Lesson 11: Percentages and double number lines

Goals

- Comprehend a phrase like "A% of B" (in written and spoken language) to refer to the value that makes a ratio with B that is equivalent to A : 100.
- Explain (orally) how to use a double number line diagram or table to solve problems such as A% of B is ? and A% of ? is C.
- State explicitly what one is finding the percentage of.

Learning Targets

- I can use double number line diagrams to solve different problems like "What is 40% of 60?" or "60 is 40% of what number?"

Lesson Narrative

In the previous lesson, students learned to find percentages of 100 and percentages of 1 in the context of money (100 pence and £1). In this lesson, they explore percentages of quantities other than 100 and 1 in a variety of contexts. All of the tasks use comparison contexts—describing one quantity relative to another quantity—rather than part-whole contexts.

Students continue to have double number lines as a reasoning tool to use if they want. In several cases the double number line is provided. There are two reasons for this. First, the equal intervals on the provided double number line are useful for reasoning about percentages. Second, using the same representation that was used earlier for other ratio and rate reasoning reinforces the idea of a percentage as a rate per 100. It is perfectly acceptable, however, for students to use strategies other than double number lines for solving percentage problems.

Addressing

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percentage.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Compare and Connect
- Think Pair Share

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Student Learning Goals

Let's use double number lines to represent percentages.

11.1 Fundraising Goal

Warm Up: 5 minutes

This warm-up is the first time students are asked to find A% of B when B is not 100 or 1.

Students may approach the problem in a few different ways, with or without filling in values on the provided double number line diagram. For example, they may understand from the fundraising context of the problem that £40 is 100% because it is the amount of the goal. From there, they may simply find half to £40 for the 50% value and add that value to £40 to find the 150% value. Other students may use equivalent ratio reasoning to calculate the value at 50% and 150%. As students work, notice the different strategies used and any misconceptions so they can be addressed during discussion.

Launch

Remind students that in the previous lesson, we found percentages of 100 and of 1 using double number lines. Explain that in this lesson we will find percentages of other numbers. Give students 2 minutes of quiet work time, and follow with a whole-class discussion. Encourage students to create a double number line to help them answer the questions if needed.

Anticipated Misconceptions

Students may be surprised by a percentage greater than 100. If they are puzzled by this, explain that Andre raised more money than the goal.

Student Task Statement

Each of three friends—Lin, Jada, and Andre—had the goal of raising £40. How much money did each person raise? Be prepared to explain your reasoning.

1. Lin raised 100% of her goal.
2. Jada raised 50% of her goal.
3. Andre raised 150% of his goal.

Student Response

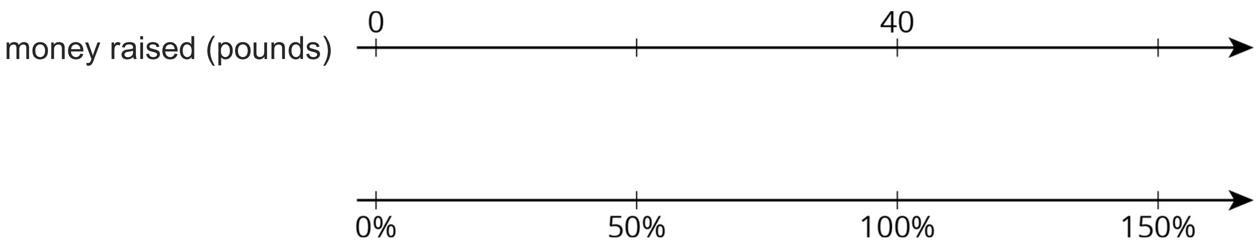
1. £40. Lin raised £40 since £40 is 100% of the goal.

2. £20. Half of 100 is 50, and half of £40 is £20, so Jada raised £20, which is 50% of £40.

3. £60. Since the first tick mark is £20, the third tick mark must be £60. This means Andre raised £60, which is 150% of the £40 goal.

Activity Synthesis

Consider displaying this double number line for all to see.



Invite a few students to share their solving strategies. One way to highlight the different techniques students use is to invite several students to explain how they calculated the amount of money raised by Andre. We can find 150% of 40 in several ways. For example, we can add the values of 50% of 40 and 100% of 40. We can also reason that since $100 \times (1.5) = 150$ then $40 \times (1.5) = 60$, which means £60 is 150% of £40.

