Lesson 17: Modelling with inequalities

Goals

- Critique (orally) the solution to an inequality, including whether fractional or negative values are reasonable.
- Determine what information is needed to solve a problem involving a quantity constrained by a maximum or minimum acceptable value. Ask questions to elicit that information.
- Write and solve an inequality of the form $px + q > r$ or $px + q < r$ to answer a question about a situation with a constraint.

Learning Targets

- I can use what I know about inequalities to solve real-world problems.

Lesson Narrative

By now, students have had plenty of experience writing and solving inequalities. This lesson focuses on the modelling process, in which students start with a question they want to answer and decide on their own how they will represent the situation mathematically.

As students apply inequalities in context, they must think about how to interpret their solutions. For instance, if they find that $x < 7$, but x represents the number of students who can go on a trip, then they should realise that x cannot be 3.25, nor can x be -2.

Addressing

- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Building Towards

- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Instructional Routines

- Information Gap Cards
- Three Reads
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master (if not using GeoGebra activity)

Info Gap: Giving Advice **Problem card 1** Noah's apartment building has a washing machine that uses a card for payment. His family likes to keep a minimum balance on the card. How many loads of laundry can Noah's family do before needing to add money to the card?	Info Gap: Giving Advice **Data card 1** • Each load of laundry costs £1.65. • Noah's family likes to make sure that the • balance on the card is at least £15, at which • point they add money to the card. • There is £50 on the card right now.
Info Gap: Giving Advice **Problem card 2** Elena is designing a rectangular picture frame with a lace border. What widths can Elena choose for the frame?	Info Gap: Giving Advice **Data card 2** • In order to fit nicely on her desk, the frame • needs to be 7 centimetres in length. • She only has 65 centimetres of lace • available to use for the border.

Required Preparation

Print and cut up copies of the blackline master for the Giving Advice activity. You will need one set of cards for every 4 students.

Student Learning Goals

Let's look at solutions to inequalities.

17.1 Possible Values

Warm Up: 5 minutes

The purpose of this warm-up is for students to interpret an inequality in a real-world situation and reason about the quantities in its solution. Some of the statements involve reasoning about how a sandwich shop sells its sandwiches, however, the focus of the discussion should be on the meaning of the solution to the inequality. Students should reason that they cannot order anything more than 13.86 sandwiches, but can order any amount less than 13.86 sandwiches.

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time followed by 1 minute to compare their responses with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may think of 13.86 sandwiches as 14 whole sandwiches because it rounds to that number, and 13.86 doesn't make sense to them in the context of sandwiches. It may be helpful for these students to use a calculator to find the cost of 14 sandwiches to see that is not a solution to the inequality. Tell these students that, although sandwich shops may not sell sandwiches in fractional pieces, the maximum amount that can be ordered is 13.86.

Student Task Statement

The stage manager of the school musical is trying to figure out how many sandwiches he can order with the £83 he collected from the cast and crew. Sandwiches cost £5.99 each, so he lets x represent the number of sandwiches he will order and writes $5.99x \leq 83$. He solves this to 2 decimal places, getting $x \leq 13.86$.

Which of these are valid statements about this situation? (Select **all** that apply.)

1. He can call the sandwich shop and order exactly 13.86 sandwiches.
2. He can round up and order 14 sandwiches.
3. He can order 12 sandwiches.
4. He can order 9.5 sandwiches.
5. He can order 2 sandwiches.
6. He can order -4 sandwiches.

Student Response

He can order 12, 2, and maybe 9.5 sandwiches.

13.86: Probably not: it is unlikely a sandwich shop would sell precisely .86 of a sandwich.

14: No. The solution of $x \leq 13.86$ means that 14 sandwiches would cost more than the £83 the group can spend.

12: Yes. The solution of $x \leq 13.86$ means that 12 sandwiches will cost less than £83.

9.5: Possibly. The sandwich shop may sell half sandwiches for half the price of a whole sandwich.

2: Yes. At a glance, 2 sandwiches will cost much less than £83.

