Lesson 4: Solving for unknown angles

Goals

- Coordinate (orally and in writing) diagrams and equations that represent the same relationship between the sizes of angles.
- Solve multi-step problems involving complementary, supplementary, and vertically opposite angles, and explain (orally) the reasoning.

Learning Targets

- I can reason through multiple steps to find the size of unknown angles.
- I can recognise when an equation represents a relationship between angles.

Lesson Narrative

In previous lessons, students solved single-step problems about supplementary, complementary, and vertically opposite angles. In this lesson, students apply these skills to find unknown angles in multi-step problems. In the info gap activity, students keep asking questions until they get all the information needed to solve the problem. Then they see that they can represent angle problems with equations.

Addressing

- Draw, construct, and describe geometrical shapes and describe the relationships between them.
- Solve real-life and mathematical problems involving the size of angles, area, surface area, and volume.
- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a diagram.

Building Towards

- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a diagram.

Instructional Routines

- Information Gap Cards
- Think Pair Share
- True or False

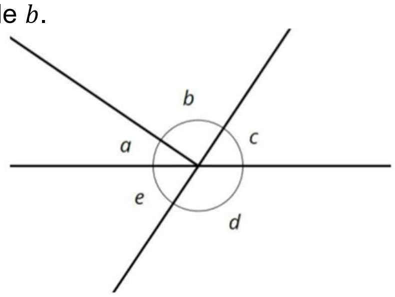
Required Materials

Pre-printed slips, cut from copies of the blackline master

Info Gap: Angle finding

Problem Card 1

Find angle b.



Info Gap: Angle finding

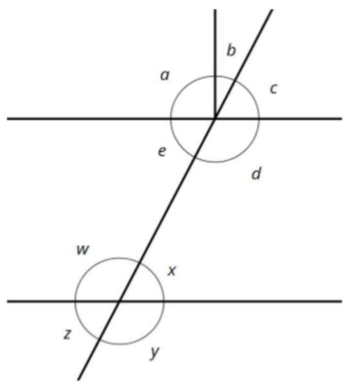
Data Card 1

- Angles c and e are vertically opposite angles
- Angles c and a are complementary angles.
- Angle $d = 124°$
- Angle $c = 56°$

Info Gap: Angle finding

Problem Card 2

Find angle b.



Info Gap: Angle finding

Data Card 2

- Angle $a = 90°$
- Angle $z = 63°$
- Angles c and z are not vertically opposite angles.
- Angles d and x are supplementary angles.

Required Preparation

Make 1 copy of the Info Gap: Angle Finding blackline master for every 2 students, and cut them up ahead of time.

Student Learning Goals

Let’s figure out some missing angles.

4.1 True or False: Length Relationships

Warm Up: 5 minutes

The purpose of this warm-up is to have students express relationships between lengths with equations, in preparation for doing the same with angles in upcoming activities.

Instructional Routines

- True or False

Launch

Remind students that we refer to a length of a line segment by naming its endpoints. For example, AB means the length of the line segment from A to B.

Display one problem at a time. Tell students to give a signal when they have decided if the equation is true or false. Give students 1 minute of quiet think time followed by a whole-class discussion.

Student Task Statement

Here are some line segments.

Decide if each of these equations is true or false. Be prepared to explain your reasoning.

$CD + BC = BD$

$AB + BD = CD + AD$

$AC - AB = AB$

$BD - CD = AC - AB$

Student Response

1. True
2. False
3. False
4. True

Activity Synthesis

Ask students to share their reasoning for each equation. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Do you agree or disagree? Why?”
- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to _____’s reasoning?”

After each true equation, ask students if they could rely on the same reasoning to determine if other similar problems are equivalent. After each false equation, ask students how the problem could be changed to make the equation true.

4.2 Info Gap: Angle Finding

20 minutes

The purpose of this info gap activity is for students to see how they can use different pieces of information to solve for the size of an unknown angle in a multi-step problem. During the whole-class discussion, students are introduced to writing and solving equations to represent the relationships between angles.