If not uncovered in students' explanations, explain that we are finding percentages of £40, since this number—not 100 pence or 1 pound—is the fundraising goal for the three friends. Since 100% of a goal of £40 is £40, the 100% and £40 are lined up on the double number line.

Students who relied on the visual similarity between, for example, £0.25 is 25% of 1 pound in the previous lesson find this strategy unworkable here (as £50 is not 50% of £40). To encourage students to use their understanding of equivalent ratios to reason about percentage problems, ask the class to explain—either when the above misconception arises or as a closing question—why £50 is not 50% of £40, but 50% of 100 pence is 50 pence.

11.2 Three-Day Cycling Trip

15 minutes

In this activity, students find percentages of a value in a non-monetary context. They begin by assigning a value to 100% and reasoning about other percentages.

The double number line is provided to communicate that we can use all our skills for reasoning about equivalent ratios to reason about percentages. Providing this representation makes it more likely that students will use it, but it would be perfectly acceptable for them to use other strategies.

Monitor for students using these strategies:

- Use a double number line and reason that since 25% is $\frac{1}{4}$ of 100%, 25% of 8 is $8 \times \frac{1}{4} = 2$. They may then skip count by the value for the first tick mark to find the values for other tick marks.
- Reason about 125% of the distance as 100% of the distance plus 25% of the distance, and add 8 and $\frac{1}{4}$ of 8.
- Reason about 75% of 8 directly by multiplying 8 by $\frac{3}{4}$ and 125% of 8 by multiply 8 by $\frac{5}{4}$.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Compare and Connect

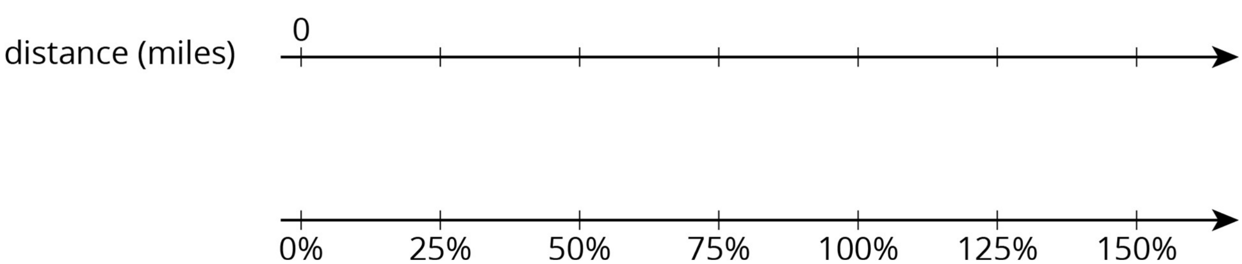
Launch

Arrange students in groups of 3–4. Provide tools for making a large visual display. Give students 2–3 minutes of quiet think time. Encourage students to use the double number line to help them answer the questions if needed. Afterwards, ask them to discuss their responses to the last two questions with their group and to create a visual display of one chosen strategy to be shared with the class.

Representation: Internalise Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

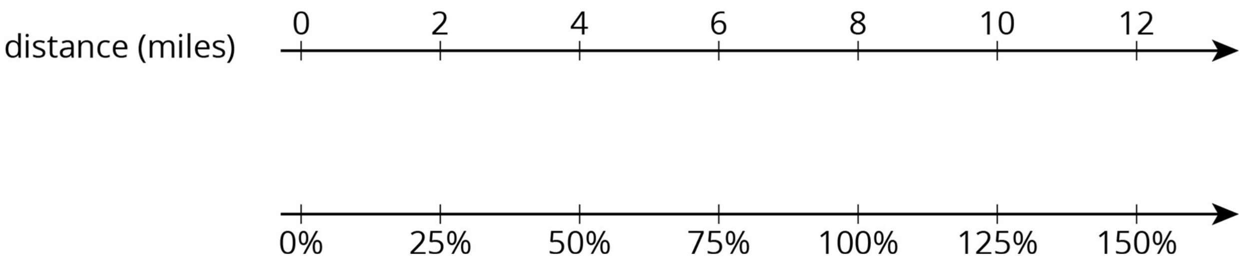
Student Task Statement

Elena cycled 8 miles on Saturday. Use the double number line to answer the questions. Be prepared to explain your reasoning.



1. What is 100% of her Saturday distance?
2. On Sunday, she cycled 75% of her Saturday distance. How far was that?
3. On Monday, she cycled 125% of her Saturday distance. How far was that?