-4: No. Though $-4 \leq 13.86$ and -4 is a numerical solution to the inequality, it does not make sense to order -4 sandwiches.

Activity Synthesis

Poll the class about whether they think each statement is valid. Ask a student to explain why the invalid statements don't work. Record and display their responses for all to see.

For each statement, students should mention the following ideas:

1. Even though 13.86 makes the inequality true, most sandwich shops would not let you order 13.86 sandwiches.
2. He doesn't have enough money to order 14 sandwiches. He has to order a number of sandwiches that is less than or equal to 13.86.
3. Works.
4. Might be okay if the shop allows you to order sandwiches in $\frac{1}{2}$-sandwich increments.
5. Works.
6. Even though -4 makes the inequality true, that value doesn't make sense in this context.

17.2 Elevator

15 minutes

This problem is an introduction to the series of modelling problems in the next activity. Here, students read a question and are prompted to think about what extra information they would need to solve it. Then they write and solve inequalities to answer the question.

The context in this problem provides an opportunity for students to think about aspects of mathematical modelling like discrete versus continuous solutions and rounding. Make sure to touch on these topics in discussion before moving on to the next activity.

Instructional Routines

- Three Reads

Launch

Ask students to close their books or devices or to leave them closed. Present this scenario verbally or display for all to see:

"A mover is loading an elevator with identical boxes. He wants to take all the boxes up the elevator at once, but he is worried about overloading the elevator. What are all the possibilities for the number of boxes the mover can take on the elevator at once?"

Give students a few minutes of quiet think time to brainstorm what information they would need to answer this question, followed by 1–2 minutes to discuss with a partner. Ask a few students to share their questions with the class and record them for all to see.

Then, ask students to open their books or devices to this activity and use the given information to help solve the problem.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “I chose to use the _____ inequality sign because...”, “Why did you...?”, “I agree/disagree because…”, or “The solution means that the mover…”
Supports accessibility for: Language; Social-emotional skills Reading; Representing: Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, have students read a display of the description of the scenario posed in the Activity Launch. Ask students, "What is this situation about?" (A mover is loading boxes in an elevator.). In the second read, ask students to brainstorm the important quantities by identifying what can be counted or measured in this situation. Sample quantities include: number of boxes, size(s) of the boxes, size of the elevator, weight of each box, weight limit of the elevator, weight of the mover. In the third read, ask students to read the actual problems and work with a partner to brainstorm strategies to write an inequality that can represent the relationship among the number of boxes, the total weight of the boxes and the mover, and the weight limit of the elevator. Invite students to sketch a diagram of these quantities. This helps students connect the language in the word problem and the reasoning needed to write an inequality for this situation.
Design Principle(s): Support sense-making

Student Task Statement

A mover is loading an elevator with many identical 48-pound boxes.
The mover weighs 185 pounds. The elevator can carry at most 2000 pounds.

1. Write an inequality that says that the mover will not overload the elevator on a particular ride. Check your inequality with your partner.

2. Solve your inequality and explain what the solution means.

3. Show the solution to your inequality on a number line.

4. If the mover asked, “How many boxes can I load on this elevator at a time?” what would you tell them?

Student Response

1. $48b + 185 \leq 2\,000$, where b is the number of identical boxes.

2. $b \leq 37.8125$ so the mover can put 37 or fewer boxes on the elevator.

3. Number line shows whole numbers 37 or less.

4. Answers vary. Sample response: 37 or fewer boxes.

Activity Synthesis

Many issues will come up in the discussion of this problem that will recur throughout the lesson. Some examples:

- "How can we represent the solution on a number line? Is 5.5 a solution?" (Not in the context of this problem; you can't have a half a box.)
- "Do we want to change the number line somehow to show this?"
  (We could plot discrete points, or we could simply leave it as is, but just know that for a problem with this context, we're only going to use integer solutions.)
- "Which type of inequality would you use to describe answers using *no more than* or *no less than*?" ($\leq$ and $\geq$, respectively.)
- "How did you know which way to round?" (Round down, otherwise you've gone over the weight limit.)
- "What other limitations do the contexts place on the solutions?"
  (You must have a positive number of boxes.)

