Lesson 5: Comparing speeds and prices

Goals

- Explain (orally and in writing) that if two ratios have the same unit rate, they are equivalent ratios.
- Justify (orally and in writing) comparisons of speeds or prices.
- Recognise that calculating how much for 1 of the same unit is a useful strategy for comparing rates. Express these rates (in spoken and written language) using the word "per" and specifying the unit.

Learning Targets

- I understand that if two ratios have the same unit rate, they are equivalent ratios.
- When measurements are expressed in different units, I can decide who is travelling faster or which item is the better deal by comparing "how much for 1" of the same unit.

Lesson Narrative

Previously, students found and used unit rates to solve problems in a context. This lesson is still about contexts, but it's more deliberately working toward the general understanding that when two ratios are associated with the same unit rate, then they are equivalent ratios. Therefore, to determine whether two ratios are equivalent, it is useful to find and compare their associated unit rates. In this lesson, we also want students to start to notice that dividing one of the quantities in a ratio by the other is an efficient way to find a unit rate, while attending to the meaning of that number in the context.

Calculating unit rates is also a common way to compare rates in different situations. For example, suppose we find that one car is travelling 30 miles per hour and another car is travelling 40 miles per hour. The different rates tell us not only that the cars are travelling at different speeds, but which one is travelling faster. Similarly, knowing that one grocery store charges £1.50 per item while another charges £1.25 for the same item allows us to select the better deal even when the stores express the costs with rates such as "2 for £3" or "4 for £5."

Building On

- Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b} = a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Addressing

- Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a : b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. Expectations for unit rates in this stage are limited to non-complex fractions.
- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Co-Craft Questions
- Discussion Supports

Required Preparation

For the activity The Best Deal on Beans, consider gathering some examples of supermarket advertisements from newspapers/weekly emails/websites for deals like “3 for £5.”

Student Learning Goals

Let’s compare some speeds and some prices.

5.1 Closest Quotient

Warm Up: 5 minutes

This warm-up prompts students to reason about the meaning of division by looking closely at the dividend and divisor. The expressions were purposely chosen to encourage more precise reasoning than roughly estimating. While some students may mentally solve each, encourage them to also think about the numbers in the problem without calculating. Ask them what would happen if the dividend or divisor increased or decreased. Expect students to think of fractions both as division and as numbers. Encourage connections between these two ideas.

Instructional Routines

- Discussion Supports

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation Speaking, Reading Representing: Discussion Supports. Display a table that shows different representations and language used for $\frac{1}{2}$, 1, and $1\frac{1}{2}$. Highlight differences between similar-looking or similar-sounding language like "one", "one half", "a half", "one and a half", etc.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

Is the value of each expression closer to $\frac{1}{2}$, 1, or $1\frac{1}{2}$?

1. $20 \div 18$

2. $9 \div 20$

3. $7 \div 5$

Student Response

1. Closer to 1. Possible strategy: $20 \div 20 = 1$ and since 18 is less than 20, the quotient is more than 1 and since $27 \div 18 = 1.5$ and 20 is less than 27, the quotient is less than 1.5. The distance from 1 and 1.5 could be reasoned about thinking about the size of the leftovers, $\frac{2}{18}$ versus $\frac{7}{18}$

2. Closer to $\frac{1}{2}$. Possible strategy: since $\frac{10}{20} = \frac{1}{2}$ and since 9 is less than 10, the quotient is less than $\frac{1}{2}$, but only a small $\frac{1}{20}$ away.

3. Closer to $1\frac{1}{2}$. Possible strategy: $\frac{5}{5} = 1$, the quotient is $\frac{2}{5}$ over 1. $\frac{2}{5} = 0.4$ which is closer to 1.5 than 1.

Activity Synthesis

Discuss each problem one at a time with this structure:

- Ask students to indicate whether they think the expression is closer to $\frac{1}{2}$, 1, or $1\frac{1}{2}$.

- If everyone agrees on one answer, ask a few students to share their reasoning, recording it for all to see. If there is disagreement on an answer, ask students with opposing answers to explain their reasoning to come to an agreement on an answer.