Student Response



1. 8 miles. She cycled 8 miles total, so 100% of her distance is 8 miles.
2. 6 miles. 75% of 8 miles is 6 miles, so she cycled 6 miles on Sunday.
3. 10 miles. Sample strategies:
   - Complete the double number line and observe that 125% aligns with 10 miles.
   - 25% of the distance is 2 miles, and 100% of the distance is 8 miles, and $2 + 8 = 10$.
   - Multiply 8 by 1.25 (or $\frac{5}{4}$).

Activity Synthesis

For each unique strategy, select one group to share their displays and explain their thinking. Sequence the presentations in the order they are presented in the Activity Narrative. If no students mention one or more of these strategies, bring them up. For example, if no one thought of 125% of the distance hiked as 100% plus 25%, present that approach and ask students to explain why it works.

Speaking, Listening: Compare and Connect. As students prepare a visual display of how they made sense of the last question, look for groups with different methods for finding 125% of 8 miles. Some groups may reason that 25% of the distance is 2 miles and 100% of the distance is 8 miles, so 125% of the distance is the sum of 2 and 8. Others may reason that the product of 100 and 1.25 is 125. Since 125 is 125% of 100, then 125% of 8 miles is the product of 8 and 1.25. As students investigate each other's work, encourage students to compare other methods for finding 125% of 8 to their own. Which approach is easier to understand with the double number line? This will promote students' use of mathematical language as they make sense of the connections between the various methods for finding 125% of a quantity.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

11.3 Puppies Grow Up

15 minutes

Previously students were asked to find various percentages given 100% of a quantity. Here they are asked to find 100% of quantities given other percentages. The context does not explicitly state that the values being sought (the adult weights of two puppies) are the values for 100%, so students will first need to make that connection.

Double number lines continue to be provided as a reasoning tool, but students may use a table of equivalent ratios or other methods. Those who use double number lines are likely to find them effective for the first question (find 100% of a quantity given 20%) but less straightforward for the second question (find 100% of a quantity given 30%). Since 100 is not a multiple of 30, students may use strategies such as subdividing the double number line into intervals of 10% and scaling up from there to find the value of 100%.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 3–4 minutes of quiet think time and then time to share their responses with their partner. Encourage students to refer to diagrams in previous activities if they are not sure how to get started. Students may need help interpreting the question to understand that 100% corresponds to the puppy's adult weight.

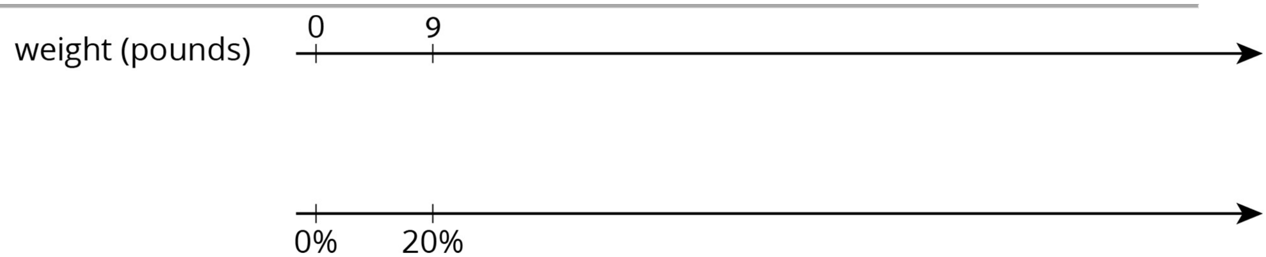
Writing, Speaking, Listening: Stronger and Clearer Each Time. After students have had the opportunity to determine the adult weight of Jada's puppy, ask students to write a brief explanation of their process. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "Can you explain how…?", "You should expand on...", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their explanation and learn about other methods for finding the adult weight of a puppy.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Anticipated Misconceptions

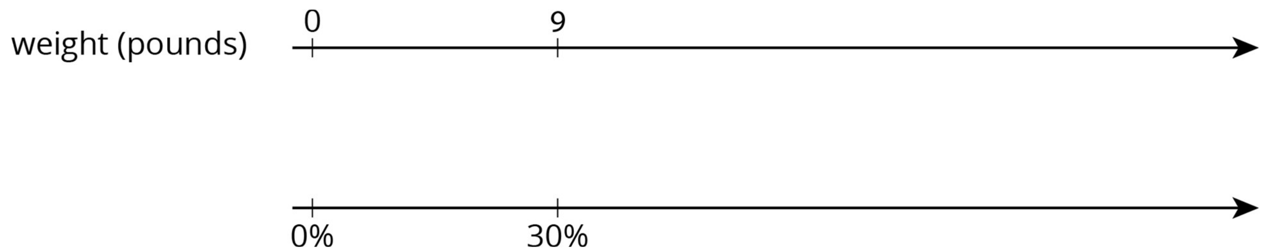
Students may stop before they reach 100% or go further than 100%. If this happens, explain that in this situation, the adult weight is at exactly 100%. Students may not use equal-sized increments between the tick marks they draw and label.