The info gap structure requires students to make sense of problems by determining what information is necessary. This may take several rounds of discussion. Since there are many different pieces of information that *could* be used to solve the problem but are not given on the data card, consider using a variation on the typical info gap structure: Instead of the student with the problem card asking for specific pieces of information, the student with the data card chooses a piece of information to share, and the student with the problem card explains how they use that piece of information. If enough information hasn't been given, the student with the data card chooses another piece of information to share.

You will need the blackline master for this activity.

As students work, monitor for those who use different strategies or start with different pieces of information. Also, monitor for students who choose to show their reasoning by writing and solving equations.

Instructional Routines

- Information Gap Cards

Launch

If desired, explain this variation from the typical info gap: instead of the student with the problem card asking for each piece of information, the student with the data card chooses a piece of information to share. The student with the problem card still needs to explain how they can use each piece of information. If more information is needed to solve the problem, the student with the data card chooses another piece of information to share. Also, students need to listen to their partner carefully because they may be asked to explain their partner's reasoning to the class.

Arrange students in groups of 2. Distribute a problem card to one student and a data card to the other student in each group.

Action and Expression: Internalise Executive Functions. Begin with a small-group or whole-class demonstration and think aloud of a sample situation problem card and data card to remind students how to use the info gap structure. Keep the worked-out equations and angle drawing on display for students to reference as they work.
Supports accessibility for: Memory; Conceptual processing Conversing: Use this modified version of *Information Gap* to give students an opportunity to discuss the information necessary to solve for an unknown angle. Display the follow sentence frames to support student discussion: "Can you tell me . . . (specific piece of information)", "Why do you need to know . . . (that piece of information)?", and "I can use this information to . . . ."
Design Principle(s): Cultivate conversation

Anticipated Misconceptions

For the second set of cards, students may struggle to find the connection between the lower half of the diagram and the upper half. Remind them that supplementary angles do not need to be next to one another, but they can be.

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

1. $b = 90°$. Possible strategies:
   - $c = 56, a = 90 - 56 = 34, b = 180 - 34 - 56 = 90$

- $d = 124$, $e = 180 - 124 = 56$, $c = 56$, $a = 90 - 56 = 34$, $b = 180 - 34 - 56 = 90$
- $a + c = 90$, $a + b + c = 180$, $b = 180 - 90 = 90$

2. $b = 27°$. Possible strategies:
   - $z = 63$, $x = 63$, $d = 180 - 63 = 117$, $c = 180 - 117 = 63$, $a = 90$, $b = 180 - 90 - 63 = 27$
   - $z = 63$, $w = 180 - 63 = 117$, $x = 180 - 117 = 63$, $d = 180 - 63 = 117$, $e = 180 - 117 = 63$, $c = 63$, $a = 90$, $b = 360 - 117 - 63 - 90 - 63 = 27$

Activity Synthesis

The goal of this discussion is for students to see that writing and solving equations is an efficient strategy to show their reasoning about multi-step angle problems.

For each problem, select a student who had the information card to share their partner's reasoning. Record their reasoning using equations and display for all to see. Listen carefully and make sure the equation matches how they explain their reasoning. Here are some sample equations for the first problem.

solving for e given d

$$\begin{aligned} e + d &= 180 \\ e + 124 &= 180 \\ e &= 180 - 124 \\ e &= 56 \end{aligned}$$

solving for a given c

$$\begin{aligned} a + c &= 90 \\ a + 56 &= 90 \\ a &= 90 - 56 \\ a &= 34 \end{aligned}$$

solving for e given c

$c = 56$

$e = ce = 56$

solving for b given that a and c are complementary

$a + c = 90$

$$\begin{aligned} a + b + c &= 180 \\ b + (a + c) &= 180 \\ b + 90 &= 180 \\ b &= 180 - 90 \\ b &= 90 \end{aligned}$$

Display the equations that represent different students' strategies side by side and have students contrast the different methods, for example, the difference between how two students worked the problem if one was given the size of angle d first but the other was given the size of angle c first.

