Lesson 11: On both of the lines

Goals

- Create a graph that represents two linear relationships in context, and interpret (orally and in writing) the point of intersection.
- Interpret a graph of two equivalent lines in context.

Learning Targets

- I can use graphs to find an ordered pair that two real-world situations have in common.

Lesson Narrative

For the next several lessons, students will study systems of linear equations where the context is of the distance-versus-time variety, where there is an initial value and a rate of change. The equations in the system are in the form $y = mx + c$. Such contexts are useful in thinking about the meaning of the solution to the system (the time when two quantities are equal). The purpose of this lesson is to introduce students to the graphical interpretation of such systems. Keeping the graphs in mind will be useful as students navigate algebraic techniques for solving systems in the lessons to come.

The first activity after the warm-up focuses on the solution by drawing a graph for a ladybug's motion, marking the point on the graph where an ant and a ladybug meet, and asking the student to fill in the graph of the ant. The second activity draws attention to systems which have infinitely many solutions because the graphs of the equations are identical. This is interpreted as two runners staying together for the entire duration of a race.

Addressing

- Analyse and solve pairs of simultaneous linear equations.

Building Towards

- Analyse and solve pairs of simultaneous linear equations.

Instructional Routines

- Collect and Display
- Three Reads
- Notice and Wonder

Required Materials

Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Required Preparation

Provide students with access to straightedges for drawing accurate lines.

Student Learning Goals

Let's use lines to think about situations.

11.1 Notice and Wonder: Bugs Passing in the Night

Warm Up: 10 minutes

The purpose of this warm-up is to get students to think about a context that will be explored in the following activity and to reason about the speed, distance, and time each animal is travelling in relation to one another. In the next activity, students will write equations for the bugs and graph these relationships.

Instructional Routines

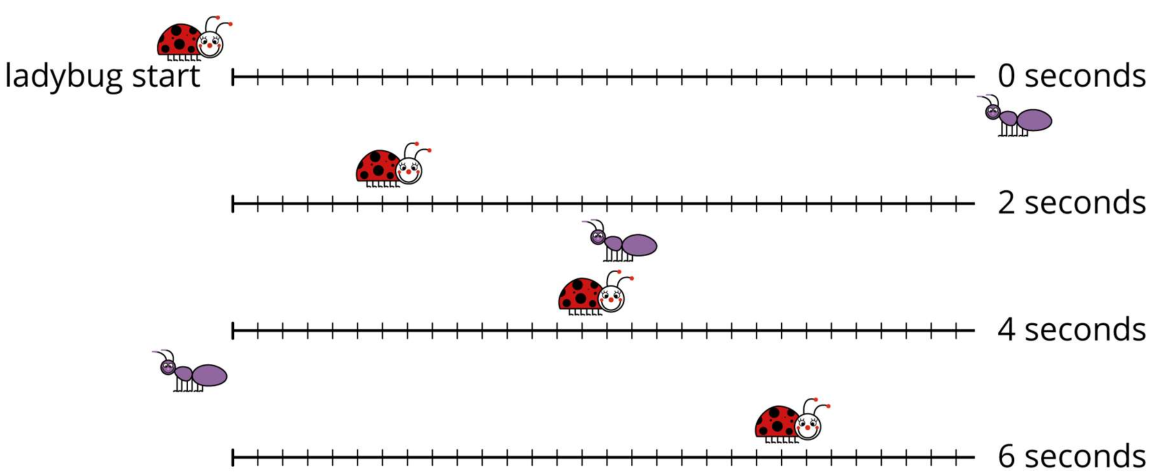
- Notice and Wonder

Launch

Tell students they will see a picture that shows a ladybug and ant travelling for 6 seconds. Tell students to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have noticed or wondered about something.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students might notice:

- The ladybug is moving from left to right and the ant is moving from right to left.
- The ladybug and the ant are each moving at a constant speed.
- The ladybug is moving 8 units every 2 seconds and the ant is moving 16 units every 2 seconds.
- In the picture at 6 seconds, the ant is no longer visible in the picture.
- At some time in between the 2 second picture and the 4 second picture, they pass each other.

Things students might wonder:

- Where did the ant go in the last picture?
- At what time did they pass each other?
- At what tick mark did they pass each other?
- Did they wave as they passed each other?

Activity Synthesis

Ask students to share things they noticed and wondered. Record and display their responses for all to see. If any students remember a similar representation from an earlier unit where both the ladybug and the ant started on the same side, you may wish to include how this situation is similar and how it is different.