5.2 More Treadmills

15 minutes

In this activity, students analyse the workouts of several people on a treadmill given time-distance ratios. The purpose of this activity is to remind students how speed contexts work and to start to nudge them toward more efficient ways to compare speeds. Students see that when such ratios can be expressed with the same number of metres per minute, the ratios are equivalent and the moving objects (people, cars, etc.) have the same speed.

Speed is typically expressed as a distance per 1 unit of time, so the task provides a familiar context for computing and using unit rates. The numbers have been chosen such that any two workouts being compared has the same time, same distance, or same speed.

Encourage students to use “per 1” and “for each” language throughout, as this language supports the development of the concept of unit rate.

As students discuss the problems, listen closely for those who use these terms as well as descriptions of speed (e.g., “same speed,” “faster,” “slower”). Also notice students who make the connections between the unit rates they calculated in the first half of the task and use them to answer questions in the second half. Invite some of these students or groups to share later.

Instructional Routines

- Co-Craft Questions

Launch

Arrange students in groups of 3. Give students 2–3 minutes of quiet think time to complete the first three questions. Then, ask them to share their responses and complete the last three questions in their groups.

Specify that, when discussing the first three questions (comparisons of pairs of runners), each student in the group should take the lead on analysing one sub-problem (i.e., sharing how the workouts of the two given runners are similar or different).

Representation: Access for Perception. Read the problem aloud. Students who both listen to and read the information will benefit from extra processing time. Check for understanding by asking 1-2 students to restate the problem in their own words.
Supports accessibility for: Language Speaking, Writing: Co-Craft Questions. Display the constant speed of Tyler, Kiran, and Mai and ask pairs of students to write possible mathematical questions about the situation. They can also ask questions about information that might be missing, or even about assumptions that they think are important. Then, invite select pairs to share their questions with the class. Look for questions that require students to make comparisons about different speeds. Finally, reveal the actual questions students are expected to work on, and students are set to work. This routine creates space for students to produce the language of mathematical questions as well as develop the language used to talk about constant speed.
Design Principle(s): Optimise output (for questioning); Cultivate conversation

Anticipated Misconceptions

If students are not sure how to begin, suggest that they try using a table or a double number line that associates metres and minutes.

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Student Task Statement

Some students did treadmill workouts, each one running at a constant speed. Answer the questions about their workouts. Explain or show your reasoning.

- Tyler ran 4 200 metres in 30 minutes.
- Kiran ran 6 300 metres in $\frac{1}{2}$ hour.
- Mai ran 6.3 kilometres in 45 minutes.

1. What is the same about the workouts done by:
   a. Tyler and Kiran?
   b. Kiran and Mai?
   c. Mai and Tyler?
2. At what rate did each of them run?
3. How far did Mai run in her first 30 minutes on the treadmill?

Student Response

a. Tyler and Kiran both ran for the same amount of time: 30 minutes. Kiran ran a greater distance in 30 minutes, so Kiran was running faster than Tyler.

b. Kiran and Mai ran the same distance, 6,300 metres, but Mai took more time than Kiran to run 6 300 metres, so Mai was running slower than Kiran.

c. Mai and Tyler both ran 140 metres per minute, so Mai and Tyler were running at the same speed. However they ran different distances and took different amounts of time to do so.

1. The tables show one possible strategy. Some students may reason with double number lines while others may simply calculate $\frac{b}{a}$ for the given ratio $a : b$.

   Tyler ran 140 metres per minute.

distance (metres)	time (minutes)
4 200	30
1 400	10
140	1

Kiran ran 210 metres per minute.

distance (metres)	time (minutes)
6 300	30

2 100	10
1 050	5
210	1

Mai ran 140 metres per minute.

distance (metres)	time (minutes)
6 300	45
140	1

2. Mai ran 4 200 metres in 30 minutes, because she is going the same speed as Tyler and that is how far Tyler ran in 30 minutes.

Are You Ready for More?

