Lesson 7: No bending or stretching

Goals

- Comprehend that, for some transformations, all pairs of “corresponding distances” and “corresponding angle” sizes in the shape and its image are the same.
- Draw and label a diagram of the image of a polygon under such a transformation, including calculating side lengths and angle sizes.
- Identify (orally and in writing) a sequence of such transformations using a drawing of a shape and its image.

Learning Targets

- I can describe the effects of one of these transformation on the lengths and angles in a polygon.

Lesson Narrative

In this lesson, students begin to see that translations, rotations, and reflections preserve lengths and angle sizes. In earlier lessons, students talked about corresponding points under a transformation. Now they will talk about **corresponding** sides and **corresponding** angles of a polygon and its image.

As students experiment with measuring corresponding sides and angles in a polygon and its image, they will need to use the structure of the grid as well as appropriate technology, including protractors, rulers, and tracing paper.

Building On

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Addressing

- Lines are taken to lines, and line segments to line segments of the same length.
- Angles are taken to angles of the same size.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display
- Discussion Supports

Required Materials

Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Student Learning Goals

Let's compare measurements before and after translations, rotations, and reflections.

7.1 Measuring Line Segments

Warm Up: 5 minutes

In this warm-up, students measure four line segments. They discuss the different aspects of making and recording accurate measurements. It is important to highlight the fractional markings and fraction and decimal equivalents used as students explain how they determined the length of the line segment.

Launch

Give students 2 minutes of quiet work time followed by whole-class discussion.

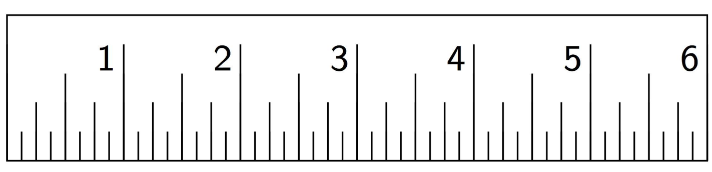
Anticipated Misconceptions

Students may struggle with the ruler that is not pre-partitioned into fractional units. Encourage these students to use what they know about eighths and tenths to partition the ruler and estimate their answer.

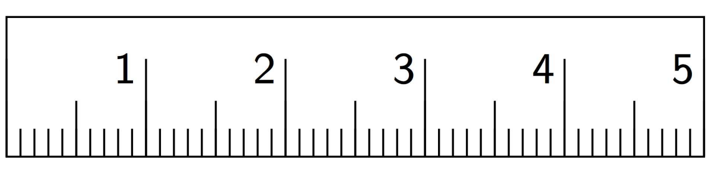
Student Task Statement

For each question, the unit is represented by the large tick marks with whole numbers.

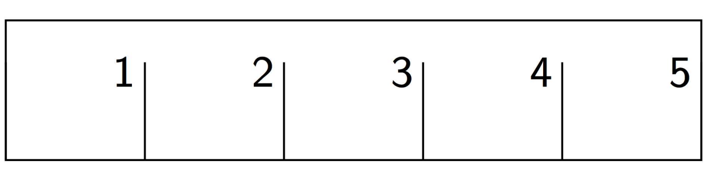
1. Find the length of this line segment to the nearest $\frac{1}{8}$ of a unit.



2. Find the length of this line segment to the nearest 0.1 of a unit.



3. Estimate the length of this line segment to the nearest $\frac{1}{8}$ of a unit.



4. Estimate the length of the line segment in the prior question to the nearest 0.1 of a unit.

Student Response

1. $4\frac{5}{8}$ units

2. 4.7 units

3. $3\frac{3}{4}$ (or $3\frac{6}{8}$ units)

4. 3.7 units (or 3.8 units)

Activity Synthesis

Invite students to share their responses and record them for all to see. Ask the class if they agree or disagree with each response. When there is a disagreement, have students discuss possible reasons for the different measurements.

Students are likely to have different answers for their measurement of the third line segment. The ruler shown is not as accurate as the question requires as it has not been pre-partitioned into fractional units. Ask 2–3 students with different answers to share their strategies for measuring the third line segment. There will be opportunities for students to use measuring strategies later in this lesson.

7.2 Sides and Angles

15 minutes (there is a digital version of this activity)

The purpose of this activity is for students to see that translations, rotations, and reflections preserve lengths and angle sizes. Students can use tracing paper to help them draw the shapes and make observations about the preservation of side lengths and angle sizes under transformations. While the grid helps measure lengths of horizontal and vertical line segments, the students may need more guidance when asked to measure diagonal lengths. It is important in the launch to demonstrate for students how to either use the tracing paper or an index card to mark off unit lengths using the grid.

Since students are creating their own measuring tool, they can only give an estimate, and some flexibility should be allowed in the response. During the discussion, highlight different reasonable answers that students find for the lengths which are not whole numbers.

As students work individually, monitor and ask them to explain how they are performing their transformations and finding the side lengths and angle sizes. During the discussion, select students who mention corresponding sides and angles, which they learned earlier in KS3 when making scaled copies, to share. Also select students who estimated the side lengths for shape C correctly using either the tracing paper or index card.