Important ideas to highlight during the discussion:

- The bugs are moving in opposite directions and at some time in between $t = 2$ and $t = 4$, they pass each other.
- The bugs are moving at a constant speed.
- The ant is moving faster than the ladybug.

11.2 Bugs Passing in the Night, Continued

10 minutes

In this task, students find and graph a linear equation given only the graph of another equation, information about the gradient, and the coordinates where the lines intersect. The purpose of this task is to check student understanding about the point of intersection in relationship to the context while applying previously learned skills of equation writing and graphing.

Identify students who use different strategies to answer the first problem to share during the whole-class discussion. For example, some students may reason about the equation from an algebraic perspective while others may start by drawing in the graph for the ant based on the provided information. Also, make note of what strategy is most common among students.

Instructional Routines

- Collect and Display

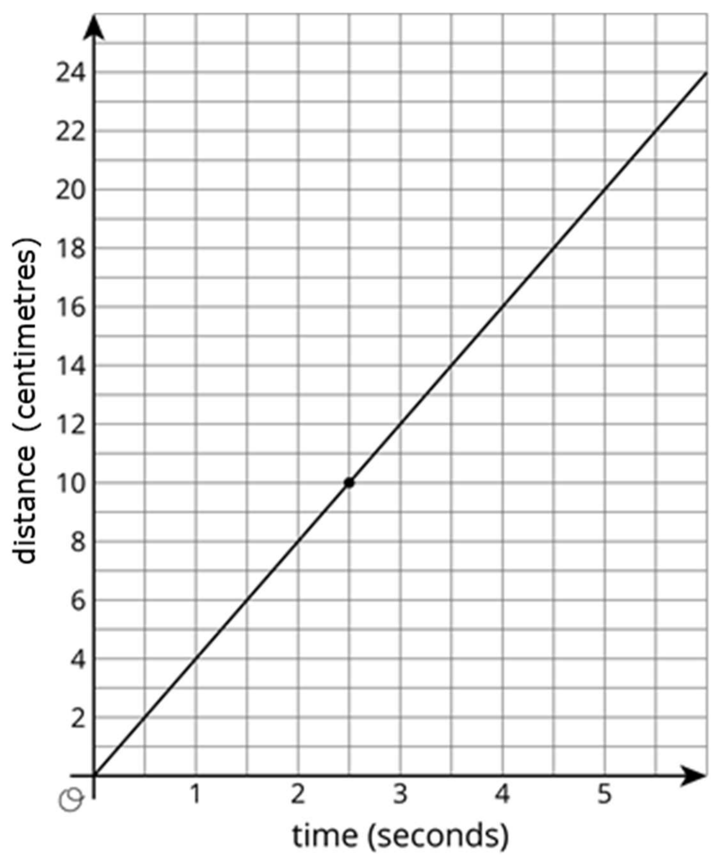
Launch

Display the graph from the task statement. Tell students that this activity is about a different ant and ladybug from the warm-up, and we are going to think about their distances using a coordinate plane. Give 4–6 minutes for students to complete the problems followed by a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge about equation writing and graphing. Incorporate explicit opportunities for review and practice if necessary.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

A different ant and ladybug are a certain distance apart, and they start walking toward each other. The graph shows the ladybug's distance from its starting point over time and the labelled point (2.5,10) indicates when the ant and the ladybug pass each other.



The ant is walking 2 centimetres per second.

