Lesson 12: Finding the percentage

Goals

- Determine what information is needed to solve a problem involving sales tax and discounts. Ask questions to elicit that information.

- Explain (orally) how to calculate the percentage, given the pound amounts before and after a sales tax, tip, or discount.

- Interpret (orally and in writing) bar models that represent situations involving a sales tax, tip, or discount.

Learning Targets

- I can find the percentage increase or decrease when I know the original amount and the new amount.

Lesson Narrative

In this lesson, students consolidate what they have learned over the last few lessons and solve a variety of multi-step percentage problems involving taxes, tips, and discounts, including problems involving fractional percentages. They continue to move towards using equations to represent problems, which enable them to see the common underlying structure behind different problems. For example, $1.2x$ can represent

- a 20% increase in x.

- the total bill when 20% tax is added.

- the total bill when a 20% tip is added.

- the retail price when the wholesale price is marked up by 20%.

Building On

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.

Addressing

- Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Building Towards

- Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- Clarify, Critique, Correct

- Information Gap Cards

- Think Pair Share

Required Materials

Four-function calculators
Pre-printed slips, cut from copies of the blackline master

Info Gap: Sporting Goods Problem Card 1 Elena went to a sporting goods store that was having a sale. She bought a tennis racket and 3 cans of tennis balls. How much will she pay for everything, including tax?	Info Gap: Sporting Goods Data Card 1 • The tennis racket normally costs £43. • All tennis rackets are marked down 15%. • One can of tennis balls normally costs £4. • The tennis balls are not marked down. • The sales tax rate is 8.5%.
Info Gap: Sporting Goods Problem Card 2 Andre went to a sporting goods store that was having a different sale. He bought a baseball glove and 2 packs of socks. What percentage of the total usual price (before tax) was his savings?	Info Gap: Sporting Goods Data Card 2 • The baseball glove normally costs £34. • The baseball glove is not discounted. • One pack of socks normally costs £6. • On sale, one pack of socks costs £4. • The sale tax rate is 7.75%.

Required Preparation

Print and cut up slips from the Info Gap: Sporting Goods blackline master. One copy of the blackline master is needed for every 4 students. A class set could be re-used if you have more than one class.

Student Learning Goals

Let's find unknown percentages.

12.1 Tax, Tip, and Discount

Warm Up: 5 minutes

In this warm-up, students are reminded of the bar model method for understanding parts of a whole. The bar models are used in the context of tips, taxes, and discounts.
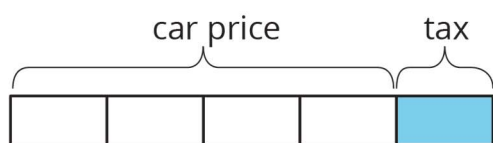
Instructional Routines

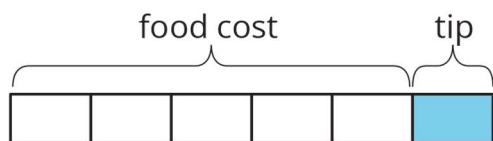- Think Pair Share

Launch

Students in groups of 2. Allow students 2 minutes quiet work time followed by partner then whole-class discussion.
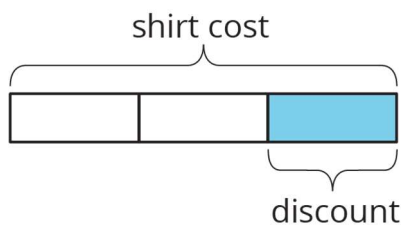
Student Task Statement

What percentage of the car price is the tax?

What percentage of the food cost is the tip?

What percentage of the shirt cost is the discount?

Student Response

The tax is 25% since it is the same size as one fourth of the entire car price.

The tip is 20% since it is the same size as one fifth of the entire food cost.

The discount is about 33% since it is the same size as one third of the entire shirt cost.

Activity Synthesis

The purpose of the discussion is for students to recognise that a bar model can be useful for working with percentages as part of a whole.

Consider asking these discussion questions:

- "The tax on the car is what fraction of the car price before tax was added on?" (One fourth)

- "With the tip added on, how is the length of the entire bar related to the length of the bar that just represents the food cost?" (It is 1.2 times as long.)

- "How does the value $\frac{2}{3}$ relate to the last bar model?" (It represents the fraction of the original cost of the shirt that is the price of the shirt after the discount.)

12.2 What Is the Percentage?

10 minutes

In this activity, students continue to practise finding percentages from pound amounts including commission, tip, and markdown. Some of the questions require multiple steps to solve for the percentage needed.

As students work, monitor for students who take different approaches to solving the multiple steps involved in these problems. For example, some students may subtract the discounted price from the original price to find the amount that the item has been marked down before finding the percentage of markdown. Other students may find the percentage of the original price used for the discounted price and then subtract that from 100%.

Instructional Routines
- Anticipate, Monitor, Select, Sequence, Connect

- Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet work time followed by partner and whole-class discussions.

Student Task Statement
1. A salesperson sold a car for £18 250 and their commission is £693.50. What percentage of the sale price is their commission?