Instructional Routines

- Discussion Supports

Launch

Tell students, “In this activity you will be performing transformations. You can use tracing paper to help you draw the images of the shapes or to check your work.”

Point students to shape C and tell them, “When you are asked to measure side lengths here, you will need to make a ruler on either tracing paper or on a blank edge of an index card.” This reinforces the strategies and estimates students made in the warm-up.

Give students 3 minutes of quiet think time. Be sure to save at least 5 minutes for the discussion.

For classrooms using the digital version of the activity, the applets contain tools for the three transformations students need. They have to choose which tool to use in each problem. Caution students that a quick click is all that is needed to select a shape. If they move the cursor away, and the image does not seem “highlighted,” it is likely they selected and de-selected the shape.

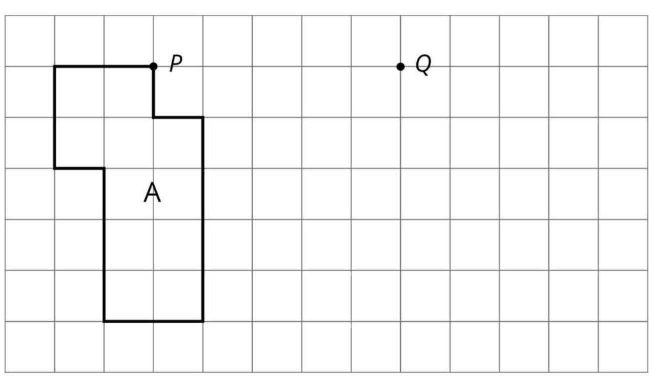
Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Look for students who make a ruler and use it appropriately in finding side lengths. Invite students to share how they found diagonal side lengths in the last problem.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

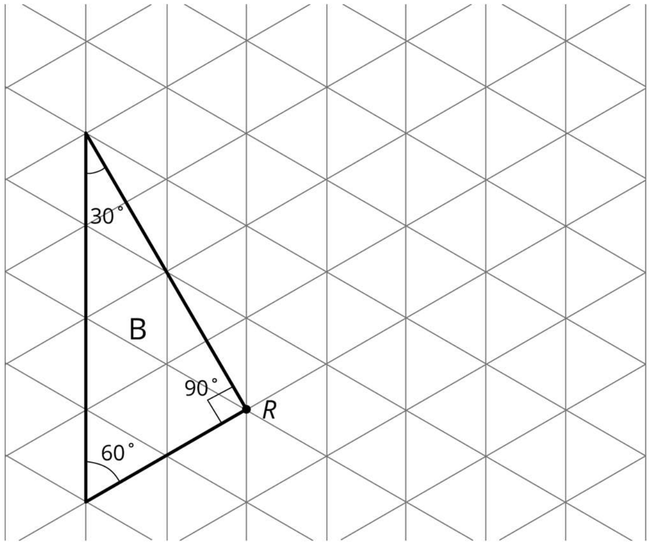
Students may try to count the grid squares on the diagonal side lengths. Remind students to measure these lengths with their tracing paper or index card. Students may also struggle estimating the diagonal side lengths on their self-marked index card or tracing paper. Remind students of how they estimated the lengths for the questions in the warm-up where the ruler was not marked.

Student Task Statement

1. Translate polygon A so point P goes to point Q. In the image, write the length of each side, in grid units, next to the side.



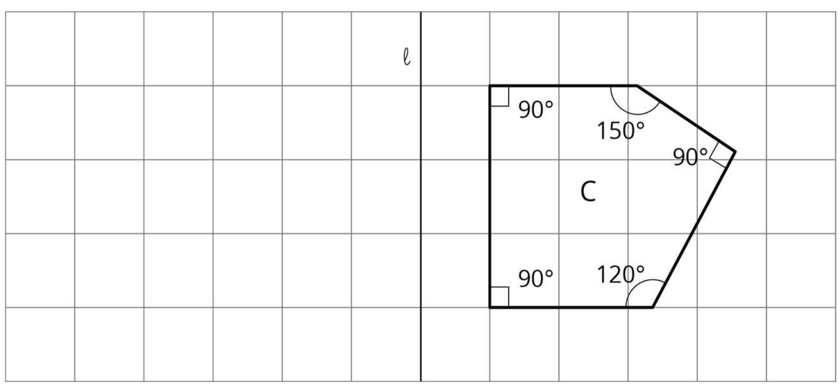
2. Rotate triangle B 90 degrees clockwise using R as the centre of rotation. In the image, write the size of each angle in its interior.



