Lesson 17: A Fermi problem

Goals

- Apply reasoning developed throughout this unit to an unfamiliar problem.
- Decide what information is needed to solve a real-world problem.
- Make simplifying assumptions about a real-world situation.

Learning Targets

- I can apply what I have learned about ratios and rates to solve a more complicated problem.
- I can decide what information I need to know to be able to solve a real-world problem about ratios and rates.

Lesson Narrative

This unit concludes with an opportunity for students to apply the reasoning developed so far to solve an unfamiliar, Fermi-type problem. Students must take a problem that is not well-posed and make assumptions and approximations to simplify the problem so that it can be solved, which requires sense making and perseverance. To understand what the problem entails, students break down larger questions into more-manageable sub-questions. They need to make assumptions, plan an approach, and reason with the mathematics they know.

Engineers, computer scientists, physicists, and economists often make simplifying assumptions as they tackle complex problems involving mathematical modelling.

Building On

- Measurement and Data
- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Addressing

- Understand ratio concepts and use ratio reasoning to solve problems.
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.

Instructional Routines

- Group Presentations
- Compare and Connect
- Discussion Supports

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Student Learning Goals

Let’s solve a Fermi problem.

17.1 Fix It!

Warm Up: 10 minutes

This activity encourages students to apply ratio reasoning to solve a problem they might encounter naturally outside a mathematics classroom. The warm up invites open-ended thinking that is validated by mathematical reasoning, which is the type of complex thinking needed to solve Fermi problems in the following activities.

Launch

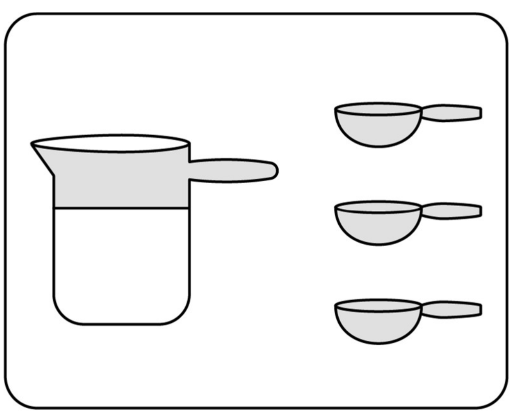
Arrange students in groups of 2. Display the image for all to see.

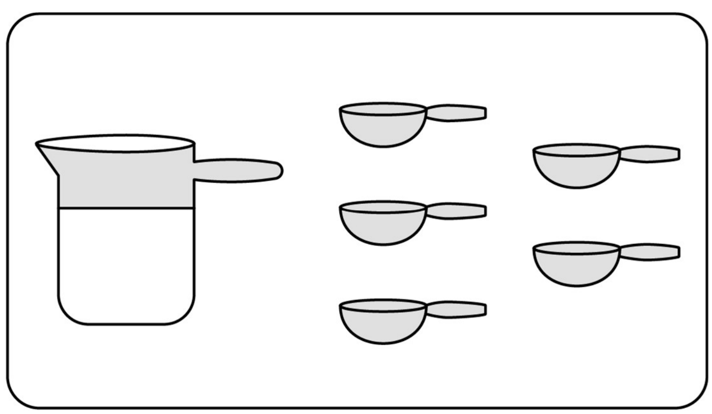
Optionally, instead of the abstract image, you could bring in a clear glass, milk, and cocoa powder. Pour 1 cup of milk into the glass, add 5 tablespoons of cocoa powder, and introduce the task that way.

Tell students to give a signal when they have an answer and a strategy. Give students 2 minutes of quiet think time.

Student Task Statement

Andre likes a hot cocoa recipe with 1 cup of milk and 3 tablespoons of cocoa. He poured 1 cup of milk but accidentally added 5 tablespoons of cocoa.





1. How can you fix Andre’s mistake and make his hot cocoa taste like the recipe?

2. Explain how you know your adjustment will make Andre's hot cocoa taste the same as the one in the recipe.

Student Response

1. Answers vary. Possible strategies: Add 1 more tablespoon of cocoa and 1 cup of milk or add $\frac{2}{3}$ cup of milk.

2. The ratios for the recipe and for the fixed mixture are equivalent.

Activity Synthesis

Invite students to share their strategies with the class and record them for all to see. After each explanation, ask the class if they agree or disagree and how they know two hot cocoas will taste the same.

17.2 Who Was Fermi?

15 minutes

In this activity, students are introduced to the type of thinking useful for Fermi problems. The purpose of this activity is not to come up with an answer, but rather to see different ways to break a Fermi problem down into smaller questions that can be measured, estimated, or calculated.