Tyler and Kiran each started running at a constant speed at the same time. Tyler ran 4 200 metres in 30 minutes and Kiran ran 6 300 metres in $\frac{1}{2}$ hour. Eventually, Kiran ran 1 kilometre more than Tyler. How much time did it take for this to happen?

Student Response

Just over 14 minutes. Kiran runs 2100 metres more than Tyler in 30 minutes. Each minute he runs 70 metres more, so it will take $\frac{1\,000}{70} = 14\frac{2}{7}$ minutes for him to run 1 kilometre more.

difference (metres)	time (minutes)
2 100	30
70	1
1 000	$\frac{1\,000}{70}$

Activity Synthesis

Focus the conversation on the questions in the second half of the activity, the idea of "same speed," and the clues that two objects are moving equally fast or slow. To begin the conversation, ask: "How can you tell when things are going the same speed?" Give a moment of quiet think time before soliciting responses. Students may say: "They keep up with one another running on a track," "same distance in the same time," or "same miles per hour in a car," etc.

Invite a few students to share their analyses of how the runners compare, starting with how Tyler's workout compares to Kiran's, and how Kiran's compares to Mai's. Descriptions such as "slower," "faster," or "higher or lower speed" should begin to emerge. After students share their analyses of Mai and Tyler's workouts, make sure to highlight that even though they ran different distances in different amounts of time, they each ran 140 metres per minute so we can say "they ran at the same speed." This also means that Mai and Tyler's original ratios—4 200 : 30 and 6 300 : 45—are equivalent ratios.

In the last problem, students need to understand that since Mai and Tyler ran at the same speed they travelled the same distance for the first 30 minutes on the treadmill. This may be difficult for students to articulate with precision, so allowing multiple students to share their thinking may be beneficial.

5.3 The Best Deal on Beans

15 minutes

Students use and compare unit rates in a shopping context as they look for "the best deal." The purpose of this activity is to remind students how unit price contexts work and to start to nudge them toward more efficient ways to compare unit prices.

While this task considers "the best deal" to mean having the lowest cost per unit, the phrase may have different meanings to students and should be discussed. For instance, students may bring up other considerations such as distance to store, store preference (e.g., some stores offer loyalty points), what else they need to purchase, and not wanting to buy in bulk when only a small quantity is needed. Discussing these real-life considerations, and choosing which to prioritise and which to disregard, is an important part of modelling with mathematics, but it is also appropriate to clarify that, for the purposes of this problem, we are looking for "the best deal" in the sense of the lowest cost per can.

As students work, monitor for students who use representations like double number lines or tables of equivalent ratios. These are useful for making sense of a strategy that divides the price by the number of cans to find the price per 1. Also monitor for students using more efficient strategies.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time

Launch

While some students may help with grocery shopping at home, it is likely many have not and will need extra information to understand what "the best deal" means.

Before students begin, ask if anyone is familiar with the weekly fliers that many stores send out to advertise special deals. Show students some advertisements from local stores, if available.

Ask students to share what "a good deal" and "the best deal" mean to them. Many students are likely to interpret these in terms of low prices (per item or otherwise) or "getting more for less money," but some may have other practical or personal considerations. (Examples: it is not a good deal if you buy more than you can use before it goes bad. It is not a good deal if you have to travel a long distance to the store.) Acknowledge students' perspectives and how "messy" such seemingly simple terms can be. Clarify that in this task, we are looking for "the best deal" in the sense of lowest cost per can.

5–10 minutes of quiet work time followed by whole-class discussion.

Action and Expression: Internalise Executive Functions. Provide students with a graphic organiser to organise their problem-solving strategy. The graphic organiser should ask students to identify what they need to find out, what information is provided, how they solved the problem, and why their answer is correct.
Supports accessibility for: Language; Organisation Writing, Listening, Conversing: Stronger and Clearer Each Time. Display the ads about the special sales on 15-oz cans of baked beans. Ask students to write a brief explanation to answer the prompt, “Which store is offering the best deal? Explain your reasoning.” Ask each student to share their written explanation with 2–3 partners who will provide constructive feedback. Students can use ideas and language from each partner to refine and clarify the response through conversation, and then finally revise their original written response. Throughout this process, students should be pressed for details, and encouraged to press each other for details. This helps students to compare unit rates in a shopping context as they look for “the best deal.”
Design Principle(s): Optimise output (for explanation); Cultivate conversation

Anticipated Misconceptions

At first glance, students may look only at the number of cans in each offer or only at the price. Let students know that they need to consider the price per one can.