3. Reflect pentagon C in line ℓ.

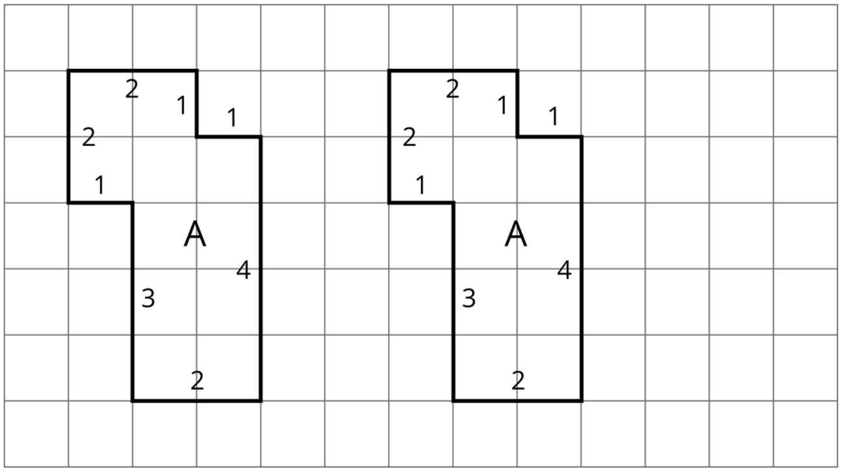
   a. In the image, write the length of each side, in grid units, next to the side. You may need to make your own ruler with tracing paper or a blank index card.

   b. In the image, write the size of each angle in the interior.



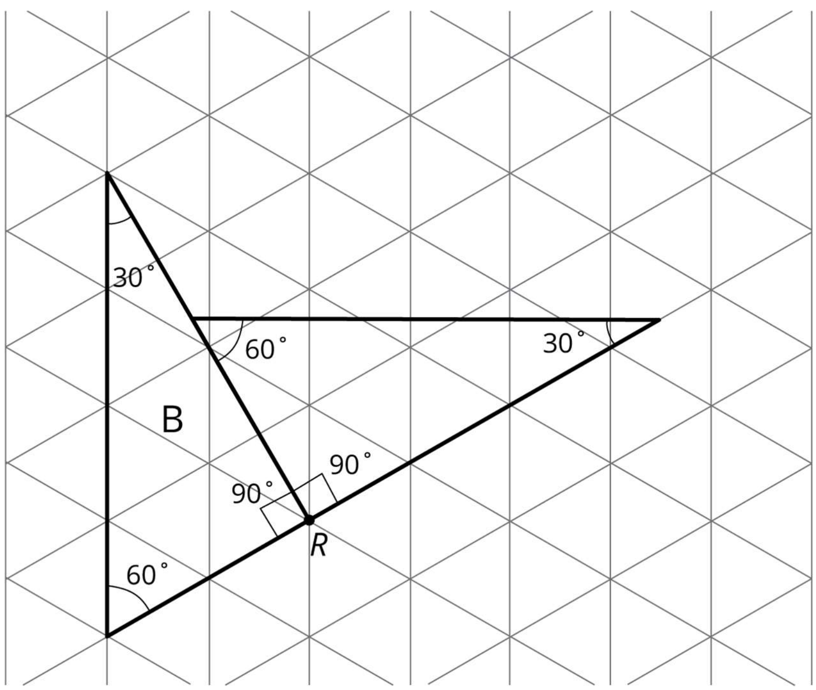
Student Response

1.

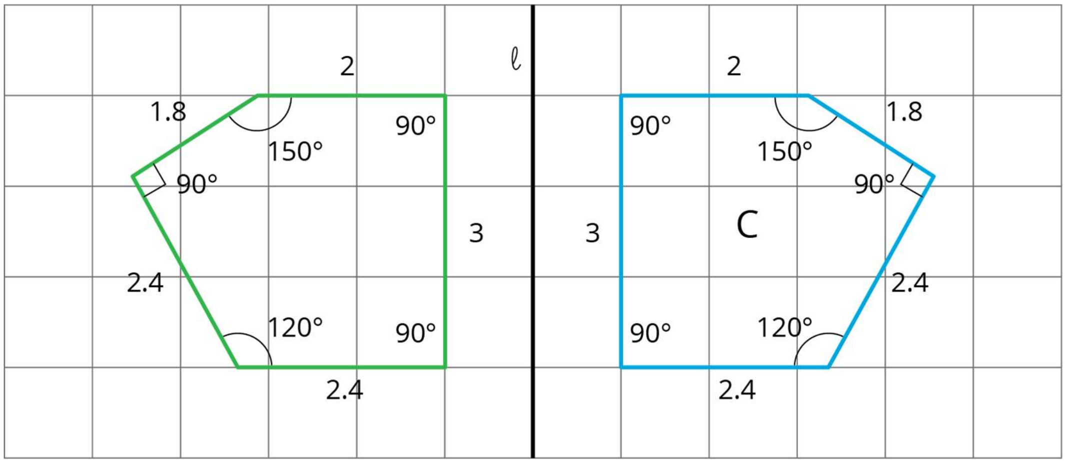


The side lengths are measured in units where one unit is the side length of the square in the grid.

2.



3.