Student Task Statement

1. Jada has a new puppy that weighs 9 pounds. The vet says that the puppy is now at about 20% of its adult weight. What will be the adult weight of the puppy?



2. Andre also has a puppy that weighs 9 pounds. The vet says that this puppy is now at about 30% of its adult weight. What will be the adult weight of Andre's puppy?

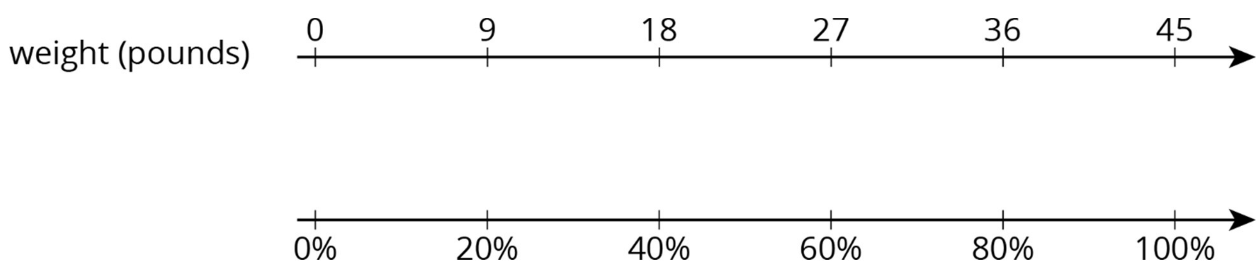


3. What is the same about Jada and Andre's puppies? What is different?

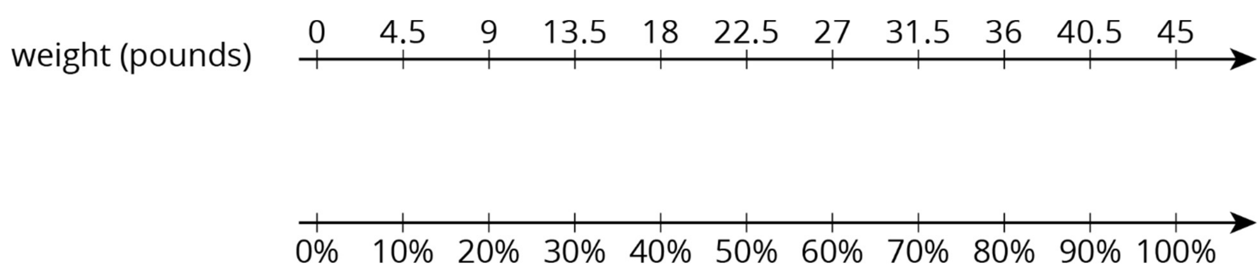
Student Response

1. 45 pounds. Possible approaches:

   - Complete the double number line with multiples of 9 and 20%:



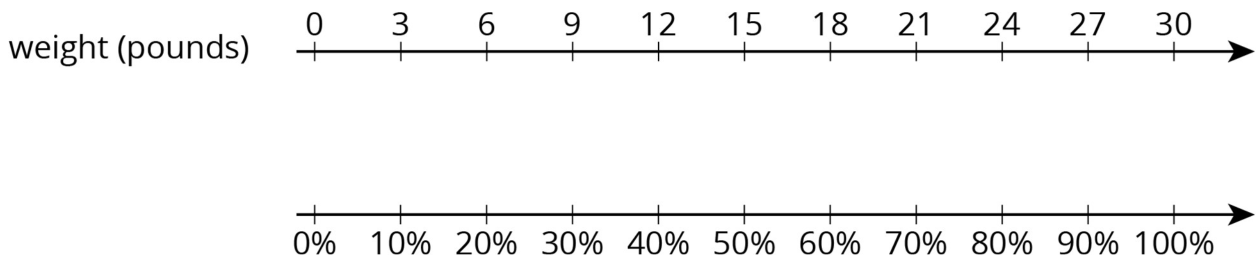
   - Complete the double number line in 10% increments:



   - 20% is $\frac{1}{5}$ of 100%, so 100% is $5 \times 9 = 45$.