4.3 What's the Match?

10 minutes

The purpose of this activity is for students to match relationships between angles in a diagram with equations that can represent those relationships. This prepares students for writing and solving equations that represent relationships between angles in the next lesson. As students explain their reasoning, monitor for students who use the appropriate vocabulary and language (i.e. vertically opposite angles are equal, supplementary angles sum to 180 degrees, etc.).

Instructional Routines

- Think Pair Share

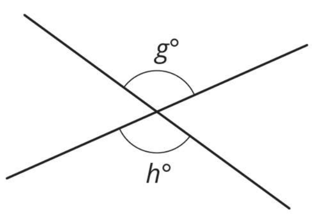
Launch

Keep students in same groups. Give students 2–3 minutes of quiet work time followed by partner and whole-class discussions.

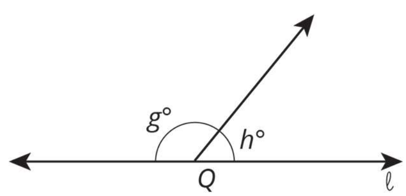
Student Task Statement

Match each diagram to an equation that represents what is seen in the diagram. For each match, explain how you know they are a match.

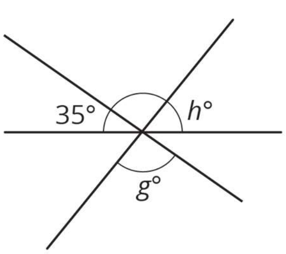
A



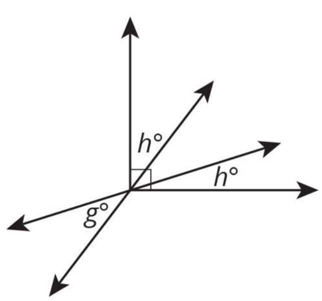
B



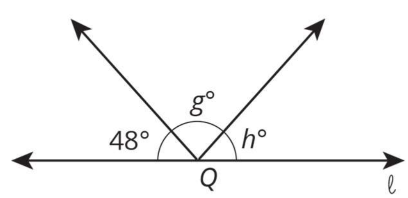
C



D



E



1. $g + h = 180$
2. $g = h$
3. $2h + g = 90$
4. $g + h + 48 = 180$
5. $g + h + 35 = 180$

Student Response

A. 2. (This is because g and h are supplementary angles, $g + h = 180$.)

B. 1. (g and h are vertically opposite angles and vertically opposite angles are equal so $g = h$.)

C. 5. (The three angles form a straight line and the angle in between the angle measuring 35° and the angle measuring h degrees is a vertically opposite angle with the angle measuring g degrees, which means $g + h + 35 = 180$.)

D. 3. (The three angles in the diagram form a right angle, and the angle that measures g degrees is vertically opposite to one of those angles, which means $2h + g = 90$.)

E. 4. (The angles all form a straight angle, which is why $g + h + 48 = 180$.)

Are You Ready for More?

1. What is the angle between the hour and minute hands of a clock at 3:00?
2. You might think that the angle between the hour and minute hands at 2:20 is 60 degrees, but it is not! The hour hand has moved beyond the 2. Calculate the angle between the clock hands at 2:20.
3. Find a time where the hour and minute hand are 40 degrees apart. (Assume that the time has a whole number of minutes.) Is there just one answer?

Student Response

1. 90 degrees
2. 50 degrees. At 2:20, the hour hand should be one-third of the way between the 2 and the 3. Since the angle between the 2 and the 3 is a 30-degree angle, the hour hand has moved 10 degrees toward the 3. Therefore, the angle is 50 degrees rather than 60.
3. 5:20 and 6:40. These can be found by guess-and-check. It may help to realise that the hour hand of a clock moves at half a degree per minute, and the minute hand of a clock moves at 6 degrees per minute.

