Lesson 5: Using negative numbers to make sense of contexts

Goals

- Interpret a table of signed numbers that represent how a quantity changed.

- Recognise that signed numbers can be useful to represent changes in a quantity in opposite directions, e.g., money received, and money paid, stock bought and stock sold, etc.

Learning Targets

- I can explain and use negative numbers in situations involving money.

- I can interpret and use negative numbers in different contexts.

Lesson Narrative

In this lesson, students are introduced to conventions for using signed numbers to represent money spent and received, as well as stock gained and lost. While money contexts can be represented without signed numbers, there are many situations that are more efficiently modelled by signed numbers. For example, if a person has £50 in the bank and writes a £20 cheque, we can represent the balance as $50 - 20$. If they had written an £80 cheque, we can still write the balance as $50 - 80$, as long as we have adopted the convention that negative numbers represent what the person owes the bank (and assuming the bank allows overdrafts). Since students do not operate on signed numbers yet, this lesson is simply an introduction to the convention of using signed numbers to represent a change in money or a change in stock, an important convention in modelling financial situations with mathematics. In a later lesson, students will be introduced to the idea of an account balance. Later in KS3, students will study addition and subtraction of signed numbers and apply those concepts in accounting situations.

Addressing

- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, height above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Instructional Routines

- Collect and Display

- Discussion Supports

- Notice and Wonder

- Think Pair Share

Student Learning Goals

Let's make sense of negative amounts of money.

5.1 Notice and Wonder: It Comes and Goes

Warm Up: 5 minutes

The purpose of this warm-up is to elicit the idea that we can represent money we get with positive numbers and money we spend with negative numbers, which will be useful when students make sense of data about money and stock in later activities. While students may notice and wonder many things about this table, interpreting the meaning of positive and negative numbers in this context is most important.

Instructional Routines
- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

Activity	amount
do my chores	30.00
babysit my cousin	45.00
buy my lunch	-10.80
get my allowance	15.00
buy a shirt	-18.69
pet my dog	0.00

What do you notice? What do you wonder?

Student Response

Things students may notice:

- When I buy things, the numbers are negative numbers.

- When I do things to get money, the numbers are positive.

- When no money is involved, the value is zero.

Things students may wonder:

- How much I have left.

- Why there are negative numbers.

- Why there is a 0.

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the table. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If representing money that we receive with positive numbers and money that we spend with negative numbers does not come up during the conversation, ask students to discuss this idea.

5.2 The Concession Stand

15 minutes

The purpose of this activity is to interpret signed numbers in a situation involving money and stock. Students reason abstractly and quantitatively when they think about change in stock and money using positive and negative numbers.

Instructional Routines
- Discussion Supports

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 8 minutes of quiet work time, 3 minutes for partner discussion, followed by whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity. *Supports accessibility for: Memory; Conceptual processing*

Student Task Statement

The manager of the concession stand keeps records of all of the supplies she buys and all of the items she sells. The table shows some of her records for Tuesday.

item	quantity	value in pounds
doughnuts	-58	37.70
straws	3 000	-10.35
hot dogs	-39	48.75
pizza	13	-116.87

apples	-40	14.00
french fries	-88	132.00

1. Which items did she sell? Explain your reasoning.

2. How can we interpret -58 in this situation?

3. How can we interpret -10.35 in this situation?

4. On which item did she spend the most amount of money? Explain your reasoning.

Student Response
1. She sold doughnuts, hot dogs, apples, and french fries. Sample explanation: Those items are associated with negative quantities, which are less than 0.

2. 58 doughnuts went out, or she sold 58 doughnuts.

3. -10.35 means that she spent £10.35 on straws.

4. She spent the most money on pizza. Sample explanation: The pound amount for pizza is negative or less than 0, which suggests money being paid out. The digits represent a number in the hundreds, which is more than the other payment for straws.

Activity Synthesis

Ask students to compare their responses with their partner and work to reach agreement. Monitor student discussions and select students to share their partner's reasoning in whole-class discussion. The key takeaway in the discussion is that positive and negative numbers are useful for describing change. If an amount goes up, then the change is positive. If an amount goes down, that means the change is negative.

Speaking, Listening: Discussion Supports. To support students to interpret this table and to produce statements about the values in the table, provide sentence frames such as "In this situation, -58 means ___" and "-58 is related to 37.70 because ___." This will help students make the connection that a negative number in the "quantity" column means that the items have been sold which generates a positive number in the "value in pounds" column.
Design Principle(s): Support sense-making

5.3 Drinks for Sale

15 minutes

Students interpret positive and negative numbers in the context of a changing stock.

Instructional Routines
- Collect and Display

- Think Pair Share

Launch

Keep students in the same groups of 2. Give students 8 minutes of quiet work time and 3 minutes of partner discussion. Follow with whole-class discussion.

