Grades 11-12 (AS)

Duration: 20 min

Tools: one 16 pcs Set

Individual / Group work

Keywords: Probability, Favourable outcome, Total outcome, Conditional probability

620 - Conditional Probability

MATHS / PROBABILITY

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

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DESCRIPTION

Students put aside the repeated blocks of the 16 pcs Set and work with the remaining 10 blocks. They choose two blocks at random. They recall the definition of conditional probability, and answer the following questions:

a) What is the probability that one of the chosen blocks is 223, given that the selected pair of blocks can be joined to form a regular prism of height 5?

b) What is the probability that one of the chosen blocks is 223, given that the selected pair of blocks can be joined to form a regular prism of height 4? (See the diagram for a favourable outcome.)

c) What is the probability that one of the chosen blocks is 113, given that the selected pair of blocks can be joined to form a regular prism of height 5?

d) What is the probability that one of the chosen blocks is 113, given that the selected pair of blocks can be joined to form a regular prism of height 4?

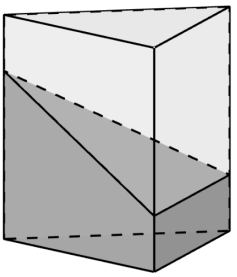
They then consider the following “reverse” conditional probability questions, and discuss the difference between these and the previous questions.

e) What is the probability that the selected pair of blocks can be joined to form a regular prism of height 5, given that one of the chosen blocks is 223?

f) What is the probability that the selected pair of blocks can be joined to form a regular prism of height 4, given that one of the chosen blocks is 223?

g) What is the probability that the selected pair of blocks can be joined to form a regular prism of height 5, given that one of the chosen blocks is 113?

h) What is the probability that the selected pair of blocks can be joined to form a prism of height 4, given that one of the chosen blocks is 113?



SOLUTIONS / EXAMPLES

There are 10 different blocks in the 16 pieces set. Hence the total number of outcomes is the number of ordered pairs, which is $10 \times 9 = 90$.

The following notations are used in the solution:

A = one of the chosen blocks is 223
B = one of the chosen blocks is 113
C = the selected pair of blocks can be joined to form a regular prism of height 5
D = the selected pair of blocks can be joined to form a regular prism of height 4

The numbers of favourable outcomes are listed for these events below:

- The events A and B has both $9 \times 2 = 18$ outcomes, because one of the picked blocks is fixed and the other can be any of the remaining 9 blocks, and the two blocks can be picked in both orders, this gives the factor 2.

- C={(233, 223), (223, 233)}, hence the number of favourable outcomes is 2.

- D={(111, 333), (333, 111), (113, 133), (133, 113), (233, 112), (112, 233), (223, 122), (122, 223)}, hence the number of favourable outcomes is 8.

To calculate the conditional probabilities, we also need the favourable outcomes of the intersections of the events:

- $A \cap C = C = \{(233, 223), (223, 233)\}$
- $A \cap D = \{(223, 112), (112, 223)\}$
- $B \cap C = \{\}$
- $B \cap D = \{(113, 133), (133, 113)\}$

The solutions are the following conditional probabilities:

a) $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{2/90}{2/90} = 1$

b) $P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{2/90}{8/90} = \frac{1}{4}$

c) $P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0}{2/90} = 0$

d) $P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{2/90}{8/90} = \frac{1}{4}$

e) $P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{2/90}{18/90} = \frac{1}{9}$

f) $P(D|A) = \frac{P(A \cap D)}{P(A)} = \frac{2/90}{18/90} = \frac{1}{9}$

g) $P(C|B) = \frac{P(B \cap C)}{P(B)} = \frac{0}{18/90} = 0$

h) $P(D|B) = \frac{P(B \cap D)}{P(B)} = \frac{2/90}{18/90} = \frac{1}{9}$

Observe that the conditional probabilities differ from the “reverse” conditional probabilities, for example $P(A|C) \neq P(C|A)$. The reason is that the conditions differ, hence the denominators differ. Nevertheless, $P(B|C) = P(C|B) = 0$, because the numerators are 0 in this case. In fact, the events B and C are mutually exclusive events.

PRIOR KNOWLEDGE

The traditional model of probability, Conditional probability.

RECOMMENDATIONS / COMMENTS

Exercise 603 - Pairing 16pcs is recommended before this exercise to find the possible pairs of blocks that can be combined into a regular prism of height 4 or 5.
Exercise 616 - Pick a Pair is recommended before this exercise to calculate the unconditional probabilities.
Different questions or pairs of questions can be given to different students. The number of favourable outcomes can be discussed with the whole class. After calculating the conditional probabilities, we recommend discussing the connection between the results for the different questions.