2. The bill for a meal was £33.75. The customer left £40.00. What percentage of the bill was the tip?

3. The original price of a bicycle was £375. Now it is on sale for £295. What percentage of the original price was the markdown?
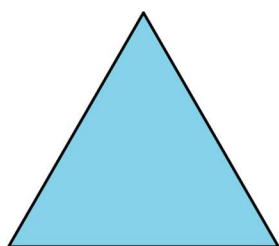
Student Response
1. 3.8% since $693.50 \div 18\,250 = 0.038$.

2. about 18.52% since the tip is £6.25 and $6.25 \div 33.75 \approx 0.1852$.

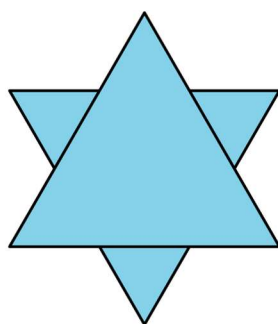3. about 21.33% since the price was marked down by £80 and $80 \div 375 \approx 0.2133$.

Are You Ready for More?
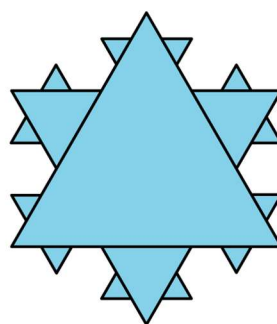
To make a Koch snowflake,

- Start with an equilateral triangle. This is step 1.

- Divide each side into 3 equal pieces. Construct a smaller equilateral triangle on the middle third. This is step 2.

- Do the same to each of the newly created sides. This is step 3.

- Keep repeating this process.

step 1   step 2   step 3

By what percentage does the perimeter increase at step 2? Step 3? Step 10?

Student Response

The perimeter increases by a factor of $\frac{4}{3}$ at each step, so the percent increase at each step is $33\frac{1}{3}$%. In step 2, the perimeter has increased by about 33.3%.

In step 3, the perimeter has increased by about 77.8% since the perimeter is $\frac{4}{3}$ of the perimeter in step 2 which is, in turn, $\frac{4}{3}$ of the perimeter in step 1, so the step 3 is $\frac{4}{3} \times \frac{4}{3} = \frac{16}{9} \approx 1.778$ of the length of the perimeter in step 1.

In step 10, the perimeter has increased by about $1\,675.8$% since it increases by $\frac{4}{3}$ at each step, so by step 10 the perimeter is $\left(\frac{4}{3}\right)^{10} = \frac{1\,048\,576}{59\,049} \approx 17.758$ of the length of the perimeter in step 1.

Activity Synthesis

The purpose of the discussion is for students to share methods for finding percentages in problems that require multiple steps.

Select the previously identified students to share their different methods for solving the last 2 questions. Sequence so strategies using diagrams come first, and those using equations come second.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as "First, I ____ because . . . ", "I noticed ____ so I . . . ", "Why did you . . . ?", "I agree/disagree because . . . ." *Supports accessibility for: Language; Social-emotional skills Writing: Clarify, Critique, Correct.* Present an incorrect response to the third question that reflects a possible misunderstanding from the class. For example, "It's 27.12% because 375/295 is 1.2712." To prompt students to identify the error, ask, "Do you agree with the author's reasoning? Explain why or why not? and write a correct version." Amplify mathematical language that students use, drawing attention to where precise language helps improve understanding. This can help students to further understand how to calculate the percentage of a markdown when given the original and sale prices.
Design Principle(s): Support sense-making

12.3 Info Gap: Sporting Goods

20 minutes

The purpose of this info gap activity is for students to identify the essential information needed to determine the total savings after various discounts are applied to different items.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Instructional Routines
- Information Gap Cards

Launch

Provide access to calculators. Tell students they will continue to work with percentages and their similarities to proportional relationships. Explain the Info Gap structure and consider demonstrating the protocol if students are unfamiliar with it. There are step-by-step instructions in the student task statement.

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the

activity.

Supports accessibility for: Memory; Organisation Conversing: Display questions or question starters for students who need a starting point such as: "Can you tell me . . . (specific piece of information)", and "Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate Conversation

Anticipated Misconceptions

Students might fail to notice that Elena and Andre buy multiple cans of tennis balls and packs of socks, respectively. Ask students to figure out how much 2 packs of socks (or 3 cans of tennis balls) will cost.

If students automatically give the 15% discount on all of Elena's purchases, ask students which of Elena's items fall under the discount.

Students might apply the discount after the adding in the sales tax. Remind students that the discount gets applied to the subtotal before the tax is calculated.

Some students may include the sales tax when calculating the percentage of Andre's savings. Remind them that the problem specifies "before tax."

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.

2. Ask your partner for the specific information that you need.

3. Explain how you are using the information to solve the problem.

    Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.

5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.

2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

    If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.

3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.

4. Read the *problem card* and solve the problem independently.

5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

1. Elena will pay £52.68. Sample explanation: Elena spent 0.85×43, or £36.55 on the tennis racket and 3×4, or £12 on the cans of tennis balls. The total price before tax was £48.55, because $36.55 + 12 = 48.55$. The total cost including tax is 1.085×48.55, or about £52.68.