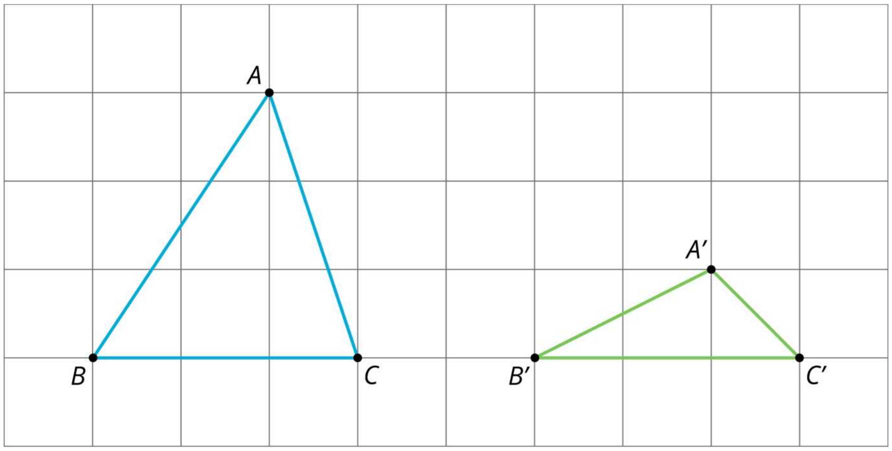


The lengths are measured in grid units. The sides that are not whole numbers have been rounded to the nearest tenth.

Activity Synthesis

Ask selected students to share how they performed the given transformation for each question. After each explanation, ask the class if they agree or disagree. Introduce students to the idea of **corresponding sides** and **corresponding angles**. Ask students to identify the corresponding angles in the first question and the corresponding side lengths in the second since they were not asked about these attributes the first time. The point here is not to find the actual values but to note that the corresponding measurements are equal. Since it is sometimes not possible to measure angles or side lengths exactly, student estimates for these values (both corresponding sides and corresponding angles) may be slightly different.

Point out that for each of the transformations in this activity, the lengths of the sides of the original shape equal the lengths of the corresponding sides in the image, and the sizes of the angles in the original shape equal the sizes of the corresponding angles in the image. For this reason, we consider these transformations special: they behave as if we are moving the shapes around without stretching, bending, or breaking them. An example of a different sort of transformation is one that compresses a shape vertically, like this:



Tell them that these transformations are ones where all pairs of corresponding distances and angle sizes in the shape and its image are equal. It turns out that translations, reflections, and rotations are the building blocks for *all* transformations that maintain the sizes of corresponding lengths and angles, and we will explore that next.

Speaking: Discussion Supports. As students describe their approaches, press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example of their process. Connect the terms corresponding sides and corresponding angles to students' explanations multi-modally by using different types of sensory inputs, such as demonstrating the transformation or inviting students to do so, using the images, and using gestures. This will help students to produce and make sense of the language needed to communicate their own ideas.
Design Principle(s): Optimise output (for explanation)

7.3 Which One?

10 minutes (there is a digital version of this activity)

The purpose of this activity is to decide if there is a sequence of translations, rotations, and reflections that take one shape to another and, if so, to produce one such sequence. Deciding whether or not such a sequence is possible uses the knowledge that translations, rotations, and reflections do not change side lengths or angle sizes. The triangles ABC and CFG form part of a large pattern of images of triangle ABC that will be examined more closely in future lessons.

Monitor for students who use different transformations to take triangle ABC to triangle CFG and select them to share during the discussion. (There are two possible sequences in the Possible Responses section, but these are not the only two.)

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display

Launch

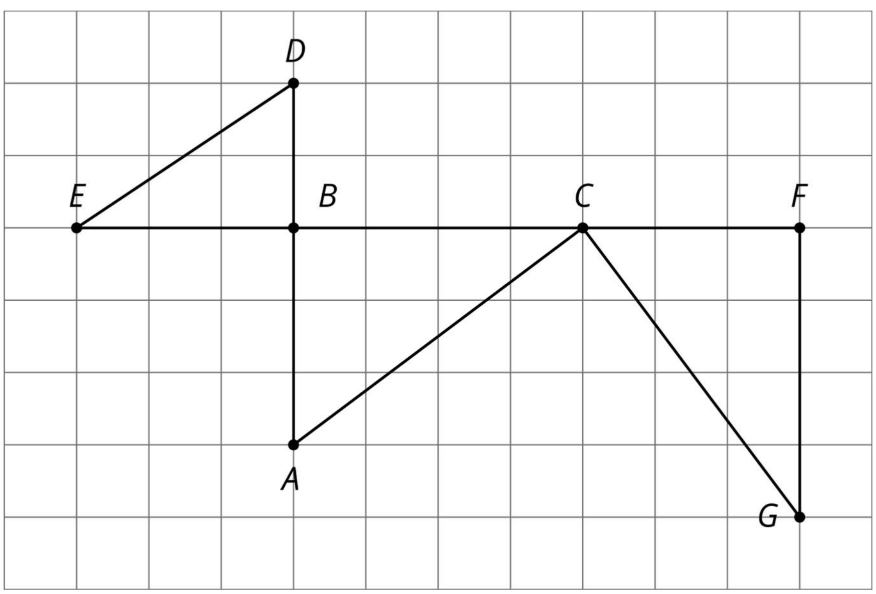
Provide access to geometry toolkits. Give students 4 minutes quiet work time, 2 minutes to discuss with partner, and then time for a whole-class discussion.