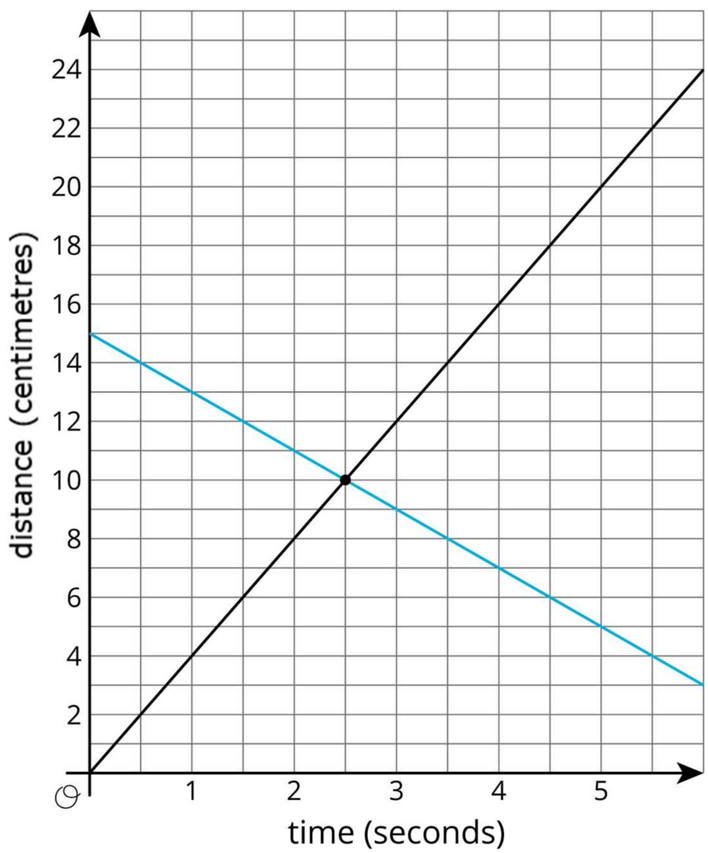
1. Write an equation representing the relationship between the ant's distance from the ladybug's starting point and the amount of time that has passed.

2. If you haven't already, draw the graph of your equation on the same coordinate grid.

Student Response

1. $d = \text{-}2t + 15$ or $d = 15 - 2t$. In 2.5 seconds the ant will have walked 5 centimetres ($2 \times 2.5 = 5$). Therefore, the ant started a distance of 15 centimetres away from the ladybug ($10 + 5 = 15$). This yields the equation $d = \text{-}2t + 15$ or $d = 15 - 2t$.

2.



Activity Synthesis

The purpose of this discussion is to ensure all students understand both how the labelled point in the task statement relates to the context and how to write and graph an equation from the given information.

Select previously identified students to share their strategy for the first problem, starting with the most common strategy used in the class. Record and display in one place the equations students write. While students may come up with equations like $\text{-}2 = \frac{d-10}{t-2.5}$, let them know that this approach is valid, but that an equation of the form $d = \text{-}2t + 15$ will be easier to work with today. Ask each student who shares how they knew to use the point (2.5,10) when making the equation for the ant.

If students struggled to graph the ant's path, you may wish to conclude the discussion by asking students for different ways to add the graph of the ant's distance onto the coordinate grid. For example, some students may say to use the equation figured out in the first problem to plot points and then draw a line through them. Other students may suggest starting from the known point, (2.5,10), and "working backwards" to figure out that 1 second earlier at 1.5 seconds, the ant would have to be 12 centimetres away since $10 + 2 = 12$.

Speaking, Listening: Collect and Display. Before the whole-class discussion, give groups the opportunity to discuss the first question. Circulate through the room and record the language students use to talk about the situation. Listen for words or phrases such as "rate of change," "ordered pair," "increasing/decreasing," and "initial value." Organise and group similar strategies in the display for students to reference throughout the lesson. For example, group strategies that reference the point (2.5,10) and strategies that involved working backwards from the graph to write the equation. This will help students solidify their understanding about how to write an equation using the point of intersection on a graph.
Design Principle(s): Support sense-making

11.3 A Close Race

15 minutes

In previous lessons, students encountered equations with a single variable that had infinitely many solutions. In this activity, students interpret a situation with infinitely many solutions. A race is described using different representations (a table and a description in words). Students graph the relationships given by the descriptions and notice that the lines overlap so that both relationships are true for any pair of values along the graphed line.

Instructional Routines

- Three Reads

Launch

Allow students 7–10 minutes of silent work time followed by a whole-class discussion.