Activity Synthesis

The goal of this discussion is for students to articulate the angle relationships they noticed in each diagram and equation. Display the diagrams for all to see. Select previously identified students to share their explanations for each diagram. If not mentioned in students' explanations be sure that students see the vertical, supplementary, straight, and right angle relationships in the diagrams.

Lesson Synthesis

- If you know that angles a and b are vertical, what equation could you use to represent this angle relationship? ($a = b$)
- If you know that angles c and d are complementary, what equation could you use to represent this angle relationship? ($c + d = 90$)
- If you know that angles e and f are supplementary, what equation could you use to represent this angle relationship? ($e + f = 180$)

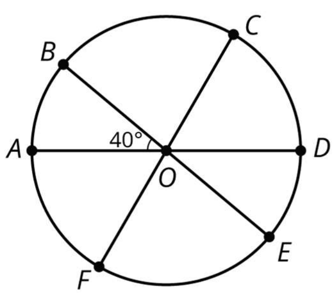
As the situations become more intricate, you can use equations to keep track of what you know so that you can revisit them when you learn more information.

4.4 Missing Circle Angles

Cool Down: 5 minutes

Student Task Statement

AD, BE, and CF are all diameters of the circle. The size of angle AOB is 40 degrees. The size of angle DOF is 120 degrees.



Find angles:

1. BOC
2. COD

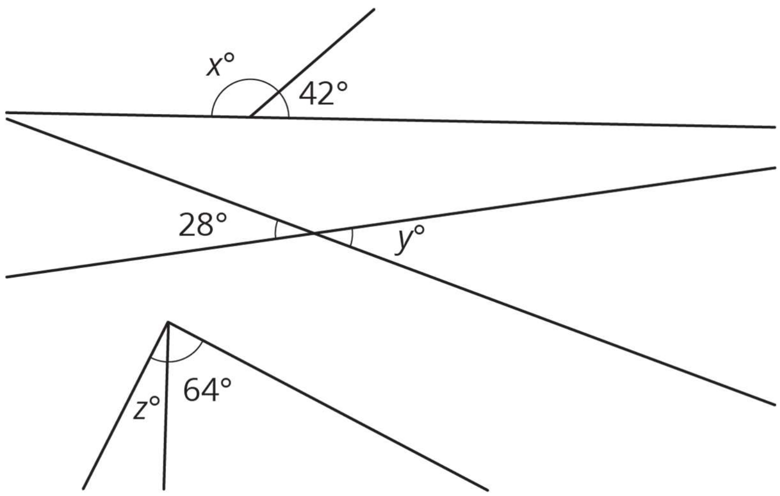
Student Response

1. Angle $BOC = 80°$. Given angle $DOF = 120°$, angle $AOC = 120°$ because they are congruent vertically opposite angles. Consequently, angles $AOB + BOC = 120°$ because they are adjacent.

2. Angle $COD = 60°$. Angle COD sums to $180°$ with angles DOF because the two are supplementary angles.

Student Lesson Summary

We can write equations that represent relationships between angles.

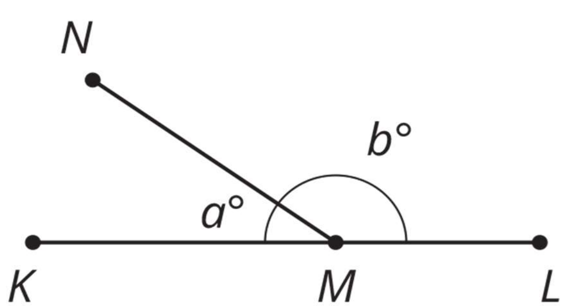


- The first pair of angles are supplementary, so $x + 42 = 180$.
- The second pair of angles are vertically opposite angles, so $y = 28$.
- Assuming the third pair of angles form a right angle, they are complementary, so $z + 64 = 90$.