   - 10% is half of 20%, so 10% is 4.5, and then 100% is 45.

2. 30 pounds. The strategies for this problem are similar to the previous ones, although they have to multiply 9 by $\frac{10}{3}$ to go directly from 9 pounds to the puppy's adult weight, which might not occur to most students.



3. Both puppies weigh the same right now. The puppies will weigh different amounts when they are adults.

Are You Ready for More?

A loaf of bread costs £2.50 today. The same size loaf cost 20 pence in 1955.

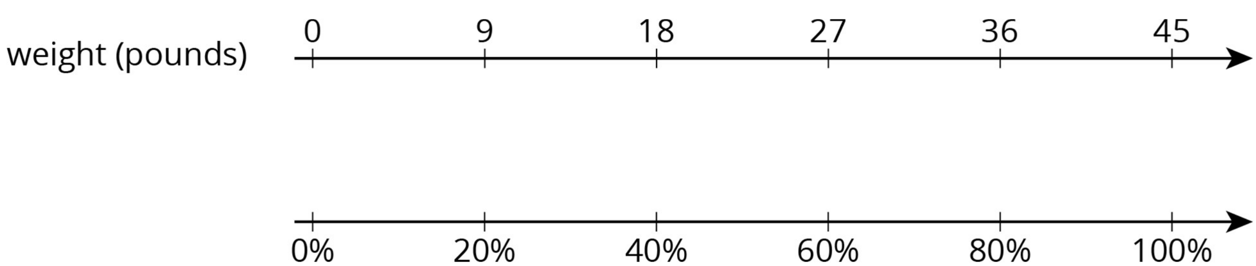
1. What percentage of today's price did someone in 1955 pay for bread?
2. A job pays £10.00 an hour today. If the same percentage applies to income as well, how much would that job have paid in 1955?

Student Response

1. 8%
2. £0.80 an hour

Activity Synthesis

Invite previously identified students to share their work. Start with someone who solved the first question using a double number line as follows, and follow with increasingly efficient strategies. Keep the number line displayed for all to see and to refer to throughout discussion.



If no students reasoned with a table, display this abbreviated table, or illustrate one student's approach and organise the steps in a table.

	weight (pounds)	percentage	
	9	20	
× 5	45	100	× 5

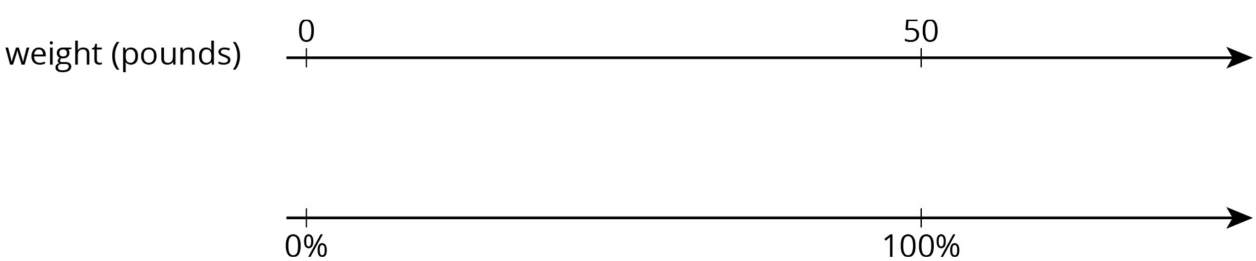
Follow a similar flow when discussing strategies for solving the second problem: start with a double number line and, if not mentioned by students, discuss how a table such as this one can be an efficient tool.

	weight (pounds)	percentage	
	9	30	
$\times \frac{1}{3}$	3	10	$\times \frac{1}{3}$
× 10	30	100	× 10

Representation: Internalise Comprehension. Use colour and annotations to illustrate connections between representations. For example, on a display, illustrate one student's approach on both a double number line and a table. Support connections by highlighting how each step appears in each representation.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Lesson Synthesis

If you know the percentage and 100%, then you can find the percentage with a double number line by putting the value assigned to 100% opposite the tick mark labelled 100%. For example, if we want to find some percentage of 50 pounds, we can label 100% and 50 like this:



Display the table for all to see. Questions for discussion:

