frac{1}{3}\left(\frac{3}{5}\left(\frac{3}{4}x - 1\right) - 3\right) = 15$. To solve this equation, we would multiply by 3, add 3, multiply by $\frac{5}{3}$, add 1, and then multiply by $\frac{4}{3}$. Compare these steps to the steps we took to solve with the diagram to see they are the same. Note that divide by 3, then multiply by 5 is the same as multiply by $\frac{5}{3}$.

Activity Synthesis

The purpose of the discussion is to let students voice their reasoning about the direction of the inequality by reasoning about the context. Ask students to share their reasons for choosing the direction of inequality in their solutions. Some students may notice that the algebra in both problems involves multiplying or dividing by a negative number. Respect this observation, but again, the goal is not to turn this observation into a rule for students to memorise and follow. Interpreting the meaning of the solution in the context should remain at the forefront.

As students model real-world situations, questions about the interpretation of the mathematical solution should continue to come up in the conversation. For instance, the amount of the granola bar discount cannot be £3.5923, even though this is a solution to the inequality $x \geq 3$. The value -10 is a solution to Kiran's inequality, even though he can't withdraw -10 pounds. Students can argue that negative values for x simply don't make sense in this context. Some may argue that we should interpret $x =$ -10 to mean that Kiran deposits £10 every month.

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.

Supports accessibility for: Language; Social-emotional skills; Attention Representing, Writing, Conversing: Discussion Supports. Use this routine to support whole-class discussion. After a student shares their reasons for the direction of the inequality in their solution, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. This will help students understand how context determines the direction of inequality.
Design Principle(s): Support sense-making

Lesson Synthesis

By the time students have finished this lesson, they have reasoned about solutions to several inequalities, all of which involve some kind of final decision about the direction of inequality. Return to the ideas in the warm-up for the previous lesson. Draw the number line showing solutions to $x > 1$ on the board. Ask students to name some values of x that satisfy the inequality. For each of those values of x, plot the value of $-x$ on the number line together (perhaps in a different colour). What inequality did we just graph?

14.4 Colder and colder

Cool Down: 5 minutes

Student Task Statement

It is currently 0 degrees outside, and the temperature is dropping 4 degrees every hour. The temperature after h hours is $-4h$.

1. Explain what the equation $-4h = -14$ represents.
2. What value of h makes the equation true?
3. Explain what the inequality $-4h \leq -14$ represents.
4. What values of h make the inequality true?

Student Response

1. After h hours, the temperature has dropped to -14 degrees.
2. $h = 3.5$. It takes 3.5 hours for the temperatures to drop to -14 degrees.
3. After h hours, the temperature is less than or equal to -14 degrees.
4. $h \geq 3.5$. After reaching a temperature of -14 degrees at 3.5 hours, the temperature will continue to decrease.

Student Lesson Summary

Suppose Elena has £5 and sells pens for £1.50 each. Her goal is to save £20. We could solve the equation $1.5x + 5 = 20$ to find the number of pens, x, that Elena needs to sell in

order to save *exactly* £20. Adding -5 to both sides of the equation gives us $1.5x = 15$, and then dividing both sides by 1.5 gives the solution $x = 10$ pens.

What if Elena wants to have some money left over? The inequality $1.5x + 5 > 20$ tells us that the amount of money Elena makes needs to be *greater* than £20. The solution to the previous equation will help us understand what the solutions to the inequality will be. We know that if she sells 10 pens, she will make £20. Since each pen gives her more money, she needs to sell *more* than 10 pens to make more than £20. So the **solution to the inequality** is $x > 10$.

Glossary

- solution to an inequality

Lesson 14 Practice Problems

1. **Problem 1 Statement**

   The solution to $5 - 3x > 35$ is either $x >$ -10 or -10 $> x$. Which solution is correct? Explain how you know.

   Solution

   $x <$ -10. Sample reasoning: If I try -100 in place of x, I get $305 > 35$, which is true. Any value of x that is less than -10 makes the inequality true. -10 $> x$ refers to all values of x that are less than -10.

