Lesson 7: Comparing relationships with tables

Goals

- Calculate and compare the quotients of the values in each row of a given table.
- Generate a different recipe for lemonade and describe (orally) how it would taste in comparison to a given recipe.
- Justify (orally) whether the values in a given table could or could not represent a proportional relationship.

Learning Targets

- I can decide if a relationship represented by a table could be proportional and when it is definitely not proportional.

Lesson Narrative

In the next two lessons students compare proportional and non-proportional relationships. In this lesson, students examine tables and explain whether the relationships represented are proportional, not proportional, or possibly proportional. At this point in the unit, students should be comfortable using the terms “proportional relationship,” "is proportional to," and “constant of proportionality.” By the end of the next lesson, students should understand that equations of the form $y = kx$ with $k > 0$ characterise proportional relationships.

As students look at data from a context and reason about whether it makes sense quantitatively for the data to represent a proportional relationship, they are engaging in making viable arguments.

Building On

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.

Addressing

- Recognise and represent proportional relationships between quantities.

Building Towards

- Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- Co-Craft Questions
- Think Pair Share

Required Materials

Four-function calculators

Required Preparation

Calculators can optionally be made available to take the focus off computation.

Student Learning Goals

Let's explore how proportional relationships are different from other relationships.

7.1 Adjusting a Recipe

Warm Up: 5 minutes

This activity encourages students to reason about equivalent ratios in a context. During the discussion, emphasise the use of ratios and proportions in determining the effect on the taste of the lemonade.

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet think time.

Optionally, instead of the abstract recipe description, you could bring in a clear glass, measuring implements, and the lemonade ingredients. Pour in the ingredients and introduce the task that way.

Student Task Statement

A lemonade recipe calls for the juice of 5 lemons, 2 cups of water, and 2 tablespoons of honey.

Invent four new versions of this lemonade recipe:

1. One that would make more lemonade but taste the same as the original recipe.
2. One that would make less lemonade but taste the same as the original recipe.
3. One that would have a stronger lemon taste than the original recipe.
4. One that would have a weaker lemon taste than the original recipe.

Student Response

The base recipe has a ratio of number of lemons to cups of water to tablespoons of honey of 5:2:2. Answers vary. Sample responses:

1. 10 lemons, 4 cups of water, 4 tablespoons honey
2. $2\frac{1}{2}$ lemons, 1 cup water, 1 tablespoon honey
3. 8 lemons, 2 cups water, 2 tablespoons honey
4. 2 lemons, 2 cups water, 2 tablespoons honey

Activity Synthesis

Invite students to share their versions of the recipe with the class and record them for all to see. After each explanation, ask the class if they agree or disagree and how the new lemonade would taste. After recording at least 3 responses for each, ask students to describe any patterns they notice concerning how the recipe was adjusted. If students do not mention ratios in their descriptions, be sure to ask them how the ratios changed in their new recipe.

7.2 Visiting the State Park

15 minutes

This activity provides the first example in this unit of a relationship that is not proportional. The second question focuses students' attention on the unit rates. If the relationship was proportional then regardless of the number of people in a vehicle, the cost per person would be the same. The question about the bus is to show students that they can't just scale up from 10. Students who write an equation also see that it is not of the form $y = kx$. In a later lesson students will learn that only equations of this form represent proportional relationships.

Monitor for students who approached this problem using different representations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Keep students in the same groups of 2. Give 5 minutes of quiet work time, followed by 5 minutes of students discussing responses with a partner, followed by a whole-class discussion.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have accounted for the cost of the vehicle in their calculations of the total entrance cost for 4 people and 10 people.
Supports accessibility for: Memory; Organisation

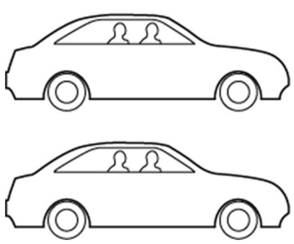
Anticipated Misconceptions

Some students may not account for the cost of the vehicle. They will get the following table with incorrect values and will need to be prompted to include the cost of the vehicle.