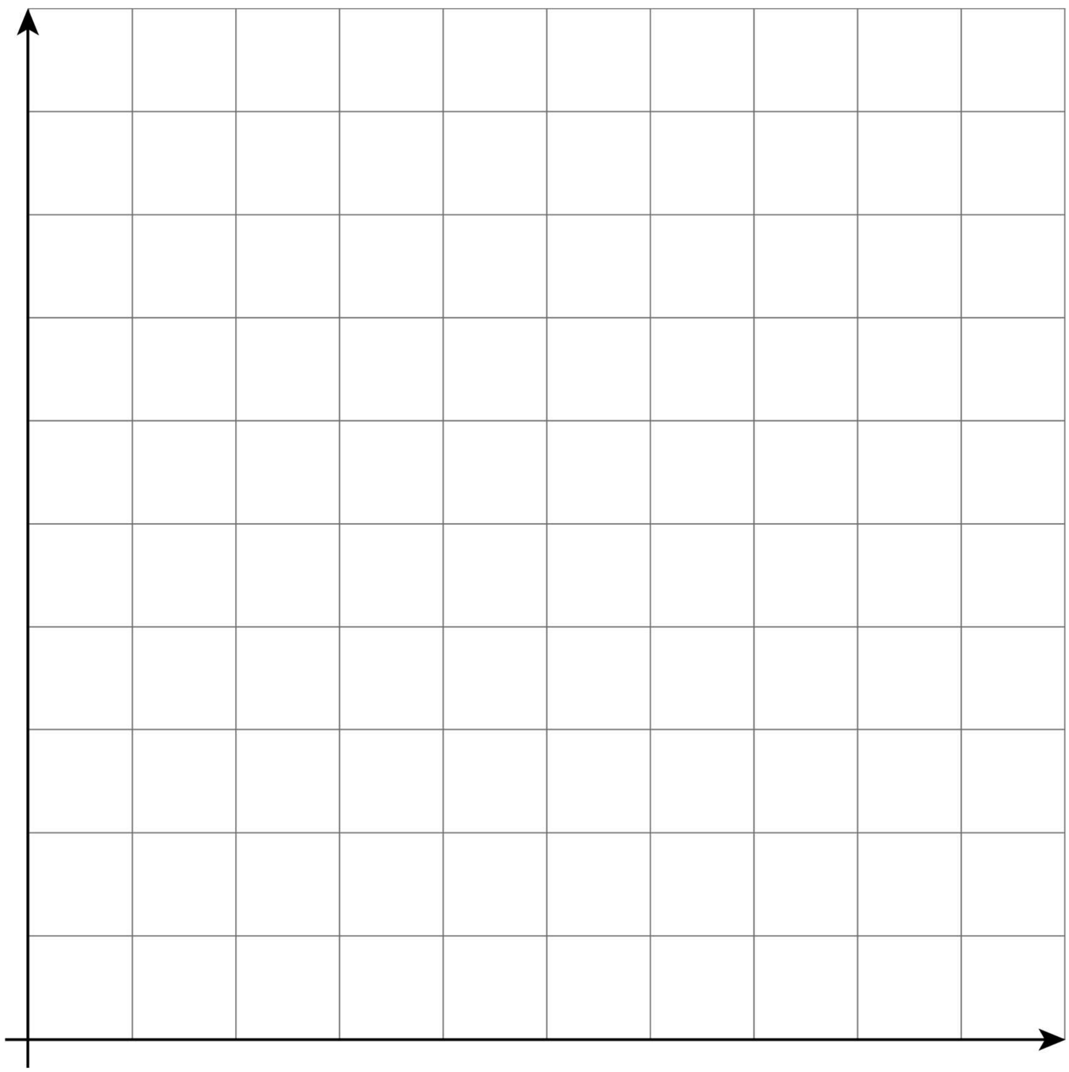
Reading: Three Reads. Use this routine to support reading comprehension of Elena and Jada's bike race scenario with the table. In the first read, students read the problem with the goal of comprehending the situation (e.g., Elena and Jada are racing their bikes.). In the second read, ask students to look for quantities represented (e.g., total distance is 100 metres, Jada rode 36 metres in 6 seconds, etc.). In the third read, ask students to brainstorm possible strategies to answer the question, "Who won the race?" This will help students interpret a situation in which there are infinitely many solutions, given a description and table.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

Elena and Jada were racing 100 metres on their bikes. Both racers started at the same time and rode at constant speed. Here is a table that gives information about Jada's bike race:

time from start (seconds)	distance from start (metres)
6	36
9	54

1. Graph the relationship between distance and time for Jada's bike race. Make sure to label and scale the axes appropriately.



2. Elena travelled the entire race at a steady 6 metres per second. On the same set of axes, graph the relationship between distance and time for Elena's bike race.

3. Who won the race?

Student Response

1. The line $d = 6t$ should be graphed. The horizontal axis should have the title "time (seconds)" and the vertical axis should have the title "distance (metres)."

2. Elena's graph is the same line as Jada's.

3. The race ends in a tie since both racers travelled the same distance at the same speed.

Activity Synthesis

The key point for discussion is to connect what students observed about the graph they made to the concept of "infinitely many solutions" encountered in earlier lessons. Graphically, students see that there are situations where two lines align "on top of each other." We can interpret each point on the line as representing a solution to both Elena's and Jada's equations. If t is the time from start and y is the distance from the start, the equation for Elena is $y = 6t$. The equation for Jada is also $y = 6t$. Every solution to Elena's equation is also a solution to Jada's equation, and every solution to Jada's equation is also a solution to Elena's equation. In this way, there are infinitely many points that are solutions to both equations at the same time.