17.3 Info Gap: Giving Advice

15 minutes

In this activity, students set up and solve inequalities that represent real-life situations. Students will think about how to interpret their mathematical solutions. For example, if they use w to represent width in centimetres and find $w < 25.5$, does that mean $w = -10$ is a solution to the inequality?

Instructional Routines

- Information Gap Cards

Launch

Tell students they will practice using their knowledge of inequalities to think about specific situations and interpret what their solutions mean in those situations. Ask students to represent their solution using words, an inequality, and a graph. Arrange students in groups of 2. If necessary, demonstrate the protocol for an Info Gap activity. In each group, distribute a problem card to one student and a data card to the other student. Give students who finish early a different pair of cards and ask them to switch roles.

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organisation Conversing: This activity uses *Information Gap* to give students a purpose for discussing information necessary for solving problems involving inequalities. Display questions or question starters for students who need a

starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate conversation

Anticipated Misconceptions

If students do not know where to start, suggest that they first identify the quantity that should be a variable and choose a letter to represent it.
In Elena’s problem, it may help to remind students that they know how to write a formula for the area of a rectangle.

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *“What specific information do you need?”* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.
3. Before sharing the information, ask “*Why do you need that information?*”

   Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

a. $-1.65x + 50 \geq 15, x \leq 21\frac{7}{33}$.

b. Number lines may vary. Some may have a closed (or open?) circle at a point representing $21\frac{7}{33}$, with an arrow extending to the left. Some may have closed dots on each integer less than 22.

c. Noah's family can wash 21 or fewer loads of laundry before having to add more money to the card (assuming they don't also use the card to pay for drying the clothes).

a. $14 + 2w \leq 65, w \leq 25.5$.

b. All number lines will have a closed circle at $w = 25.5$. Some may have arrows extending indefinitely to the left. Others may extend an arrow to the left, but stop at zero.

c. Elena can choose widths between very close to zero and 25.5 centimetres.

Are You Ready for More?

In a day care group, nine babies are five months old and 12 babies are seven months old. How many full months from now will the average age of the 21 babies first pass 20 months old?

Student Response

14 months. Right now, the sum of all their ages is 129, because $9 \times 5 + 12 \times 7 = 129$. After x months, the sum of all the babies' ages will have increased by $21x$. For 21 babies to have an average age of 20 months, the sum of all their ages would need to be 420 months, because $\frac{420}{21} = 20$. Solving the inequality $129 + 21x > 420$ we get $x > 13.86$.

Activity Synthesis

As the groups report on their work, encourage other students to think and ask questions about whether the answers are plausible. If students do not naturally raise these questions, consider asking:

- In Noah's problem, is 1.5 loads of laundry a solution to the inequality?
- In Elena's problem, can the width of the frame be -10 centimetres? Can the width of the frame be 0 centimetres? How about 0.1 centimetres?

Other questions for discussion:

- Which situations are discrete (have only whole-number solutions)?
- In Noah's problem, should we round up or down?

Lesson Synthesis

In the last few lessons, students have seen a variety of situations in which inequalities described situations. Ask students to think about a career they might be interested in pursuing and have them write a few sentences about the usefulness of inequalities in the work of that profession, including at least one example. Ask them to think about whether inequalities are sometimes more helpful than equations, and if so, why.

17.4 Movies on a Hard Drive

Cool Down: 5 minutes

This cool-down checks through error analysis to determine whether students can interpret solutions to an inequality in context.

Student Task Statement

Elena is trying to figure out how many movies she can download to her hard drive. The hard drive is supposed to hold 500 gigabytes of data, but 58 gigabytes are already taken up by other files. Each movie is 8 gigabytes. Elena wrote the inequality $8x + 58 \geq 500$ and solved it to find the solution $x \geq 55.25$.

