Problemas – Tema 8

Problemas resueltos - 3 - indeterminación 0 dividido 0 en cociente de polinomios y raíces

1. Calcula los siguientes límites:

a) $\lim_{x\to 1}\frac{x^3+2x^2-3x}{x^3+4x^2+x-6}=\frac{1+2-3}{1+4+1-6}=\frac{0}{0}$ → Indeterminación

$$\lim_{x\to 1}\frac{x^3+2x^2-3x}{x^3+4x^2+x-6}=\lim_{x\to 1}\frac{(x-1)(x^2+3x)}{(x-1)(x^2+5x+6)}=\lim_{x\to 1}\frac{x^2+3x}{x^2+5x+6}=\frac{1+3}{1+5+6}=\frac{4}{12}=\frac{1}{3}$$

b) $\lim_{x\to -3}\frac{x^3+2x^2-3x}{x^3+4x^2+x-6}=\frac{-27+18+9}{-27+36-3-6}=\frac{0}{0}$ → Indeterminación

$$\lim_{x\to -3}\frac{x^3+2x^2-3x}{x^3+4x^2+x-6}=\lim_{x\to -3}\frac{(x+3)(x^2-x)}{(x+3)(x^2+x-2)}=\lim_{x\to -3}\frac{x^2-x}{x^2+x-2}=\frac{9+3}{9-3-2}=\frac{12}{4}=3$$

c) $\lim_{x\to 3}\frac{\sqrt{x+6}-3}{3-x}=\frac{\sqrt{9}-3}{3-3}=\frac{0}{0}$ → Indeterminación

$$\lim_{x\to 3}\frac{\sqrt{x+6}-3}{3-x}=\lim_{x\to 3}\frac{(\sqrt{x+6}-3)(\sqrt{x+6}+3)}{(3-x)(\sqrt{x+6}+3)}=\lim_{x\to 3}\frac{x+6-9}{(3-x)(\sqrt{x+6}+3)}$$

Operar numerador.

$$\lim_{x\to 3}\frac{x-3}{(3-x)(\sqrt{x+6}+3)}=\lim_{x\to 3}\frac{-1}{\sqrt{x+6}+3}=\frac{-1}{\sqrt{9}+3}=\frac{-1}{6}$$

d) $\lim_{x\to 4}\frac{2x-8}{\sqrt{x^2+9}-5}=\frac{2\cdot 4-8}{\sqrt{25}-5}=\frac{0}{0}$ → Indeterminación

$$\lim_{x\to 4}\frac{2x-8}{\sqrt{x^2+9}-5}=\lim_{x\to 4}\frac{(2x-8)(\sqrt{x^2+9}+5)}{(\sqrt{x^2+9}-5)(\sqrt{x^2+9}+5)}$$ → Operar denominador

$$\lim_{x\to 4}\frac{(2x-8)(\sqrt{x^2+9}+5)}{x^2-16}=\lim_{x\to 4}\frac{(2x-8)(\sqrt{x^2+9}+5)}{(x+4)(x-4)}$$ → Operar numerador

$$\lim_{x\to 4}\frac{(2x-8)(\sqrt{x^2+9}+5)}{(x+4)(x-4)}=\lim_{x\to 4}\frac{2(x-4)(\sqrt{x^2+9}+5)}{(x+4)(x-4)}=\lim_{x\to 4}\frac{2(\sqrt{x^2+9}+5)}{(x+4)}=\frac{2(\sqrt{25}+5)}{4+4}=\frac{20}{8}=\frac{5}{2}$$

e) $\lim_{x \to 2} \frac{3-\sqrt{2x^2+1}}{3x-6} = \frac{0}{0}$ → Indeterminación

$$\lim_{x \to 2} \frac{3-\sqrt{2x^2+1}}{3x-6} = \lim_{x \to 2} \frac{(3-\sqrt{2x^2+1})(3+\sqrt{2x^2+1})}{(3x-6)(3+\sqrt{2x^2+1})}$$ → Operar numerador