2. Andre saved about 8.7%. Sample explanation: The usual price for the baseball glove and two packs of socks is £46, because $34 + 6 + 6 = 46$. Andre saved £4, because each pack of socks was discounted by £2, he bought 2 packs, and $2 \times 2 = 4$. This savings is about 8.7% of the usual price, because $4 \div 46 \approx 0.087$.

Activity Synthesis

The purpose of the discussion is for students to recognise what information may be needed to solve problems involving percentages for prices of items.

After students have completed their work, share the correct answers and ask students to discuss the different ways they solved this problem. Some guiding questions:

*   "What information did you and your partner have to figure out?"

*   "How did you determine the cost of Elena's tennis racket?" (Multiply the original cost by 0.85 or multiply by 0.15 and subtract from the original cost.)

*   "How did you determine the total cost after tax for Elena's purchases?" (Multiply the total by 1.085 or multiply by 0.085 and add to the original cost.)

*   "What different calculations did you have to make for Andre and Elena's situations?"

*   "Was there information given that you did not need to use?"

Lesson Synthesis

In this lesson, students found the percentage increase or decrease given the original and final amounts.

*   "When the original price and discounted prices are known, how can we find the percent markdown?" (Find the difference and then find the percentage that difference is of the original price.)

- "When you know the original price of an item and the price you paid at a register, how can you find the tax rate?" (Find the difference and then find the percentage that difference is of the original price.)

12.4 Shoes on Sale

Cool Down: 5 minutes

In this cool-down, students are assessed on their ability to find the percentage discount on a pair of shoes given the original and discounted prices.

Student Task Statement

With a coupon, you can get a pair of shoes that normally costs £84 for only £72. What percentage was the discount?

Student Response

Approximately 14.3% since the discount takes £12 off the price of shoes and $12 \div 84 \approx 0.143$.

Student Lesson Summary

To find a 30% increase over 50, we can find 130% of 50.

$1.3 \times 50 = 65$

To find a 30% decrease from 50, we can find 70% of 50.

$0.7 \times 50 = 35$

If we know the initial amount and the final amount, we can also find the percent increase or percent decrease. For example, a plant was 12 inches tall and grew to be 15 inches tall. What percent increase is this? Here are two ways to solve this problem:

The plant grew 3 inches, because $15 - 12 = 3$. We can divide this growth by the original height, $3 \div 12 = 0.25$. So the height of the plant increased by 25%.

The plant's new height is 125% of the original height, because $15 \div 12 = 1.25$. This means the height increased by 25%, because $125 - 100 = 25$.

Here are two ways to solve the problem: A rope was 2.4 metres long. Someone cut it down to 1.9 metres. What percentage decrease is this?

The rope is now $2.4 - 1.9$, or 0.5 metres shorter. We can divide this decrease by the original length, $0.5 \div 2.4 = 0.208\dot{3}$. So the length of the rope decreased by approximately 20.8%.

The rope's new length is about 79.2% of the original length, because $1.9 \div 2.4 = 0.7916\dot{}$. The length decreased by approximately 20.8%, because $100 - 79.2 = 20.8$.

Lesson 12 Practice Problems

Problem 1 Statement

A music store marks up the instruments it sells by 30%.

    a.   If the store bought a guitar for £45, what will be its store price?

    b.   If the price tag on a trumpet says £104, how much did the store pay for it?

    c.   If the store paid £75 for a clarinet and sold it for £100, did the store mark up the price by 30%?

Solution

    a.   £58.50

    b.   £80.00

    c.   No. The store marked the price up by $\frac{1}{3}$ or about 33.3% (rounded to the nearest tenth of a percent). The store needed to sell it for £97.50 to have a 30% markup.

Problem 2 Statement

A family eats at a restaurant. The bill is £42. The family leaves a tip and spends £49.77.

    a.   How much was the tip in pounds?

    b.   How much was the tip as a percentage of the bill?

Solution

    a.   £7.77

    b.   18.5%

Problem 3 Statement

The price of gold is often reported per ounce. At the end of 2005, this price was £513. At the end of 2015, it was £1 060. By what percentage did the price per ounce of gold increase?

Solution

About 107% ($1\,060 - 513 = 547$ and $547 \div 513 \approx 1.07$)

Problem 4 Statement

A phone keeps track of the number of steps taken and the distance travelled. Based on the information in the table, is there a proportional relationship between the two quantities? Explain your reasoning.

number of steps	distance in kilometres
950	1
2 852	3
4 845	5.1

Solution

No, there is not a proportional relationship. Since the first row shows that there are 950 steps in 1 kilometre, there should be 2 850 steps in 3 kilometres (since $950 \times 3 = 2\,850$), but the table shows 2 852 steps.

Problem 5 Statement

Noah picked 3 kg of cherries. Mai picked half as many cherries as Noah. How many total kg of cherries did Mai and Noah pick?

    a. $3 + \frac{1}{2}$

    b. $3 - \frac{1}{2}$

    c. $\left(1 + \frac{1}{2}\right) \times 3$

    d. $1 + \frac{1}{2} \times 3$

Solution C