Much of the appeal of Fermi problems is in making estimates for things that in modern times we *could* easily look up. To make this lesson more fun and interesting, challenge students to work without performing any internet searches.

As students work, notice the range of their estimates and the sub-questions they formulate to help them answer the large questions. Some examples of productive sub-questions might be:

- What information do we *know*?
- What information can be *measured*?
- What information cannot be measured but can be *calculated*?
- What *assumptions* should we make?

Instructional Routines

- Discussion Supports

Launch

Open the activity with one or two questions that your students may find thought-provoking. Some ideas:

- "How many times does your heart beat in a year?"

- "How many hours of television do you watch in a year?"
- "Some research has shown that it takes 10 000 hours of practice for a person to achieve the highest level of performance in any field—sports, music, art, chess, programming, etc. If you aspire to be a top performer in a field you love—as Michael Jordan in basketball, Tiger Woods in golf, Maya Angelou in literature, etc., how many years would it take you to meet that 10 000-hour benchmark if you start now? How old would you be?"

Give students a moment to ponder a question and make a rough estimate. Then, share that the questions above are called "Fermi problems," named after Enrico Fermi, an Italian physicist who loved to think up and discuss problems that are impossible to measure directly, but can be roughly estimated using known facts and calculations. Here are some other examples of Fermi problems:

- "How long would it take to paddle across the Pacific Ocean?"
- "How much would it cost to replace all the windows on all the buildings in the United Kingdom?"

Share the questions above or select a few other Fermi-type questions that are likely to intrigue your students. Have some resources on hand to support the investigation on your chosen questions (e.g., have a globe handy if the question about paddling across the Pacific is on your short list). As a class, decide on one question to pursue. For this activity, consider giving students the option to either work independently or in groups of two.

Representation: Provide Access for Perception. Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting students to rephrase directions in their own words.
Supports accessibility for: Language; Memory Conversing, Representing: Discussion Supports. Use this routine to support student understanding of Fermi problems. After discussing some of the different Fermi problems presented in the launch, present nonexamples such as "How many students are in our classroom right now?" and "How tall is a stack of 20 pennies?" Ask pairs of students to select and critique one of the questions, and then collaborate to write a new version that represents a Fermi-type question. Invite students to share their new Fermi questions, and ask the class to identify the changes that made them Fermi questions.
Design Principle(s): Cultivate conversation; Support sense-making

Student Task Statement

1. Record the Fermi question that your class will explore together.
2. Make an estimate of the answer. If making an estimate is too hard, consider writing down a number that would definitely be too low and another number that would definitely be too high.
3. What are some smaller sub-questions we would need to figure out to reasonably answer our bigger question?

4. Think about how the smaller questions above should be organised to answer the big question. Label each smaller question with a number to show the order in which they should be answered. If you notice a gap in the set of sub-questions (i.e., there is an unlisted question that would need to be answered before the next one could be tackled), write another question to fill the gap.

Student Response

Answers vary depending on the question explored. For "How long would it take to paddle across the Pacific Ocean?" some sub-questions might be:

- What is the distance across the Pacific Ocean?
- At what speed can you paddle a boat?
- Do we assume that someone paddles continuously, or that they take breaks to sleep?

Activity Synthesis

First, ask students to share their estimates. Note the lowest and highest estimates, and point out that it is perfectly acceptable for an estimate to be expressed as a range of values rather than a single value.

Ask students to share some of their smaller questions. Then, discuss how you might come up with answers to these smaller questions, which likely revolve around what information is known, can be measured, or can be computed. Also, discuss how our assumptions about the situation affect how we solve the problem.

17.3 Researching Your Own Fermi Problem

30 minutes

This activity asks students to choose or pose a Fermi problem and solve it, with the aim of promoting the reasoning and tools developed in this unit. Students brainstorm potential problems, choose one, and—after your review—use a graphic organiser to help them formulate the sub-questions that could support their problem solving. They go on to solve their chosen Fermi problem.

To encourage ratio reasoning and the use of tools such as double number lines and tables, look for problems that involve *two* quantities. Questions that involve one quantity can be solved with multi-step multiplication and without ratio reasoning (e.g., "How many pens are there at the school?" involves only one quantity—the number of pens). But a problem such as "How much would it cost to replace all the windows on all buildings in the UK?" or "How long would it take to paddle across the Pacific Ocean?" involves accounting for two quantities at the same time (cost and number of windows, or time and distance across the Pacific) and is more likely to elicit ratio reasoning. Keep this in mind as you help students sift through their ideas.