1. Explain how you know Elena made a mistake based on her solution.

2. Fix Elena's inequality and explain what each part of the inequality means.

Student Response

1. $x \geq 55.25$ means Elena has to put more than 55 movies on her hard drive. This doesn't make sense because there should be a maximum limit on movies rather than a minimum limit.

2. The correct inequality is $8x + 58 \leq 500$. The number 8 represents the size of each movie. The variable x represents the number of movies that Elena downloads. The $\leq$ 500 represents that the number of gigabytes can't exceed 500.

Student Lesson Summary

We can represent and solve many real-world problems with inequalities. Whenever we write an inequality, it is important to decide what quantity we are representing with a variable. After we make that decision, we can connect the quantities in the situation to write an expression, and finally, the whole inequality.

As we are solving the inequality or equation to answer a question, it is important to keep the meaning of each quantity in mind. This helps us to decide if the final answer makes sense in the context of the situation.

For example: Han has 50 centimetres of wire and wants to make a square picture frame with a loop to hang it that uses 3 centimetres for the loop. This situation can be represented by $3 + 4s = 50$, where s is the length of each side (if we want to use all the wire). We can also use $3 + 4s \leq 50$ if we want to allow for solutions that don't use all the wire. In this case, any positive number that is less or equal to 11.75 cm is a solution to the inequality. Each solution represents a possible side length for the picture frame since Han can bend the wire at any point. In other situations, the variable may represent a quantity that increases by whole numbers, such as with numbers of magazines, loads of laundry, or students. In those cases, only whole-number solutions make sense.

Lesson 17 Practice Problems

1. **Problem 1 Statement**

   28 students travel on a field trip. They bring a van that can seat 12 students. Elena and Kiran's teacher asks other adults to drive cars that seat 3 children each to transport the rest of the students.

   Elena wonders if she should use the inequality $12 + 3n > 28$ or $12 + 3n \geq 28$ to figure out how many cars are needed. Kiran doesn't think it matters in this case. Do you agree with Kiran? Explain your reasoning.

   Solution

   Sample explanation: Yes, it doesn't matter. In this case n represents a number of cars, so only whole number values of n make sense for the situation, and there can't be fractions of cars. $12 + 3n = 28$ has the solution $n = \frac{16}{3}$, so the number of cars needed is 6.

2. **Problem 2 Statement**

   a. In the cafeteria, there is one large 10-seat table and many smaller 4-seat tables. There are enough tables to fit 200 students. Write an inequality whose solution is the possible number of 4-seat tables in the cafeteria.

   b. 5 barrels catch rainwater in the schoolyard. Four barrels are the same size, and the fifth barrel holds 10 litres of water. Combined, the 5 barrels can hold at least 200 litres of water. Write an inequality whose solution is the possible size of each of the 4 barrels.

   c. How are these two problems similar? How are they different?

   Solution

   a. $10 + 4n \geq 200$

   b. $10 + 4n \geq 200$

   c. Solutions to the first inequality must be whole numbers greater or equal to 47.5 because a solution represents a number of tables. Solutions to the second inequality can be any number greater or equal to 47.5 because a solution represents the volume of a bucket, which can be a whole number or not.

3. **Problem 3 Statement**

Solve each equation.

a. $5(n - 4) = -60$

b. $-3t + -8 = 25$

c. $7p - 8 = -22$

d. $\frac{2}{5}(j + 40) = -4$

e. $4(w + 1) = -6$

Solution

a. $n = -8$

b. $t = -11$

c. $p = -2$

d. $j = -50$

e. $w = \frac{-10}{4}$ (or equivalent)

4. **Problem 4 Statement**

Select **all** the inequalities that have the same graph as $x < 4$.

a. $x < 2$

b. $x + 6 < 10$

c. $5x < 20$

d. $x - 2 > 2$

e. $x < 8$

Solution ["B", "C"]

5. **Problem 5 Statement**

A 200 pound person weighs 33 pounds on the Moon.

a. How much did the person's weight decrease?

b. By what percentage did the person's weight decrease?

Solution

a. 167 pounds

b. About 84% ($167 \div 200 = 0.835$)