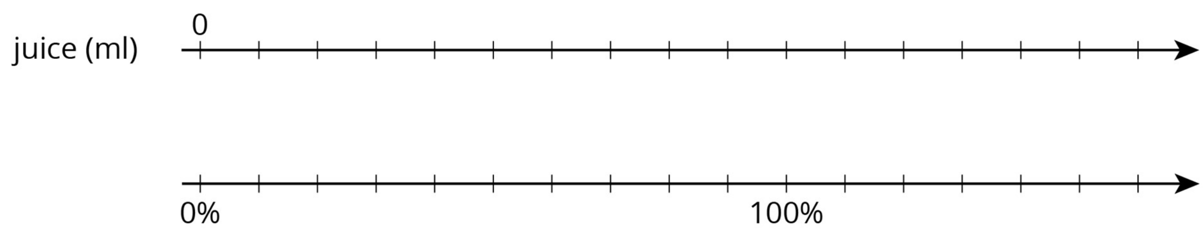
- What situation might this double number line represent?
- This says that 100% of 50 is 50. Where can we place some other percentages of 50?
- (If no one mentions a percentage greater than 100%) What about 110% of 50? Where would we place it? How would it be labelled?

11.4 A Medium Bottle of Juice

Cool Down: 5 minutes

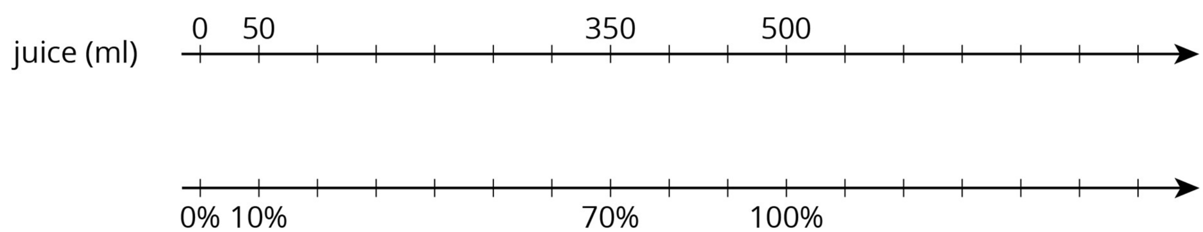
Student Task Statement

A large bottle of juice contains 500 millilitres of juice. A medium bottle contains 70% as much juice as the large bottle. How many millilitres of juice are in the medium bottle?



Student Response

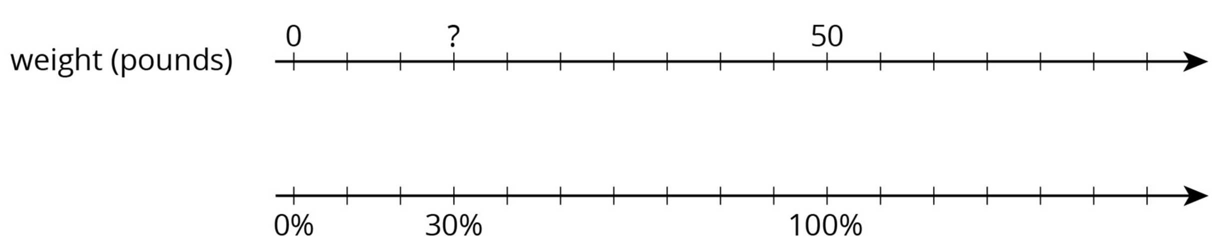
350 ml. Possible strategies: 10% of 500 is 50, so 70% of 500 is $7 \times 50 = 350$.



juice (ml)	percentage
500	100
50	10
350	70

Student Lesson Summary

We can use a double number line to solve problems about percentages. For example, what is 30% of 50 pounds? We can draw a double number line like this:

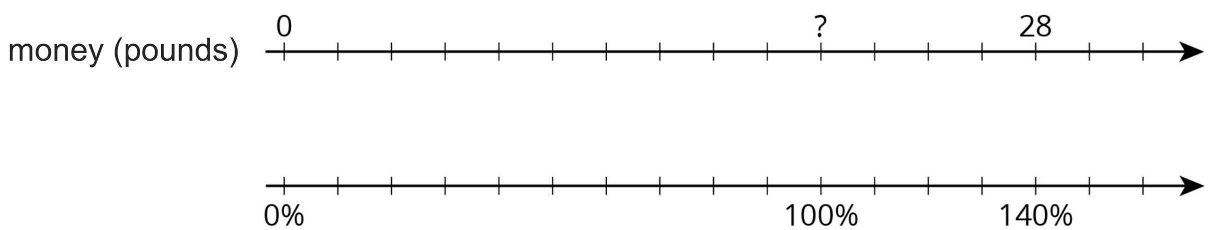


We divide the distance between 0% and 100% and that between 0 and 50 pounds into ten equal parts. We label the tick marks on the top line by counting by 5s ($50 \div 10 = 5$) and on the bottom line counting by 10% ($100 \div 10 = 10$). We can then see that 30% of 50 pounds is 15 pounds.