If using the digital activity, have a brief discussion of the previous activity to highlight the transformations that the students used. Then give students 4 minutes of individual work time, 2 minutes to discuss with a partner, and then time for a whole-class discussion.

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, “Triangle _____ is a transformation of triangle ABC because...,” “I agree/disagree because...,” or “Another transformation is _____ because….”
Supports accessibility for: Language; Organisation Conversing: Collect and Display. As students discuss their work with a partner, listen for and collect the language students use to describe each transformation. Record students’ words and phrases on a visual display (e.g., "Rotate triangle ABC 90 degrees anti-clockwise around point C," "Translate triangle ABC 7 units right," etc.), and update it throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This will help students read and use mathematical language during paired and whole-group discussions.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Student Task Statement

Here is a grid showing triangle ABC and two other triangles.



You can use a transformation to take triangle ABC to *one* of the other triangles.

1. Which one? Explain how you know.

2. Describe a transformation that takes ABC to the triangle you selected.

Student Response

1. It’s triangle CFG. Triangle DBE is smaller than $\triangle ABC$, so no sequence of these transformations can take $\triangle ABC$ to triangle DBE.

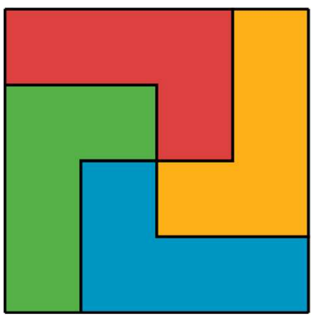
2. Answers vary. Here are two possible sequences:

- Translate triangle ABC 7 units right so that B matches up with F. Then rotate 90 degrees clockwise around F.

- Rotate triangle ABC 90 degrees anti-clockwise around point C, and then rotate 180 degrees around the midpoint of M of line segment CG.

Are You Ready for More?

A square is made up of an L-shaped region and three transformations of the region. If the perimeter of the square is 40 units, what is the perimeter of each L-shaped region?



Student Response

25 units.

Activity Synthesis

Ask a student to explain why triangle ABC *cannot* be taken to triangle DBE. (We are only using translations, rotations and reflections and therefore the corresponding lengths have to be equal and they are not.) If a student brings up that they think triangle DBE is a scale drawing of ABC, bring the discussion back to translations, rotations, and reflections, rather than talking about how or why triangle DBE isn’t actually a scale drawing of ABC.

Offer as many methods for transforming triangle ABC as possible as time permits, selecting previously identified students to share their methods. Include at least two different sequences of transformations. Make sure students attend carefully to specifying each transformation with the necessary level of precision. For example, for a rotation, that they specify the centre of rotation, the direction, and the angle of rotation.

If time allows, consider asking the following questions:

- "Can triangle ABC be taken to triangle CFG with only a translation?" (No, since CFG is rotated.)

- "What about with only a reflection?" (No, because they have the same orientation.)
- "What about with a single rotation?" (The answer is yes, but this question does not need to be answered now as students will have an opportunity to investigate this further in a future lesson.)

Lesson Synthesis

Remind students that translations, rotations and reflections are transformations for which all pairs of **corresponding** lengths and angle sizes in the original shape and its image are equal. Sequences of these are as well—for example, if you translate a shape then reflect the image, the side lengths and angle sizes stay the same.

Ask students to think of ways they could look at two shapes and tell that one is not the image of the other under a translation, rotation or reflection. Give a moment of quiet think time, and then invite students to share their ideas.

When there is a transformation taking one shape to another, there are many ways to do this. Ask students:

- "What are some good ways to tell whether one shape can be taken to another with a sequence of translations, rotations or reflections?" (Measure all of the side lengths and angle size and ensure that corresponding measurements are equal. Use tracing paper to see if one shape matches up exactly with the other.)
- "What are the three basic types of transformations that maintain the sizes of corresponding lengths and angles?" (rotations, translations, and reflections)

7.4 Translated Trapezium

Cool Down: 5 minutes

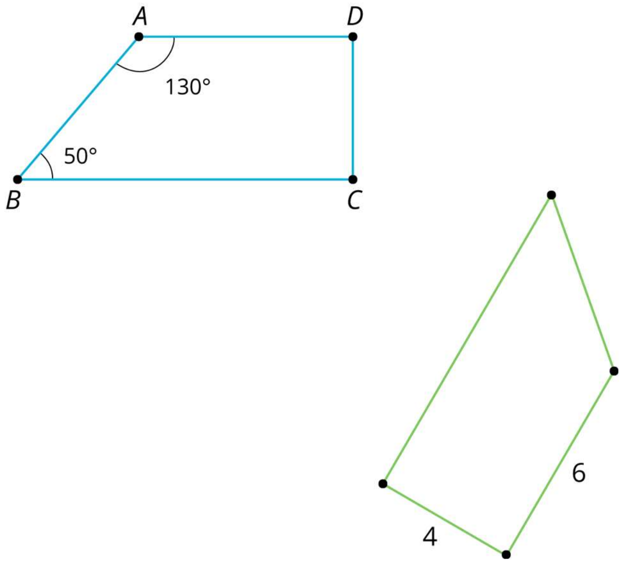
Students use key defining properties of these transformations, namely that they preserve side lengths and angle sizes, in order to calculate side lengths and angle sizes in a polygon and its image under such a transformation.