Student Task Statement

Four different stores posted ads about special sales on 15-oz cans of baked beans.

1. Which store is offering the best deal? Explain your reasoning.

2. The last store listed is also selling 28-oz cans of baked beans for £1.40 each. How does that price compare to the other prices?

Student Response

1. 8 for £6 is the best deal at £0.75 per can. 2 for £3 is the worst deal at £1.50 per can. Possible strategies:

   – 8 for £6

price (pounds)	cans
6	8
3	4
0.75	1

   – £10 for 10 cans means £1 per can.

   – 2 for £3:

price (pounds)	cans
3	2
1.50	1

   – 80p per can is the same as £0.80 per can.

2. The last store's 28 oz can for £1.40 is the same price per ounce as the first store's 15 oz can for £0.75, because $0.75 \div 15 = 0.05$ and $1.40 \div 28 = 0.05$.

Activity Synthesis

If any students used a representation like a double number line or table to support their reasoning, select these students to share their strategy first. Keep these representations visible. Follow with explanations from students who used more efficient strategies, and use the representations to make connections to more efficient strategies. Highlight the use of division to calculate the price per can and the use of “per 1” language. The purpose of this activity and this discussion is help students see that computing and comparing the price per 1 is an efficient way to compare rates in a unit price context.

Lesson Synthesis

Previously, students compared rates of different ratios by showing that they are or are not equivalent, and by using diagrams and scale factors. In prior lessons students found unit rates as a way to determine equivalent ratios. Here unit rates, in the form of speed and unit price, are deliberately calculated so that they can be compared.

To help students summarise their thinking, display a list of the stated ratios in each activity and how they would be written in “per 1” rate language, as shown here:

rate as given	unit rate
4 200 metres in 30 minutes	140 metres per minute
6 300 metres in 30 minutes	210 metres per minute
6 300 metres in 45 minutes	140 metres per minute
–	–
8 cans for £6	£0.75 per can
10 cans for £10	£1.00 per can
2 cans for £3	£1.50 per can
80p per can	£0.80 per can

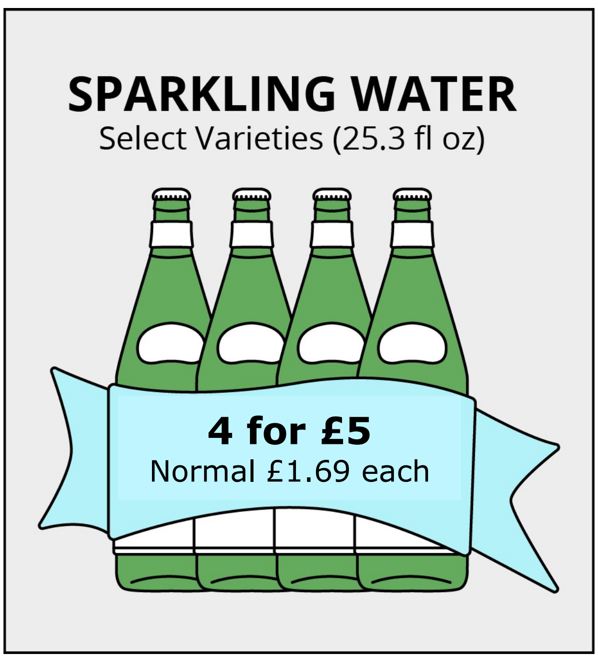
Give students some quiet time to read through the list. Then, ask 2–3 students to share which rate they prefer for comparing and why ("I prefer the unit rates because I can just compare two numbers, since the 1 is the same.").