2. **Problem 2 Statement**

   The school band director determined from past experience that if they charge t pounds for a ticket to the concert, they can expect attendance of $1000 - 50t$. The director used this model to figure out that the ticket price needs to be £8 or greater in order for at least 600 to attend. Do you agree with this claim? Why or why not?

   Solution

   No. Explanations vary. Sample response: If ticket prices are higher, fewer people will attend (this can be seen by trying some different values of t in $1000 - 50t$). 8 is the solution to $1000 - 50t = 600$, but they need to charge £8 or less if they want 600 people or more to attend.

3. **Problem 3 Statement**

   Which inequality is true when the value of x is -3?

   a. $-x - 6 <$ -3.5

   b. $-x - 6 > 3.5$

   c. $-x - 6 >$ -3.5

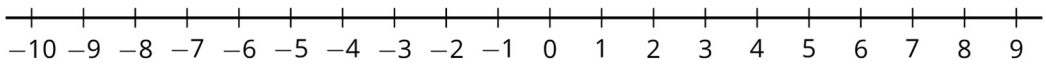
   d. $x - 6 >$ -3.5

Solution C

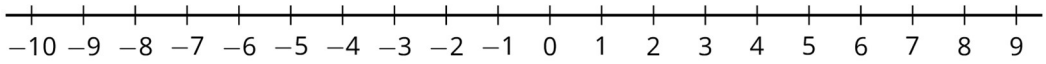
4. **Problem 4 Statement**

Draw the solution set for each of the following inequalities.

a. $x \leq 5$

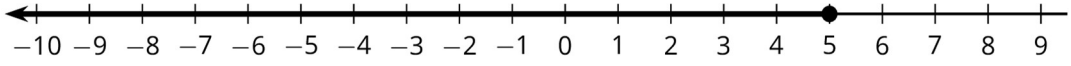


b. $x < \frac{5}{2}$

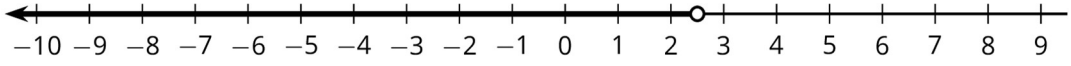


Solution

a.

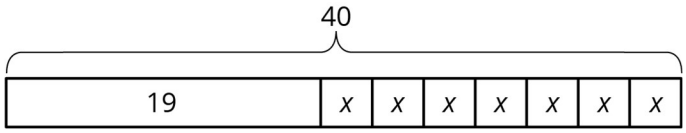


b.



5. **Problem 5 Statement**

Write three different equations that match the bar model.



Solution

Answers vary. Sample responses:

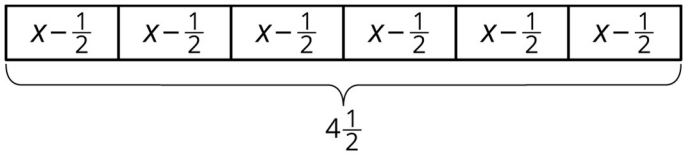
a. $7x + 19 = 40$

b. $40 = 7x + 19$

c. $7x = 40 - 19$

6. **Problem 6 Statement**

A baker wants to reduce the amount of sugar in his cake recipes. He decides to reduce the amount used in 1 cake by $\frac{1}{2}$ cup. He then uses $4\frac{1}{2}$ cups of sugar to bake 6 cakes.



a. Describe how the bar model represents the story.

b. How much sugar was originally in each cake recipe?

Solution

a. Answers vary. Sample response: The six equal parts of the diagram represent the 6 cakes the baker bakes. The label $x - \frac{1}{2}$ in each part represents the amount of sugar, measured in number of cups, that the baker used in each cake. x represents the original amount of sugar used in each cake and $x - \frac{1}{2}$ represents the original number of cups reduced by $\frac{1}{2}$ cup. $4\frac{1}{2}$ is the total amount of sugar, measured in cups, used for the 6 cakes.

b. $1\frac{1}{4}$ cups

7. **Problem 7 Statement**

One year ago, Clare was 4 feet 6 inches tall. Now Clare is 4 feet 10 inches tall. By what percentage did Clare's height increase in the last year?

Solution

About 7% (4 feet 6 inches is 54 inches and she grew 4 inches: $\frac{4}{54} \approx 0.07$)