Launch

Provide access to a geometry toolkit.

Student Task Statement

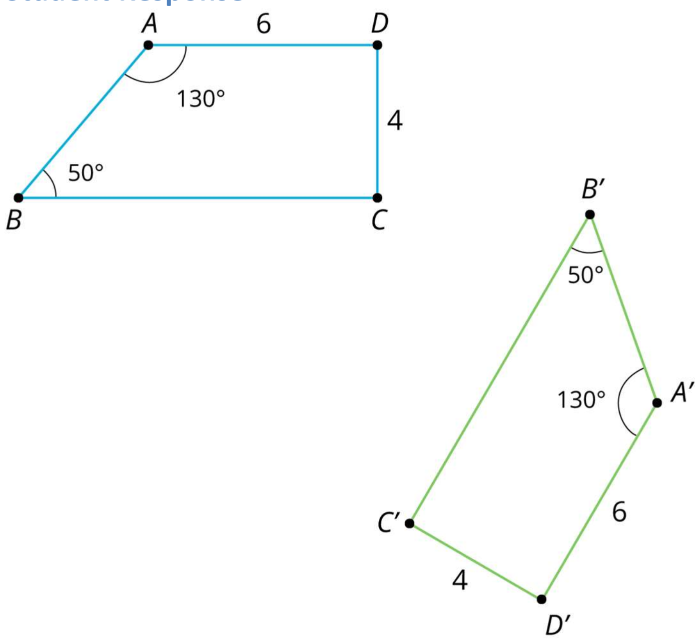
Trapezium $A'B'C'D'$ is the image of trapezium $ABCD$ under a transformation.



1. Label all vertices on trapezium $A'B'C'D'$.

2. On both shapes, label all known side lengths and angle sizes.

Student Response

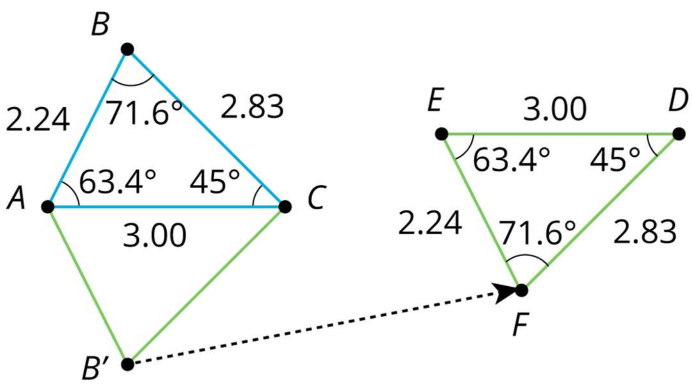


Student Lesson Summary

The transformations we've learned about so far, translations, rotations, reflections, and sequences of these motions, are all examples of transformations that don't change measurements on any shape.

Earlier, we learned that a shape and its image have corresponding points. With these transformation, shapes like polygons also have **corresponding** sides and corresponding angles. These corresponding parts have the same measurements.

For example, triangle EFD was made by reflecting triangle ABC in a horizontal line, then translating. Corresponding sides have the same lengths, and corresponding angles have the same sizes.



measurements in triangle ABC	corresponding measurements in image EFD
$AB = 2.24$	$EF = 2.24$
$BC = 2.83$	$FD = 2.83$
$CA = 3.00$	$DE = 3.00$
$\angle ABC = 71.6^\circ$	$\angle EFD = 71.6^\circ$
$\angle BCA = 45.0^\circ$	$\angle FDE = 45.0^\circ$
$\angle CAB = 63.4^\circ$	$\angle DEF = 63.4^\circ$

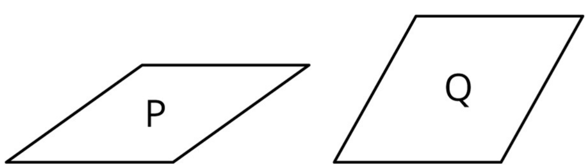
Glossary

- corresponding

Lesson 7 Practice Problems

1. **Problem 1 Statement**

   Is there a translation, rotation or reflection taking rhombus P to rhombus Q? Explain how you know.