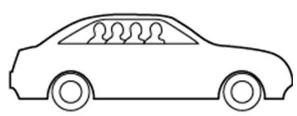
number of people in vehicle	total entrance cost in pounds
2	£10
4	£20
10	£50

Teachers will want to circulate around the room keeping an eye out for this mistake and address it as soon as possible so that students spend most of their work time analysing the non-proportional relationship. These diagrams may be helpful in illustrating to them that their resulting prices are including more than one vehicle. This gives them an opportunity to make sense of problems and persevere in solving them.

What you are paying for:



What you want to pay for:



Student Task Statement

Entrance to a state park costs £6 per vehicle, plus £2 per person in the vehicle.

1. How much would it cost for a car with 2 people to enter the park? 4 people? 10 people? Record your answers in the table.

number of people in vehicle	total entrance cost in pounds
2	
4	
10	

2. For each row in the table, if each person in the vehicle splits the entrance cost equally, how much will each person pay?

3. How might you determine the entrance cost for a bus with 50 people?

4. Is the relationship between the number of people and the total entrance cost a proportional relationship? Explain how you know.

Student Response

number of people in vehicle	total entrance cost in pounds
2	£10
4	£14
10	£26

1. With 2 people, £5.00. With 4 people, £3.50. With 10 people, £2.60.

2. £106. It still costs £6 for the vehicle, plus £2 for each of 50 people.

3. No. Explanations vary. Sample responses:

   - Considering the ratio of people in the vehicle to total entrance cost, these are not equivalent ratios.

   - The cost per person is not the same for different numbers of people.

   - Each number of people and corresponding total entrance cost is not characterised by the same unit rate.

Are You Ready for More?

What equation could you use to find the total entrance cost for a vehicle with any number of people?

Student Response

Let p be number of people and c be total entrance cost in pounds. $c = 6 + 2p$.

Activity Synthesis

Select students to explain why they think the relationship is or is not proportional. Reasons they may give:

- The cost per person is different for different number of people in a vehicle, i.e. the quotients of the entries in each row are not equal for all rows of the table.

- The ratio of people in the vehicle to total entrance cost are not equivalent ratios. You can't just multiply the entries in one row by the same constant to get the entries in another row.

- Each number of people and corresponding total entrance cost is not characterised by the same unit rate. You can't multiply the entries in the first column by the same number (constant of proportionality) to get the numbers in the second column.

Students who found an equation will also note that the equation is not of the same form as other equations, but they can't use this as a criterion until the class has established that only equations of this form represent proportional relationships. (This part of the discussion should come at the end of the next lesson, after students have analysed lots of different equations.)

7.3 Running Laps

15 minutes

The purpose of this activity is to understand that discrete values in a table can be used as evidence that a relationship is proportional and can be used to know for sure that a relationship is not proportional, but can't be used to know for sure whether a relationship is definitely proportional. This activity builds on previous ones involving constant speed, but analyses pace (minutes per lap) rather than speed (laps per minute). Explaining why the information given in the table is enough to conclude that Han didn't run at a constant pace but is not enough to know for sure whether Clare ran at a constant pace requires students to make a viable argument.

Instructional Routines

- Co-Craft Questions
- Think Pair Share

Launch

Keep students in the same groups of 2. Give 5 minutes of quiet work time, followed by 5 minutes of students discussing responses with a partner, followed by a whole-class discussion.