We can also use a table to solve this problem.

weight (pounds)	percentage
50	100
5	10
15	30

Suppose we know that 140% of an amount is £28. What is 100% of that amount? Let's use a double number line to find out.



We divide the distance between 0% and 140% and that between £0 and £28 into fourteen equal intervals. We label the tick marks on the top line by counting by 2s and on the bottom line counting by 10%. We would then see that 100% is £20.

Or we can use a table as shown.

money (pounds)	percentage
28	140
2	10
20	100

Glossary

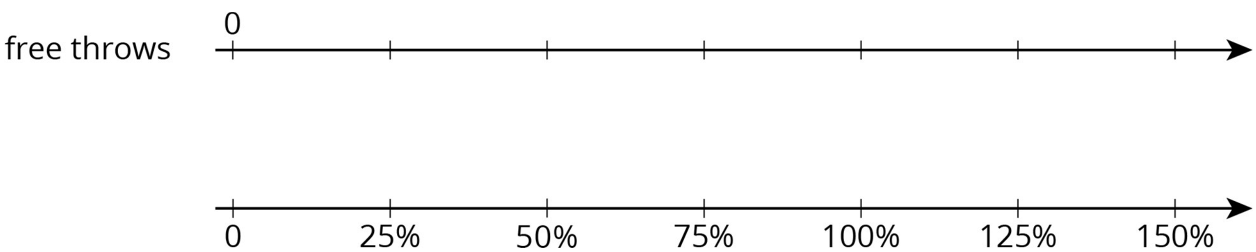
- Percentage

Lesson 11 Practice Problems

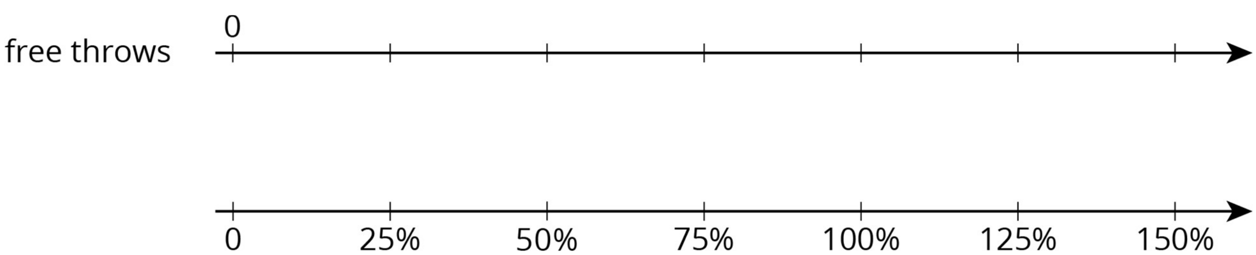
Problem 1 Statement

Solve each problem. If you get stuck, consider using the double number lines.

a. During a basketball practice, Mai attempted 40 free throws and was successful on 25% of them. How many successful free throws did she make?



b. Yesterday, Priya successfully made 12 free throws. Today, she made 150% as many. How many successful free throws did Priya make today?