5.4 A Sale on Sparkling Water

Cool Down: 5 minutes

Student Task Statement

Bottles of sparkling water usually cost £1.69 each. This week they are on sale for 4 bottles for £5. You bought one last week and one this week. Did you pay more or less for the bottle this week? How much more or less?



Student Response

I paid £0.44 less this week. Possible strategy: Since 4 bottles cost £5, each bottle costs $£5 \div 4$, or £1.25 this week. The difference is £0.44, because $1.69 - 1.25 = 0.44$. This is, of course, assuming I can buy just one bottle and have this lower rate apply.

Student Lesson Summary

Diego ran 3 kilometres in 20 minutes. Andre ran 2 550 metres in 17 minutes. Who ran faster? Since neither their distances nor their times are the same, we have two possible strategies:

- Find the time each person took to travel the *same distance*. The person who travelled that distance in less time is faster.
- Find the distance each person travelled in the *same time*. The person who travelled a longer distance in the same amount of time is faster.

  It is often helpful to compare distances travelled in *1 unit* of time (1 minute, for example), which means finding the speed such as metres per minute.

Let's compare Diego and Andre's speeds in metres per minute.

distance (metres)	time (minutes)
3 000	20
1 500	10
150	1
distance (metres)	time (minutes)
2 550	17
150	1

Both Diego and Andre ran 150 metres per minute, so they ran at the same speed.

Finding ratios that tell us how much of quantity A per 1 unit of quantity B is an efficient way to compare rates in different situations. Here are some familiar examples:

- Car speeds in *miles per hour*.
- Fruit and vegetable prices in *pounds per pound*.

Glossary

- unit price

Lesson 5 Practice Problems

Problem 1 Statement

Mai and Priya were on scooters. Mai travelled 15 metres in 6 seconds. Priya travels 22 metres in 10 seconds. Who was moving faster? Explain your reasoning.

Solution

Mai's scooter is faster. $22 \div 10 = 2.2$, so Priya's scooter travels at a rate of 2.2 metres per second. $15 \div 6 = 2.5$, so Mai's scooter travels at a rate of 2.5 metres per second.

Problem 2 Statement

Here are the prices for cans of juice that are the same brand and the same size at different stores. Which store offers the best deal? Explain your reasoning.

Store X: 4 cans for £2.48

Store Y: 5 cans for £3.00

Store Z: 59p per can

Solution

Store Z has the best deal. $2.48 \div 4 = 0.62$ or 62p per can. $3 \div 5 = 0.6$ or 60p per can. 59p is the least expensive of the 3 options.

Problem 3 Statement

Costs of homes can be very different in different parts of the United States.

a. A 450-square-foot apartment in New York City costs £540 000. What is the price per square foot? Explain or show your reasoning.

b. A 2 100-square-foot home in Cheyenne, Wyoming, costs £110 per square foot. How much does this home cost? Explain or show your reasoning.

Solution

a. £1 200 ($540\,000 \div 450 = 1\,200$)

b. £231 000 ($2\,100 \times 110 = 231\,000$)

Problem 4 Statement

There are 33.8 fluid ounces in a litre. There are 128 fluid ounces in a gallon. About how many litres are in a gallon?

a. 2

b. 3

c. 4

d. 5

Is your estimate larger or smaller than the actual number of litres in a gallon? Explain how you know.

Solution

There are about 4 litres in a gallon. Sample explanation: This estimate is too big: $4 \times 32 = 128$, so $4 \times (33.8)$ is larger than 128.

Problem 5 Statement

Diego is 165 cm tall. Andre is 1.7 m tall. Who is taller, Diego or Andre? Explain your reasoning.

Solution

Andre is taller. 1.7 m is 170 cm, and $170 > 165$.

Problem 6 Statement

Name an object that could be about the same length as each measurement.

a. 4 inches

b. 6 feet

c. 1 metre

d. 5 yards

a. 6 centimetres

b. 2 millimetres

c. 3 kilometres

Solution

Answers vary. Sample response:

a. Pencil

b. Ladder

c. Person's leg

d. Tablecloth

e. Insect

f. Grain of rice

g. Foot race