Conversing, Reading: Co-Craft Questions. Use this routine to help students interpret the language of proportional relationships, and to increase awareness of language used to talk about proportional relationships. Display the first sentence of this problem ("Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.") and the tables of Han and Clare's run, and ask students to write down possible mathematical questions that could be asked about the situation. If needed, provide students with the question starter "Why is. . .?" Invite students to share their questions with the class before revealing the task questions. Listen for and amplify any questions involving proportional relationships or mathematical language, such as "constant pace," "not constant," "proportional," or "constant of proportionality." This will help students produce the language of mathematical questions and start noticing whether or not there is a proportional relationship in this task prior to starting the task.
Design Principle(s): Maximise meta-awareness; Support sense-making

Anticipated Misconceptions

Students are likely to answer that Clare is running at a constant pace because the minutes per lap shown in the table are the same for each lap. Because we only have four data points in a table, spaced at 5-minute intervals, Clare could still be speeding up and slowing down between the recorded times. However, given the data, it is reasonable to assume Clare is running at a constant pace for the purpose of estimating times or distances.

Student Task Statement

Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.

Han's run:

distance (laps)	time (minutes)	minutes per lap
2	4	
4	9	
6	15	
8	23	

Clare's run:

distance (laps)	time (minutes)	minutes per lap
2	5	
4	10	
6	15	
8	20	

1. Is Han running at a constant pace? Is Clare? How do you know?
2. Write an equation for the relationship between distance and time for anyone who is running at a constant pace.

Student Response

Here are the tables that have been completed correctly:

Han's run:

distance (laps)	time (minutes)	minutes per lap
2	4	2
4	9	2.25
6	15	2.5
8	23	2.875

Clare's run:

distance (laps)	time (minutes)	minutes per lap
2	5	2.5
4	10	2.5
6	15	2.5
8	20	2.5

1. Han is not running at a constant pace because the numbers in the third column are not the same; however, Clare may be. At the times recorded in the table, the minutes per lap are the same, but that does not guarantee that Clare's pace is constant between the recorded times. For example, she might stand still for half a minute, then complete a lap in 2 minutes.

2. The equation $t = 2.5d$ (where t represents time in minutes and d represents distance in laps) yields the entries in the table for Clare's times; this would be the equation if we knew that she were running at a constant pace.

Activity Synthesis

Invite students to explain why they think each person is or is not running at a constant pace. Point out to students that although the data points in the table for Clare are pairs in a proportional relationship, these four pairs of values do not guarantee that Clare ran at a constant pace. She might have, but we don't know if she was running at a constant pace between the times that the coach recorded.

Ask the following questions:

- "Can you represent either relationship with an equation?" (The answer for Han is "no" and the answer for Clare is "yes, if she really ran at a constant pace between the points in time when the times were recorded." Write the equation for Clare together: $t = 2.5d$.)

- "Are the pairs of values in the table for Clare's run still values from a proportional relationship if we calculate laps per minute instead of minutes per lap? How does that change the equation?" (Yes, $d = 0.4t$.)

Lesson Synthesis

In this lesson, we learned some ways to tell whether a table could represent a proportional relationship. Revisit one or more of the activities in the lesson, highlighting the following points:

- If the quotient is the same for each row in the table, the table *could* represent a proportional relationship.
- It can be helpful to compute and write down this quotient for each row.
- The quotient is the constant of proportionality for the relationship (if the relationship is proportional).
- If all the quotients are not the same, the table definitely does not represent a proportional relationship.
- The relationship between the two quantities in a proportional relationship can be expressed using an equation of the form $y = kx$.

7.4 Apples and Pizza

Cool Down: 5 minutes

Student Task Statement

1. Based on the information in the table, is the cost of the apples proportional to the weight of apples?

pounds of apples	cost of apples
2	£3.76
3	£5.64
4	£7.52
5	£9.40

2. Based on the information in the table, is the cost of the pizza proportional to the number of toppings?

number of toppings	cost of pizza
2	£11.99
3	£13.49
4	£14.99
5	£16.49

3. Write an equation for the proportional relationship.

Student Response

1. Possibly yes, the first table represents a proportional relationship because cost per pound of apples is the same in each row, £1.88 per pound.

2. Definitely no, the second table does not represent a proportional relationship because cost per topping is not the same in each row. (An equation is $C = 1.50T + 8.99$ but students do not need to provide an equation.)

