Grades 7-8 (A), 9-12 (S)

Duration: 20 min

Tools: one 16 pcs Set / 1-2 student

Individual work

Keywords: Mathematical Induction, Proof, Algebraic identities

309 - Triangular Square Number

MATHS / SEQUENCES

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

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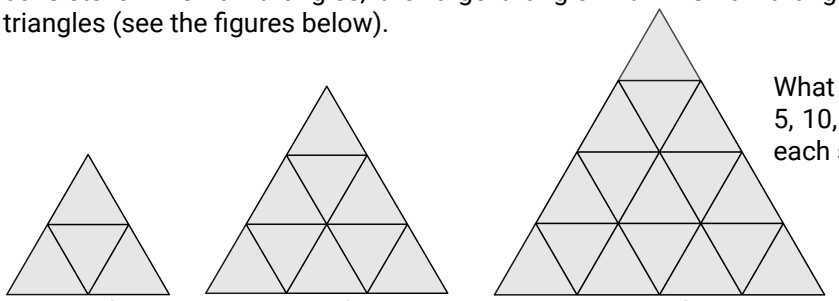
DESCRIPTION

Observe the following: if large triangles are built from congruent triangles, the large triangle with 2 small triangles on each side consists of 4 small triangles, the large triangle with 3 small triangles on each side consists of 9 small triangles, the large triangle with 4 small triangles on each side consists of 16 small triangles (see the figures below).



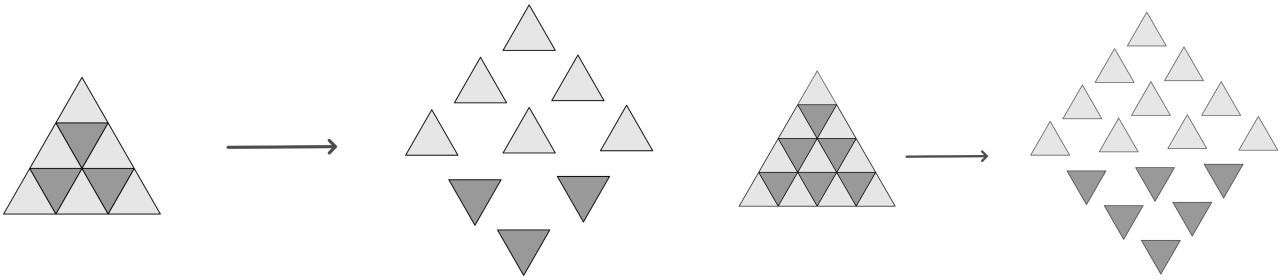
n = 2 n = 3 n = 4

What can be said about a large triangle with 5, 10, 100, or, in general, n small triangles on each side? Prove the resulting formula!

SOLUTIONS / EXAMPLES

The large triangle with n small triangles on each side consists of n^2 small triangles. It can be proven in several ways.

PROOF 1 The figures below show (on the examples n=3 and n=4) that the small triangles can be arranged into an $n \times n$ square.



PROOF 2 By induction. Observe that increasing the number of small triangles on each side from n to $n + 1$ increases the number of small triangles by $2n + 1$. Suppose that the triangle of side length n consists of n^2 small triangles. Then by the observation, the triangle with $(n + 1)$ small triangles on each side consists of $n^2 + 2n + 1$ small triangles. This equals $(n + 1)^2$, as required.

PRIOR KNOWLEDGE

PROOF 1 None

PROOF 2 Algebraic identities, Proof, Mathematical Induction

RECOMMENDATIONS / COMMENTS

This is a very natural question after laying out the large triangles using the Logifaces set.