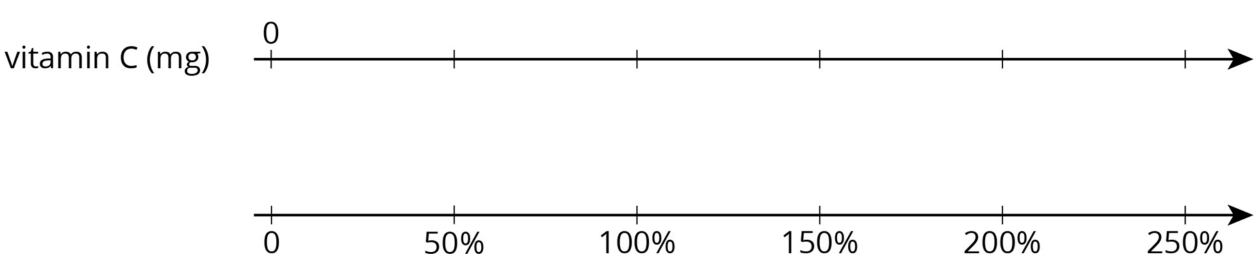
Solution

a. 10 free throws

b. 18 free throws

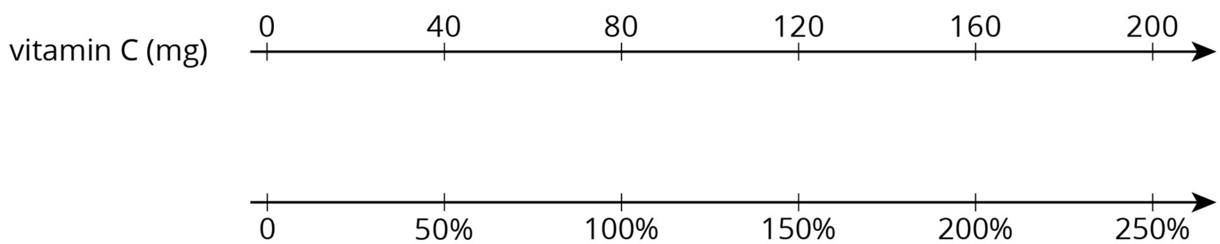
Problem 2 Statement

A 16-ounce bottle of orange juice says it contains 200 milligrams of vitamin C, which is 250% of the daily recommended allowance of vitamin C for adults. What is 100% of the daily recommended allowance of vitamin C for adults?



Solution

80 mg. Explanations vary. Sample explanation: On the double number line, place 200 above 250%. Dividing both of these by 5 gives 40 and 50%, so place 40 above 50%. Since 100% is double that, double 40 to get 80.

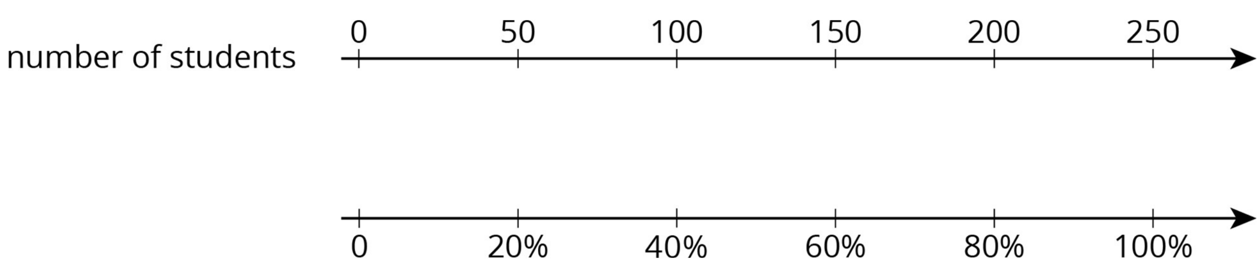


Problem 3 Statement

At a school, 40% of the year 7 students said that hip-hop is their favourite kind of music. If 100 year 7 students prefer hip hop music, how many year 7 students are at the school? Explain or show your reasoning.

Solution

250. Explanations vary. Possible explanation:



Problem 4 Statement

Diego has a skateboard, scooter, bike, and go-cart. He wants to know which vehicle is the fastest. A friend records how far Diego travels on each vehicle in 5 seconds. For each vehicle, Diego travels as fast as he can along a straight, level path.

vehicle	distance travelled
skateboard	90 feet
scooter	1 020 inches
bike	4 800 centimetres
go-cart	0.03 kilometres

a. What is the distance each vehicle travelled in centimetres?

b. Rank the vehicles in order from fastest to slowest.

Solution

a. Skateboard: 2 743.2. Scooter: 2 590.8. Bike: 4 800. Go-cart: 3 000.

b. Bike, go-cart, skateboard, scooter

Problem 5 Statement

It takes 10 pounds of potatoes to make 15 pounds of mashed potatoes. At this rate:

a. How many pounds of mashed potatoes can they make with 15 pounds of potatoes?

b. How many pounds of potatoes are needed to make 50 pounds of mashed potatoes?

Solution

a. To find the amount of mashed potatoes, multiply the amount of potatoes by $\frac{3}{2}$, $22\frac{1}{2}$ pounds of mashed potatoes (or equivalent).

b. To find the potatoes, multiply the amount of mashed potatoes by $\frac{2}{3}$, $33\frac{1}{3}$ pounds of potatoes (or equivalent).