Student Task Statement

A vending machine in an office building sells bottled drinks. The machine keeps track of all changes in the number of bottles from sales and from machine refills and maintenance. This record shows the changes for every 5-minute period over one hour.

1. What might a positive number mean in this context? What about a negative number?

2. What would a "0" in the second column mean in this context?

3. Which numbers—positive or negative—result in fewer bottles in the machine?

4. At what time was there the greatest change to the number of bottles in the machine? How did that change affect the number of remaining bottles in the machine?

5. At which time period, 8:05–8:09 or 8:25–8:29, was there a greater change to the number of bottles in the machine? Explain your reasoning.

6. The machine must be emptied to be serviced. If there are 40 bottles in the machine when it is to be serviced, what number will go in the second column in the table?

time	number of bottles
8:00–8:04	-1
8:05–8:09	+12
8:10–8:14	-4
8:15–8:19	-1
8:20–8:24	-5
8:25–8:29	-12
8:30–8:34	-2
8:35–8:39	0
8:40–8:44	0
8:45–8:49	-6
8:50–8:54	+24
8:55–8:59	0
service	

Student Response

1. A positive number means bottles being added to the machine. A negative number means bottles being dispensed or otherwise being removed from the machine.

2. A "0" would mean no activity, i.e., the machine is not being stocked with new bottles and not dispensing any bottles (or the amount stocked is equal to the amount dispensed, but this idea is not expected at this time).

3. Negative numbers lead to fewer bottles in the machine, because they mean bottles are being removed.

4. The greatest change happened at 8:50–8:54. The number of bottles in the machine increased by 24.

5. There was the same amount of change to the number of bottles in the machine. In both cases, the number of bottles changed by 12, but at 8:05–8:09 a.m. it increased by 12 and at 8:25–8:29 it decreased by 12.

6. -40

Are You Ready for More?

Priya, Mai, and Lin went to a cafe on a weekend. Their shared bill came to £25. Each student gave the server a £10 note. The server took this £30 and brought back five £1 coins in change. Each student took £1 back, leaving the rest, £2, as a tip for the server.

As she walked away from the cafe, Lin thought, "Wait—this doesn't make sense. Since I put in £10 and got £1 back, I wound up paying £9. So did Mai and Priya. Together, we paid £27. Then we left a £2 tip. That makes £29 total. And yet we originally gave the waiter £30. Where did the extra pound go?"

Think about the situation and about Lin's question. Do you agree that the numbers didn't add up properly? Explain your reasoning.

Student Response

Disagree. Sample explanations:

- It doesn't matter that the students originally paid £30. Between the bill, which is £25, and the £2 tip, the students paid £27 in total.

- Lin mistakenly thought that the £2 was in addition to the £27 the three of them paid, but the £27 actually already included the £2 tip, since the original bill was £25.

- Since the bill was £25 for three people, each person's share was $£\frac{25}{3}$ pounds. Together they left a tip of £2, which means each person pitched in $£\frac{2}{3}$. Each person then paid a total of $\frac{25}{3} + \frac{2}{3}$, which is $\frac{27}{3}$ or £9. This matches the fact that they each gave a £10 note and took £1 for the change.

Activity Synthesis

The goal of the discussion is to allow students to share their thoughts on the meaning of positive and negative numbers in context. Ask students to share their responses with their partner and work to reach agreement. Monitor groups' discussions and select students to share their partner's reasoning about what the numbers in the table mean. Ask students to summarise the story the numbers in the table tell. Tell students that tables like this (but perhaps more complicated) are used all the time to tell stories about what is happening in the world.

Speaking, Listening: Collect and Display. To highlight words students use while speaking about positive and negative numbers, record on a visual display common or important phrases you hear students say related to the values in the table (e.g., "put into the machine", "added to the machine", "taken out", "removed"). Make sure to include informal descriptions as well as more precise language. This will help students connect the everyday language that is used in mathematical situation to the more precise academic language. *Design Principle(s): Support sense-making*

Lesson Synthesis

In this lesson, students interpreted situations where positive and negative numbers were used to show changes in stock or changes in money. Here are some questions to consider while closing the lesson:

- "What did the positive and negative numbers mean in this lesson?" (Positive numbers represented a gain, like receiving money or adding bottles to the machine. Negative numbers represented a loss, like spending money by buying something or removing bottles from the machine.)

- "We saw that we could use positive and negative numbers to represent gaining and losing money. What other situations can you think of where you gain or lose an amount that you could use negative numbers to talk about? What would positive and negative changes mean in those situations?" (Responses vary. Some examples include weight, speed, number of subscribers, field position in football.)