Ensure students clearly understand that just because there are infinitely many points that are solutions, it does not mean that *any* pair of values will solve both Elena's and Jada's equations. In this example, the pair of values must still be related by the equation $y = 6x$. So, pairs of values like (1,6), (10,60), and $\left(\frac{1}{2}, 3\right)$ are all solutions, but (1,8) is not.

Lesson Synthesis

Display a set of axes for all to see. Ask each question one at a time, allowing students time to work through each problem. As students share their responses, add graphs of the lines described to the axes.

- "A line goes through the point (2,5) and has a gradient of 1.5. What is an equation for this line?" ($y = 1.5x + 2$)

- "A second line goes through the point (2,5) and has a y-intercept of (0,10). What is an equation for this line?" ($y =$ -2.5$x + 10$)

- "What does the point (2,5) represent for these lines?" (The pair of values that is true in both situations.)

- "A third line goes through this same point. How would that show up in a table representing the relationship for the third line?" (The number 2 would be in the x column right next to the number 5 in the y column.)

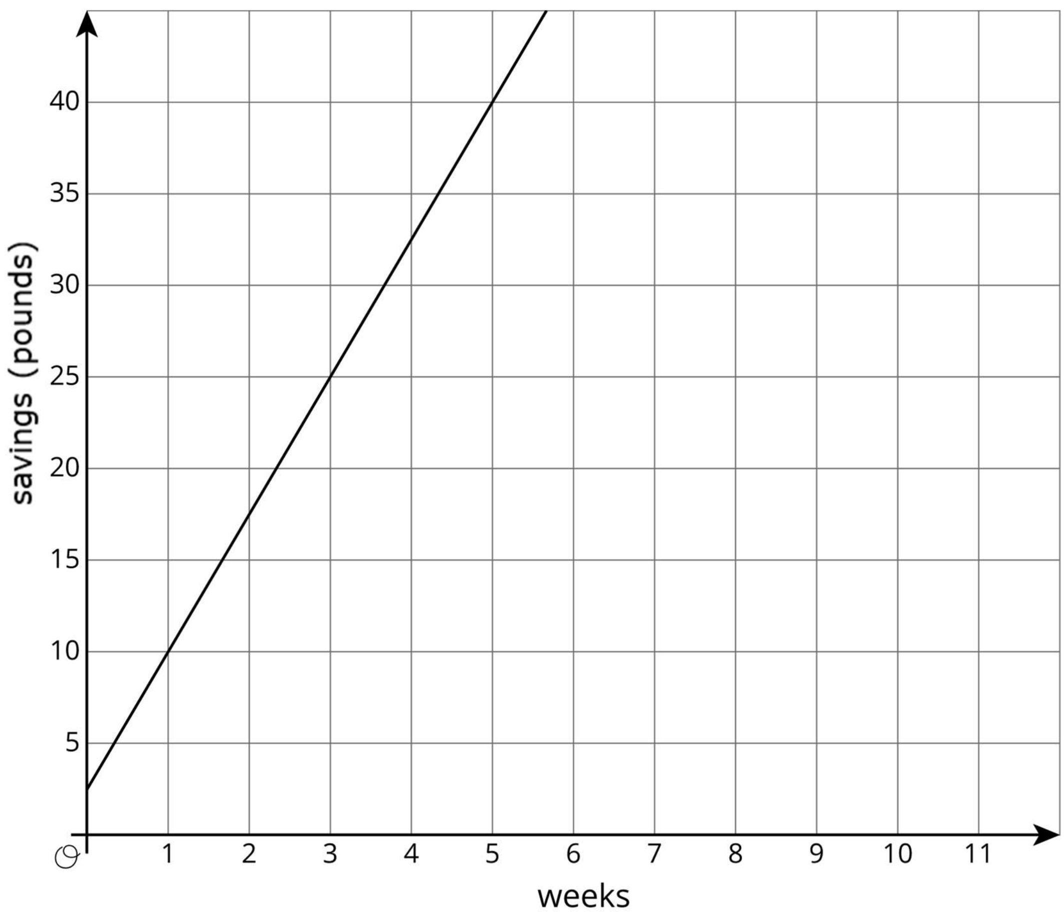
11.4 Saving Cash

Cool Down: 5 minutes

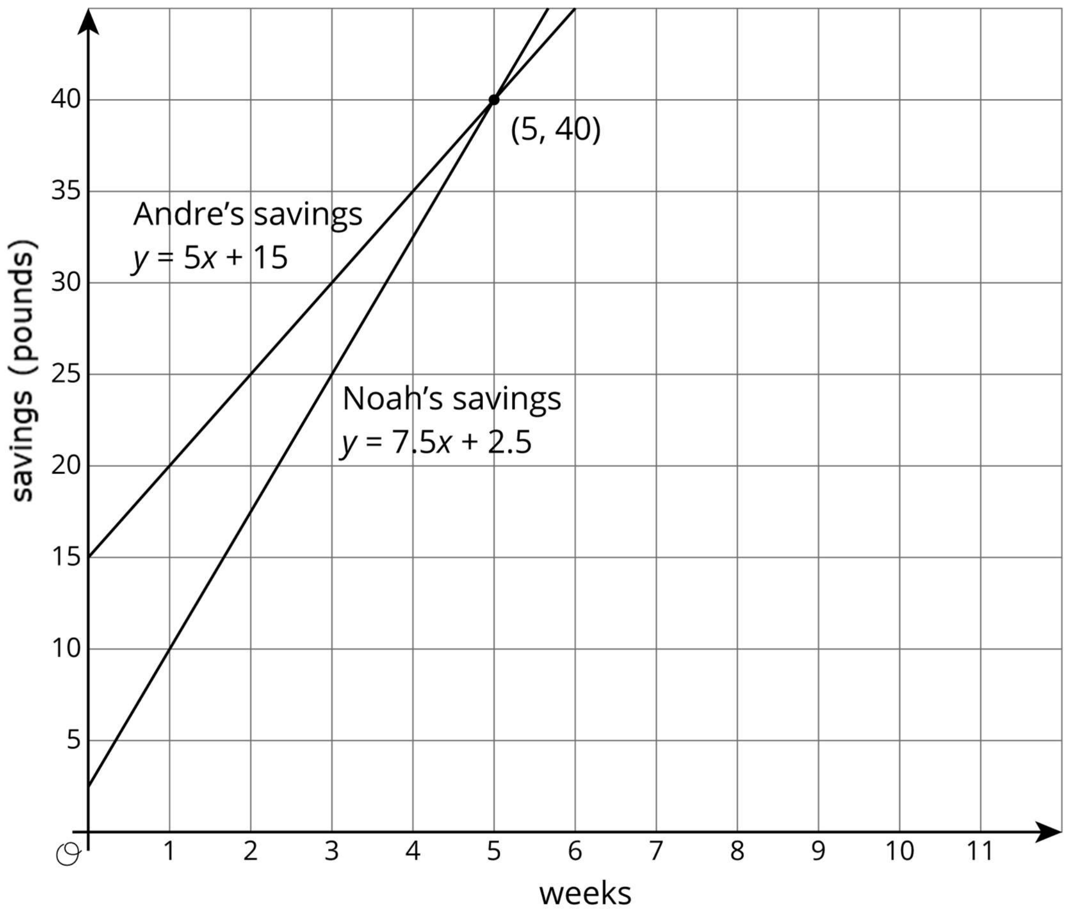
Student Task Statement

Andre and Noah started tracking their savings at the same time. Andre started with £15 and deposits £5 per week. Noah started with £2.50 and deposits £7.50 per week. The graph of Noah's savings is given and his equation is $y = 7.5x + 2.5$, where x represents the number of weeks and y represents his savings.

Write the equation for Andre's savings and graph it alongside Noah's. What does the intersection point mean in this situation?



Student Response



In this situation, the intersection at $(5,40)$ means that after 5 weeks, Noah and Andre each have £40.