Lesson 4 Practice Problems

1. **Problem 1 Statement**

   M is a point on line segment KL. NM is a line segment. Select **all** the equations that represent the relationship between the angles in the diagram.



   a. $a = b$

   b. $a + b = 90$

   c. $b = 90 - a$

d. $a + b = 180$

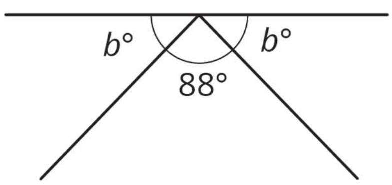
e. $180 - a = b$

f. $180 = b - a$

Solution ["D", "E"]

2. **Problem 2 Statement**

Which equation represents the relationship between the angles in the diagram?



a. $88 + b = 90$

b. $88 + b = 180$

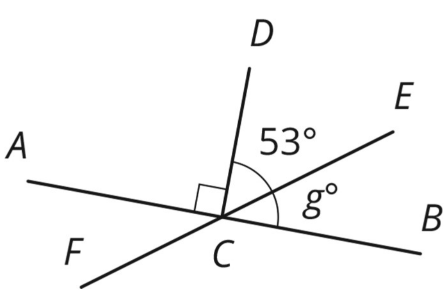
c. $2b + 88 = 90$

d. $2b + 88 = 180$

Solution D

3. **Problem 3 Statement**

Line segments AB, EF, and CD intersect at point C, and angle ACD is a right angle. Find the value of g.



Solution

37

4. **Problem 4 Statement**

Select **all** the expressions that are the result of decreasing x by 80%.

a. $\frac{20}{100}x$

b. $x - \frac{80}{100}x$

c. $\frac{100-20}{100}x$

d. $0.80x$

e. $(1 - 0.8)x$

Solution ["A", "B", "E"]

5. **Problem 5 Statement**

Andre is solving the equation $4\left(x + \frac{3}{2}\right) = 7$. He says, “I can subtract $\frac{3}{2}$ from each side to get $4x = \frac{11}{2}$ and then divide by 4 to get $x = \frac{11}{8}$.” Kiran says, “I think you made a mistake.”

a. How can Kiran know for sure that Andre’s solution is incorrect?

b. Describe Andre’s error and explain how to correct his work.

Solution

Answers vary. Sample responses:

a. He can substitute Andre's solution into the equation. If the solution is correct, the resulting equation will be true. $4\left(\frac{11}{8} + \frac{3}{2}\right)$ is $11\frac{1}{2}$, not 7, so the solution is incorrect.

b. Andre subtracted $\frac{3}{2}$ from each side, but that doesn’t remove the $\frac{3}{2}$ from the equation because $\frac{3}{2}$ is part of an expression multiplied by 4. Andre could divide each side by 4 to get $x + \frac{3}{2} = \frac{7}{4}$ and then subtract $\frac{3}{2}$ on each side to get $x = \frac{1}{4}$. (Or, he could use the distributive property to write $4x + 6 = 7$, subtract 6 from each side to get $4x = 1$, and then divide by 4 on each side to get $x = \frac{1}{4}$.)

6. **Problem 6 Statement**

Solve each equation.

$\frac{1}{7}a + \frac{3}{4} = \frac{9}{8}$

$\frac{2}{3} + \frac{1}{5}b = \frac{5}{6}$

$\frac{3}{2} = \frac{4}{3}c + \frac{2}{3}$

$0.3d + 7.9 = 9.1$

$11.03 = 8.78 + 0.02e$

Solution

a. $a = \frac{21}{8}$

b. $b = \frac{5}{6}$

c. $c = \frac{5}{8}$

d. $d = 4$

e. $e = 112.5$

7. **Problem 7 Statement**

A train travels at a constant speed for a long distance. Write the two constants of proportionality for the relationship between distance travelled and elapsed time. Explain what each of them means.

time elapsed (hr)	distance (mi)
1.2	54
3	135
4	180

Solution

45. The train travels 45 miles in 1 hour

$\frac{1}{45}$. It takes $\frac{1}{45}$ hours for the train to travel 1 mile