3. An equation relating the cost c to pounds of apples p is $c = 1.88p$.

Student Lesson Summary

Here are the prices for some smoothies at two different smoothie shops:

Smoothie Shop A

smoothie size (oz)	price (£)	pounds per ounce
8	6	0.75
12	9	0.75
16	12	0.75
s	$0.75s$	0.75

Smoothie Shop B

smoothie size (oz)	price (£)	pounds per ounce
8	6	0.75
12	8	0.67
16	10	0.625
s	???	???

For Smoothie Shop A, smoothies cost £0.75 per ounce (oz) no matter which size we buy. There could be a proportional relationship between smoothie size and the price of the smoothie. An equation representing this relationship is $p = 0.75s$ where s represents size in ounces and p represents price in pounds. (The relationship could still not be proportional, if there were a different size on the menu that did not have the same price per ounce.)

For Smoothie Shop B, the cost per ounce is different for each size. Here the relationship between smoothie size and price is definitely *not* proportional.

In general, two quantities in a proportional relationship will always have the same quotient. When we see some values for two related quantities in a table and we get the same quotient when we divide them, that means they might be in a proportional relationship—but if we can't see all of the possible pairs, we can't be completely sure. However, if we know the relationship can be represented by an equation is of the form $y = kx$, then we are sure it is proportional.

Lesson 7 Practice Problems

1. **Problem 1 Statement**

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would the constant of proportionality be?

a. How loud a sound is depending on how far away you are.

distance to listener (ft)	Sound level (dB)
5	85
10	79
20	73
40	67

b. The cost of fountain drinks at Hot Dog Hut.

volume (fluid ounces)	cost (£)
16	£1.49
20	£1.59
30	£1.89

Solution

a. Not proportional since the ratio of distance to listener to sound level is not always the same.

b. Not proportional since the ratio of volume to cost is not always the same.

2. **Problem 2 Statement**

A taxi service charges £1.00 for the first $\frac{1}{10}$ mile then £0.10 for each additional $\frac{1}{10}$ mile after that.

Fill in the table with the missing information then determine if this relationship between distance travelled and price of the trip is a proportional relationship.

distance travelled (mi)	price (pounds)
$\frac{9}{10}$	
2	
$3\frac{1}{10}$	
10	

Solution

distance travelled (mi)	price (pounds)
$\frac{9}{10}$	1.80
2	2.90
$3\frac{1}{10}$	4.00
10	10.90

This is not a proportional relationship since the ratio of price to distance travelled is not always the same.

3. **Problem 3 Statement**

A rabbit and turtle are in a race. Is the relationship between distance travelled and time proportional for either one? If so, write an equation that represents the relationship.

Turtle's run:

distance (metres)	time (minutes)
108	2
405	7.5
540	10
1 768.5	32.75

Rabbit's run:

distance (metres)	time (minutes)
800	1
900	5

1 107.5	20
1 524	32.5

Solution

The distance might be proportional to the time for the turtle. The equation would be $d = 54 \times t$, where d represents the distance travelled in metres and t is the time in minutes.

4. **Problem 4 Statement**

For each table, answer: What is the constant of proportionality?

a	b
2	14
5	35
9	63
$\frac{1}{3}$	$\frac{7}{3}$

a	b
3	360
5	600
8	960
12	1 440

a	b
75	3
200	8
1 525	61
10	0.4

a	b
4	10
6	15
22	55
3	$7\frac{1}{2}$

Solution

a. 7

b. 120

c. $\frac{1}{25}$ or equivalent

d. $2\frac{1}{2}$ or equivalent

5. **Problem 5 Statement**

Kiran and Mai are standing at one corner of a rectangular field of grass looking at the diagonally opposite corner. Kiran says that if the field were twice as long and twice as wide, then it would be twice the distance to the far corner. Mai says that it would be more than twice as far, since the diagonal is even longer than the side lengths. Do you agree with either of them?

Solution

Kiran is correct. If we scale the length and width of a rectangle by a factor of 2, then the diagonal will also scale by a factor of 2.