Instructional Routines

- Group Presentations
- Compare and Connect

Launch

Explain to students that they will now brainstorm some Fermi problems they are interested in answering and select one to solve. Consider sharing a few more examples of Fermi problems to jumpstart their thinking:

- How much would it cost to charge all the students' mobile phones in the school for a month?
- How much does it cost to operate a car for a year?
- How long would it take to make a sandwich for everyone living in our town?
- How long would it take to read the dictionary out loud?
- How long would it take to give every dog in America a bath?

Tell students that once they have a few good ideas, they should pause and get your attention so that you could help to decide on the one problem to pursue.

Arrange students in groups of 2, if desired. Provide tools for creating a visual display.

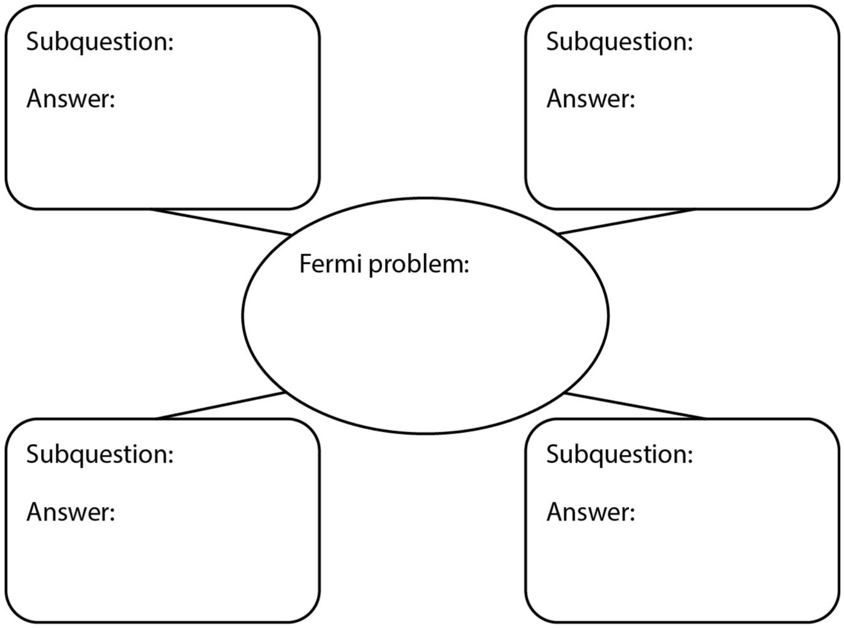
Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "How much would it cost to . . .?" or "How long would it take to . . .?"
Supports accessibility for: Language; Social-emotional skills

Anticipated Misconceptions

Students may think of problems that do not lend themselves to ratio reasoning because they only involve one quantity. If they have trouble coming up with any good options, offer them some examples. It may also be helpful to have a list of sample problems that students could refer to in creating their own problem.

Student Task Statement

1. Brainstorm at least five Fermi problems that you want to research and solve. If you get stuck, consider starting with "How much would it cost to . . .?" or "How long would it take to . . .?"
2. Pause here so your teacher can review your questions and approve one of them.
3. Use the graphic organiser to break your problem down into sub-questions.



4. Find the information you need to get closer to answering your question. Measure, make estimates, and perform any necessary calculations. If you get stuck, consider using tables or double number line diagrams.

5. Create a visual display that includes your Fermi problem and your solution. Organise your thinking so it can be followed by others.

Student Response

Answers vary.

Activity Synthesis

Display students' posters or visual presentations throughout the classroom. Consider asking some students (or all, if time permits) to present their problems and solutions to the class. Notice and highlight instances of ratio and rate reasoning, particularly productive use of double number lines or tables.

Representing, Conversing: Compare and Connect. Use this routine to prepare students for the whole-class discussion. Invite students to quietly circulate and read at least 2 of the posters or visual presentations in the room. Give students quiet think time to consider what is the same and what is different about the questions and displays. Next, ask students to find a partner to discuss what they noticed. Listen for, and amplify observations that include mathematical language and reasoning about double number lines or tables.
Design Principle(s): Cultivate conversation

Lesson Synthesis

The debrief and presentation of student work provides opportunities to summarise takeaways from this lesson. Aside from opportunities to point out how ratio reasoning and the use of representations can help us tackle difficult problems, this lesson makes explicit some aspects of mathematical modelling. Highlight instances where students had to make an estimate in order to proceed, figured out what additional information they would need to make progress, or made simplifying assumptions.