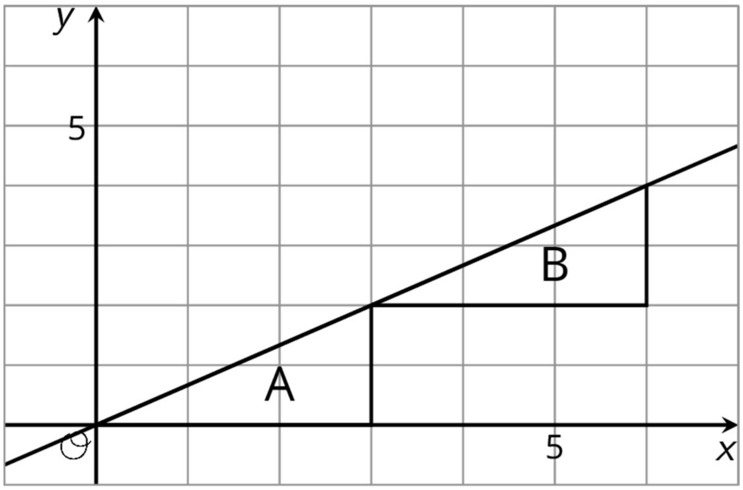


Solution

No, because the angle sizes of the two polygons are different, and a translation, rotation or reflection must preserve all lengths and angle sizes.

2. **Problem 2 Statement**

Describe a translation, rotation or reflection that takes triangle A to triangle B.

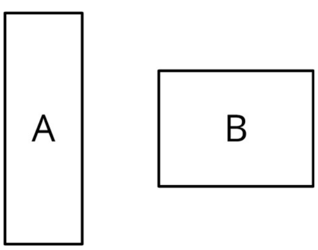


Solution

Translate three units right and two units up.

3. **Problem 3 Statement**

Is there a translation, rotation or reflection taking rectangle A to rectangle B? Explain how you know.



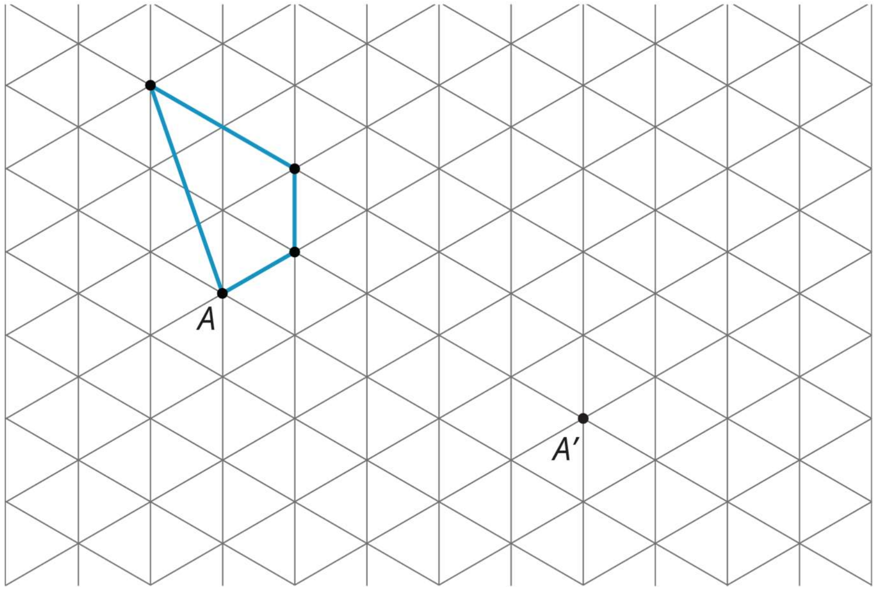
Solution

No, because the side lengths of the two rectangles are different, and a translation, rotation or reflection must preserve all lengths and angle sizes.