$$\lim_{x \to 2} \frac{9-(2x^2+1)}{(3x-6)(3+\sqrt{2x^2+1})} = \lim_{x \to 2} \frac{-2x^2+8}{(3x-6)(3+\sqrt{2x^2+1})} = \lim_{x \to 2} \frac{-2(x^2-4)}{(3x-6)(3+\sqrt{2x^2+1})}$$

$$\lim_{x \to 2} \frac{-2(x+2)(x-2)}{(3x-6)(3+\sqrt{2x^2+1})}$$ → Operar denominador

$$\lim_{x \to 2} \frac{-2(x+2)(x-2)}{3(x-2)(3+\sqrt{2x^2+1})} = \lim_{x \to 2} \frac{-2(x+2)}{3(3+\sqrt{2x^2+1})} = \frac{-2(2+2)}{3(3+\sqrt{2(2)^2+1})} = \frac{-8}{18} = \frac{-4}{9}$$

2. Resuelve:

a) $\lim_{x \to -3} \frac{x^3+2x^2-3x}{x^3+4x^2+x-6} = \frac{-27+18+9}{-27+36-3-6} = \frac{0}{0}$ → Indeterminación

$$\lim_{x \to -3} \frac{x^3+2x^2-3x}{x^3+4x^2+x-6} = \lim_{x \to -3} \frac{(x+3)(x^2-x)}{(x+3)(x^2+x-2)} = \lim_{x \to -3} \frac{x^2-x}{x^2+x-2} = \frac{9+3}{9-3-2} = \frac{12}{4} = 3$$

b) $\lim_{x \to -2} \frac{3x^2+5x-2}{x^2-4} = \frac{12-10-2}{4-4} = \frac{0}{0}$ → Indeterminación

$$\lim_{x \to -2} \frac{3x^2+5x-2}{x^2-4} = \lim_{x \to -2} \frac{(x+2)(3x-1)}{(x+2)(x-2)} = \lim_{x \to -2} \frac{3x-1}{x-2} = \frac{3(-2)-1}{-2-2} = \frac{7}{4}$$

c) $\lim_{x \to 3} \frac{\sqrt{x+6}-3}{3-x} = \frac{\sqrt{9}-3}{3-3} = \frac{0}{0}$ → Indeterminación

$\lim_{x \to 3} \frac{\sqrt{x+6}-3}{3-x} = \lim_{x \to 3} \frac{(\sqrt{x+6}-3)(\sqrt{x+6}+3)}{(3-x)(\sqrt{x+6}+3)} = \lim_{x \to 3} \frac{x+6-9}{(3-x)(\sqrt{x+6}+3)}$ → Operar numerador

$$\lim_{x \to 3} \frac{x-3}{(3-x)(\sqrt{x+6}+3)} = \lim_{x \to 3} \frac{-1}{\sqrt{x+6}+3} = \frac{-1}{\sqrt{9}+3} = \frac{-1}{6}$$

d) $\lim_{x \to 4} \frac{2x-8}{\sqrt{x^2+9}-5} = \frac{2\cdot 4-8}{\sqrt{25}-5} = \frac{0}{0}$ → Indeterminación

$\lim_{x \to 4} \frac{2x-8}{\sqrt{x^2+9}-5} = \lim_{x \to 4} \frac{(2x-8)(\sqrt{x^2+9}+5)}{(\sqrt{x^2+9}-5)(\sqrt{x^2+9}+5)}$ → Operar denominador

$$\lim_{x \to 4} \frac{(2x-8)(\sqrt{x^2+9}+5)}{(\sqrt{x^2+9}-5)(\sqrt{x^2+9}+5)} = \lim_{x \to 4} \frac{(2x-8)(\sqrt{x^2+9}+5)}{x^2+9-25}$$