5.4 Bakery Owner

Cool Down: 5 minutes

Student Task Statement

The table shows records of money-related activities of a bakery owner over a period of a week.

date	items	amount in pounds
May 1	rent	-850.00
May 2	order (birthday cake and cookies)	106.75

May 3	utilities (electricity, gas, phone)	-294.50
May 5	order (wedding cake and desserts)	240.55
May 5	baking supplies	-147.95
May 6	order (anniversary cake)	158.20
May 7	order (breads and desserts for a conference)	482.30
May 7	bakery sales	415.65

1. For which items did she receive money?

2. What does the number -147.95 mean in this context?

3. Did the bakery owner receive more or spend more money on May 5? Explain how you know.

Student Response

1. She received money on the orders for different events (birthday, weddings, etc.) and sales of bakery items.

2. The number -147.95 means she spent £147.95 on baking supplies.

3. She received more money than she spent. She received over £240 and spent under £150.

Student Lesson Summary

Sometimes we represent changes in a quantity with positive and negative numbers. If the quantity increases, the change is positive. If it decreases, the change is negative.

- Suppose 5 gallons of water is put in a washing machine. We can represent the change in the number of gallons as +5. If 3 gallons is emptied from the machine, we can represent the change as -3.

It is especially common to represent money we receive with positive numbers and money we spend with negative numbers.

- Suppose Clare gets £30.00 for her birthday and spends £18.00 buying lunch for herself and a friend. To her, the value of the gift can be represented as +30.00 and the value of the lunch as -18.00.

Whether a number is considered positive or negative depends on a person's perspective. If Clare's grandmother gives her £20 for her birthday, Clare might see this as +20, because to her, the amount of money she has increased. But her grandmother might see it as -20, because to her, the amount of money she has decreased.

In general, when using positive and negative numbers to represent changes, we have to be very clear about what it means when the change is positive and what it means when the change is negative.

Lesson 5 Practice Problems

1. **Problem 1 Statement**

   Write a positive or negative number to represent each change in the high temperature.

   a. Tuesday's high temperature was 4 degrees less than Monday's high temperature.

   b. Wednesday's high temperature was 3.5 degrees less than Tuesday's high temperature.

   c. Thursday's high temperature was 6.5 degrees more than Wednesday's high temperature.

   d. Friday's high temperature was 2 degrees less than Thursday's high temperature.

   Solution

   a. -4

   b. -3.5

   c. +6.5 or 6.5

   d. -2

2. **Problem 2 Statement**

   Decide which of the following quantities can be represented by a positive number and which can be represented by a negative number. Give an example of a quantity with the opposite sign in the same situation.

   a. Tyler's puppy gained 5 pounds.

   b. The aquarium leaked 2 gallons of water.

   c. Andre received a gift of £10.

   d. Kiran gave a gift of £10.

   e. A climber descended 550 feet.

   Solution

   Answers vary. Sample responses:

   a. Positive. Tyler's puppy lost 5 pounds.

   b. Negative. 2 gallons of water was added to the aquarium.

    c.   Positive. Andre gave a gift of £10.

    d.   Negative. Kiran received a gift of £10.

    e.   Negative. A climber ascended 550 feet.

3.   **Problem 3 Statement**

Make up a situation where a quantity is changing.

    a.   Explain what it means to have a negative change.

    b.   Explain what it means to have a positive change.

    c.   Give an example of each.

Solution

Answers vary. Sample response: They were selling sweets at the concession stand.

    a.   When they sell sweets, the change is negative.

    b.   When they get more sweets to sell, the change is positive.

    c.   For example, in one hour the number of packages of sweets changed by -5 because they sold 5, and in the next hour it changed by 20 because they got 20 more to sell.

4.   **Problem 4 Statement**

    a.   On the number line, label the points that are 4 units away from 0.
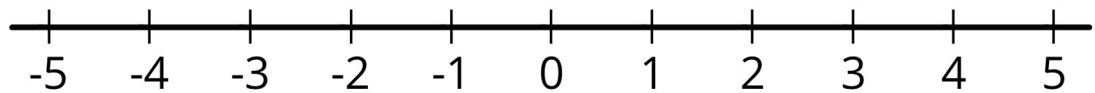
    b.   If you fold the number line so that a vertical crease goes through 0, the points you label would match up. Explain why this happens.

    c.   On the number line, label the points that are $\frac{5}{2}$ units from 0. What is the distance between these points?

Solution

    a.   On the number line, -4 and 4 should be labelled.

b. The two points match up because they are opposites; they are the same distance from 0.

c. 2.5 and -2.5 should be labelled. The distance between them is 5 units, because each one is 2.5 units away from 0.

5. **Problem 5 Statement**

Evaluate each expression.

– $2^3 \times 3$

– $\frac{4^2}{2}$

– 3^1

– $6^2 \div 4$

– $2^3 - 2$

– $10^2 + 5^2$

Solution

a. 24

b. 8

c. 3

d. 9

e. 6

f. 125