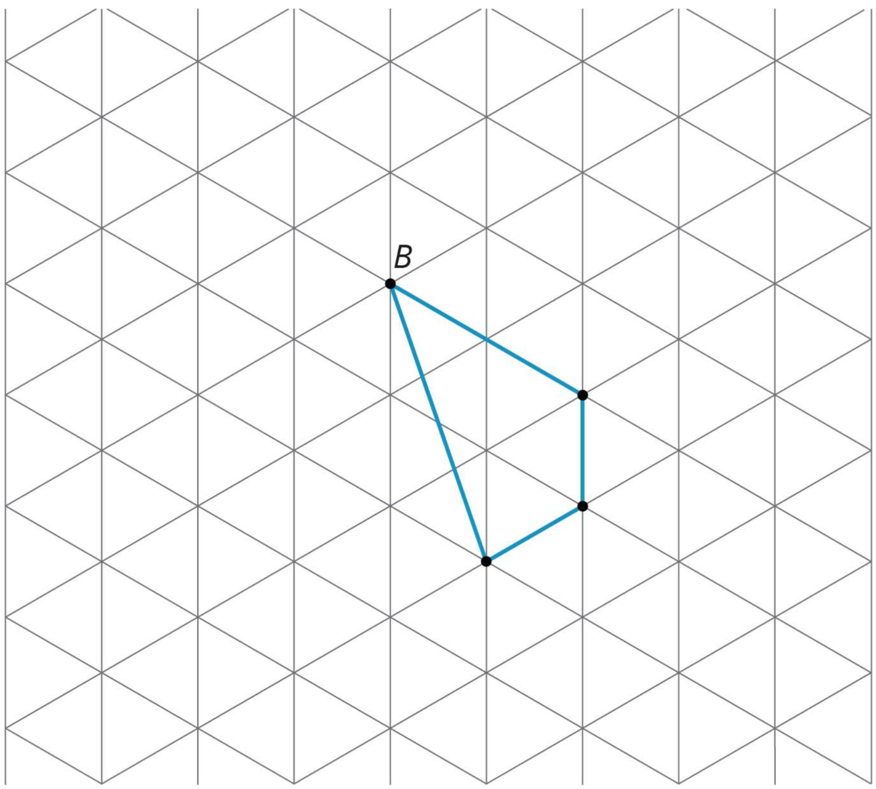
4. **Problem 4 Statement**

For each shape, draw its image after performing the transformation. If you get stuck, consider using tracing paper.

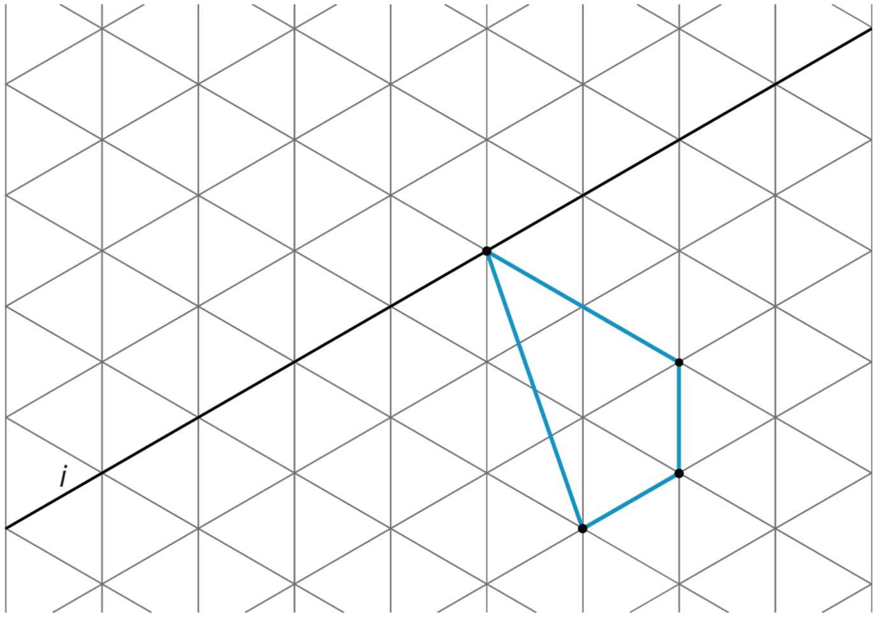
a. Translate the shape so that A goes to A'.



a. Rotate the shape 180 degrees anti-clockwise around B.

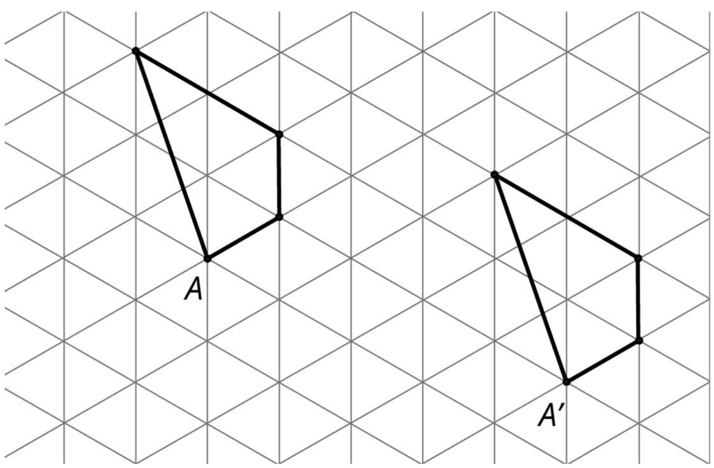


a. Reflect the shape over the line shown.

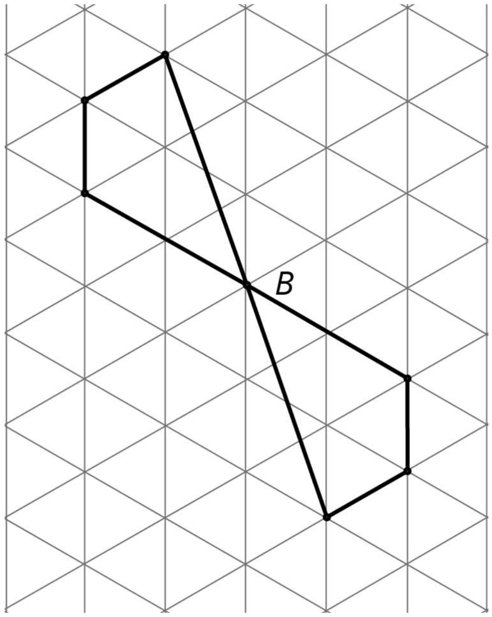


Solution

a.



b.



c.