Student Lesson Summary

The solutions to an equation correspond to points on its graph. For example, if Car A is travelling 75 miles per hour and passes a rest area when $t = 0$, then the distance in miles it has travelled from the rest area after t hours is

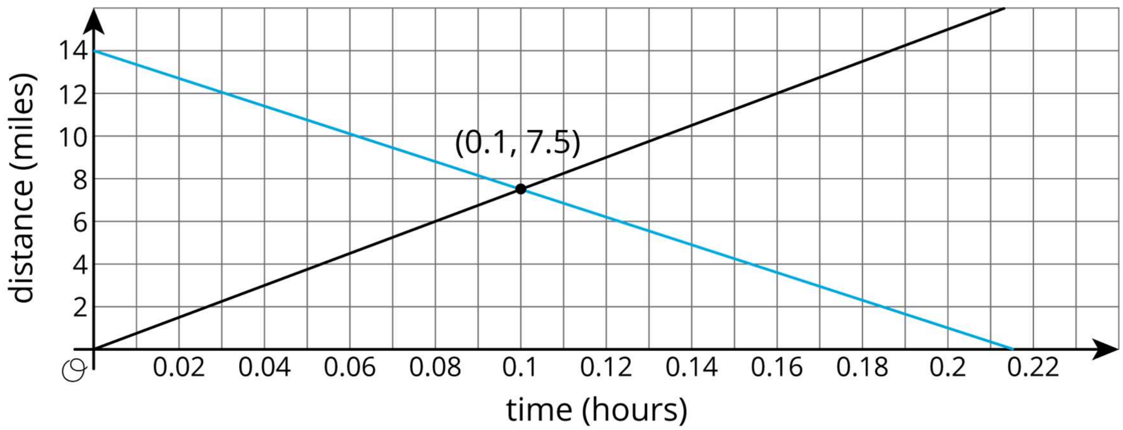
$d = 75t$

The point $(2,150)$ is on the graph of this equation because $150 = 75 \times 2$: two hours after passing the rest area, the car has travelled 150 miles.

If you have *two* equations, you can ask whether there is an ordered pair that is a solution to *both* equations simultaneously. For example, if Car B is travelling towards the rest area and its distance from the rest area is

$d = 14 - 65t$

We can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is "yes", then the solution will correspond to a point that is on both lines.



Looking at the coordinates of the intersection point, we see that Car A and Car B will both be 7.5 miles from the rest area after 0.1 hours (which is 6 minutes).

Now suppose another car, Car C, had also passed the rest stop at time $t = 0$ and travelled in the same direction as Car A, also going 75 miles per hour. It's equation would also be $d = 75t$. Any solution to the equation for Car A would also be a solution for Car C, and any solution to the equation for Car C would also be a solution for Car A. The line for Car C would land right on top of the line for Car A. In this case, every point on the graphed line is a solution to both equations, so that there are infinitely many solutions to the question "when are Car A and Car C the same distance from the rest stop?" This would mean that Car A and Car C were side by side for their whole journey.

When we have two linear equations that are equivalent to each other, like $y = 3x + 2$ and $2y = 6x + 4$, we will get two lines that are "right on top" of each other. Any solution to one equation is also solution to the other, so these two lines intersect at infinitely many points.

Lesson 11 Practice Problems

1. **Problem 1 Statement**

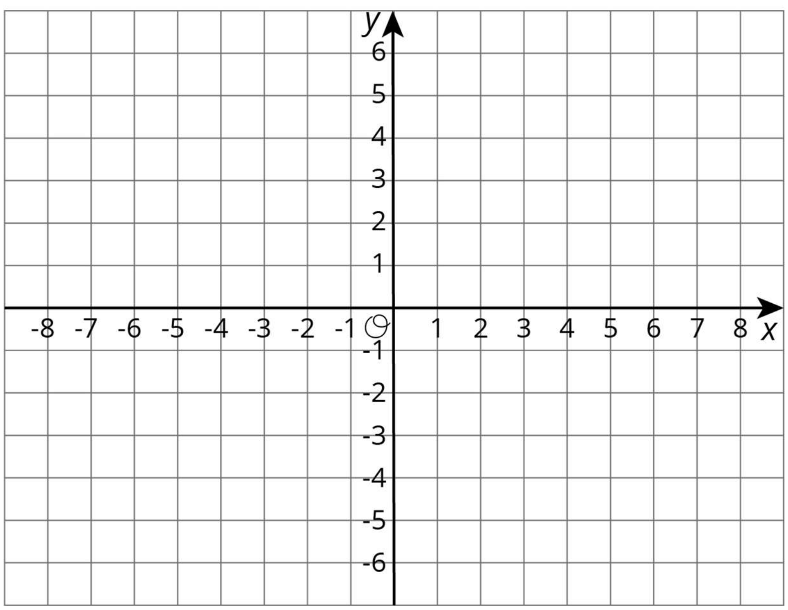
Diego has £11 and begins saving £5 each week toward buying a new phone. At the same time that Diego begins saving, Lin has £60 and begins spending £2 per week on supplies for her art class. Is there a week when they have the same amount of money? How much do they have at that time?