$\lim_{x \to 4} \frac{(2x-8)(\sqrt{x^2+9}+5)}{x^2-16} = \lim_{x \to 4} \frac{(2x-8)(\sqrt{x^2+9}+5)}{(x+4)(x-4)}$ → Operar numerador

$$\lim_{x \to 4} \frac{(2x-8)(\sqrt{x^2+9}+5)}{(x+4)(x-4)} = \lim_{x \to 4} \frac{2(x-4)(\sqrt{x^2+9}+5)}{(x+4)(x-4)} = \lim_{x \to 4} \frac{2(\sqrt{x^2+9}+5)}{(x+4)} = \frac{2(\sqrt{25}+5)}{4+4} = \frac{20}{8} = \frac{5}{2}$$

e) $\lim_{x \to -2} \frac{3-\sqrt{2x^2+1}}{3x-6} = \frac{3-\sqrt{2\cdot(-2)^2+1}}{3(-2)-6} = \frac{3-\sqrt{9}}{-6-6} = \frac{0}{-12} = 0$

f) $\lim_{x \to 2} \frac{3-\sqrt{2x^2+1}}{3x-6} = \frac{0}{0}$ → Indeterminación

$$\lim_{x \to 2} \frac{3-\sqrt{2x^2+1}}{3x-6} = \lim_{x \to 2} \frac{(3-\sqrt{2x^2+1})(3+\sqrt{2x^2+1})}{(3x-6)(3+\sqrt{2x^2+1})}$$ → Operar numerador

$$\lim_{x \to 2} \frac{9-(2x^2+1)}{(3x-6)(3+\sqrt{2x^2+1})} = \lim_{x \to 2} \frac{-2x^2+8}{(3x-6)(3+\sqrt{2x^2+1})} = \lim_{x \to 2} \frac{-2(x^2-4)}{(3x-6)(3+\sqrt{2x^2+1})}$$

$$\lim_{x \to 2} \frac{-2(x+2)(x-2)}{(3\text{x}-6)(3+\sqrt{2x^2+1})}$$ → Operar denominador

$$\lim_{x \to 2} \frac{-2(x+2)(x-2)}{3(x-2)(3+\sqrt{2x^2+1})} = \lim_{x \to 2} \frac{-2(x+2)}{3(3+\sqrt{2x^2+1})} = \frac{-2(2+2)}{3(3+\sqrt{2(2)^2+1})} = \frac{-8}{18} = \frac{-4}{9}$$

g) $\lim_{x \to \frac{1}{2}} \frac{4x^2+4x-3}{4x^2-1} = \frac{\frac{4}{4}+\frac{4}{2}-3}{\frac{4}{4}-1} = \frac{0}{0}$ → Indeterminación

$$\lim_{x \to \frac{1}{2}} \frac{4x^2+4x-3}{4x^2-1} = \lim_{x \to \frac{1}{2}} \frac{(x-\frac{1}{2})(4x+6)}{4(x^2-\frac{1}{4})} = \lim_{x \to \frac{1}{2}} \frac{(x-\frac{1}{2})(4x+6)}{4(x+\frac{1}{2})(x-\frac{1}{2})} = \lim_{x \to \frac{1}{2}} \frac{4x+6}{4(x+\frac{1}{2})} = \frac{\frac{4}{2}+6}{4(\frac{1}{2}+\frac{1}{2})} = 2$$

3. Resuelve:

a) $\lim_{x \to 0} \frac{\sqrt{1-x}-1}{x} = \frac{0}{0}$ → Indeterminación

$$\lim_{x \to 0} \frac{\sqrt{1-x}-1}{x} = \lim_{x \to 0} \frac{(\sqrt{1-x}-1)(\sqrt{1-x}+1)}{x(\sqrt{1-x}+1)} = \lim_{x \to 0} \frac{1-x-1}{x(\sqrt{1-x}+1)} = \lim_{x \to 0} \frac{-x}{x(\sqrt{1-x}+1)}$$

$$\lim_{x \to 0} \frac{-1}{\sqrt{1-x}+1} = \frac{-1}{2}$$

b) $\lim_{x \to 0} \frac{(2+x)^3-8}{x} = \frac{0}{0}$ → Indeterminación

$$\lim_{x \to 0} \frac{(2+x)^3-8}{x} = \lim_{x \to 0} \frac{(4+x^2+4x)(2+x)-8}{x} = \lim_{x \to 0} \frac{x^3+6x^2+12x+8-8}{x} = \lim_{x \to 0} \frac{x^3+6x^2+12x}{x}$$

$$\lim_{x \to 0} \frac{x^3+6x^2+12x}{x} = \lim_{x \to 0} x^2+6x+12 = 12$$