Solution

After 7 weeks, £46

2. **Problem 2 Statement**

Use a graph to find x and y values that make both $y = \frac{-2}{3}x + 3$ and $y = 2x - 5$ true.



Solution

(3,1)

3. **Problem 3 Statement**

The point where the graphs of two equations intersect has y-coordinate 2. One equation is $y = -3x + 5$. Find the other equation if its graph has a gradient of 1.

Solution

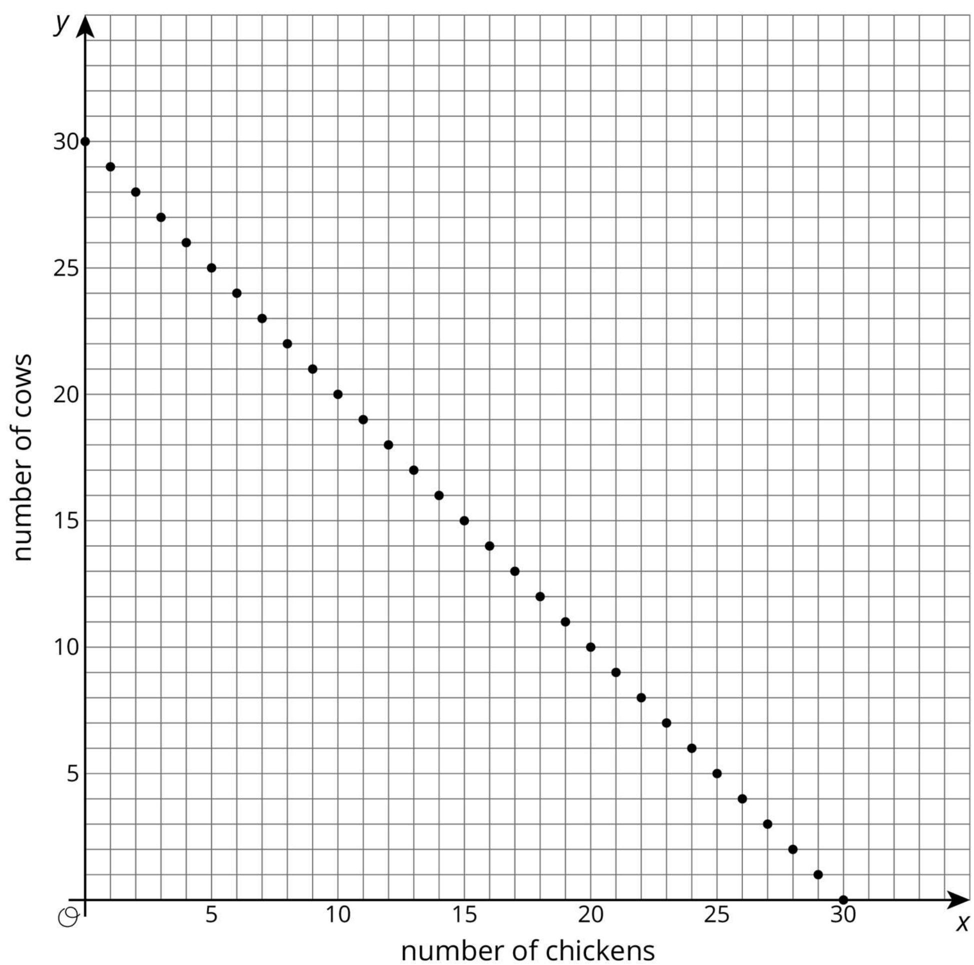
$y = x + 1$. $2 = -3x + 5$ is true when $x = 1$, so the line needed has a gradient of 1 and contains the point (1,2).

4. **Problem 4 Statement**

A farm has chickens and cows. All the cows have 4 legs and all the chickens have 2 legs. Altogether, there are 82 cow and chicken legs on the farm. Complete the table to show some possible combinations of chickens and cows to get 82 total legs.

number of chickens (x)	number of cows (y)
35	
7	
	10
19	
	5

Here is a graph that shows possible combinations of chickens and cows that add up to 30 animals:



If the farm has 30 chickens and cows, and there are 82 chicken and cow legs altogether, then how many chickens and how many cows could the farm have?

Solution

number of chickens (x)	number of cows (y)
35	3
7	17
21	10
19	11
31	5

The farm could have 19 chickens and 11 cows.