Lesson 3: Nonadjacent angles

Goals

- Comprehend the term “vertically opposite angles” (in spoken and written language) refers to a pair of angles created by two intersecting lines.
- Generalise (orally and in writing) that the vertically opposite angles created by two intersecting lines are equal angles.
- Use reasoning about angles to identify complementary or supplementary angles that are not adjacent.

Learning Targets

- I can determine if angles that are not adjacent are complementary or supplementary.
- I can explain what vertically opposite angles are in my own words.

Lesson Narrative

In this lesson, students see that angles do not need to be adjacent to be complementary or supplementary. Students are also introduced to and begin to use the term **vertically opposite angles** for describing the opposite angles formed when two lines cross. They examine multiple examples and see that the vertically opposite angles are equal. Students can relate this understanding to the fact that both angles in a pair of vertically opposite angles are supplementary to the same angle in between, but in KS3 students do not need to be able to give a formal geometric proof that vertically opposite angles must be equal.

Addressing

- Use properties of operations to generate equivalent expressions.
- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a shape.

Instructional Routines

- Collect and Display
- Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Student Learning Goals

Let's look at angles that are not right next to one another.

3.1 Finding Related Statements

Warm Up: 5 minutes

In the previous unit, students worked extensively with writing equations in equivalent forms, for example, rewriting $2x + 50 = 7$ as $2x = 7 - 50$. A new wrinkle here is that each equation has two variables. Equations with more than one variable will be studied extensively later in KS3, but here we are using the concrete context of geometry to help make sense of it.

The purpose of this warm-up is for students to use structure to reason about equivalent equations. In this unit, students will write equations to represent how angles are related to each other, and this warm-up helps prepare for that work.

All of the given statements *could* be true so students may be quick to say each of them *must* be true. Ask these students if there is a case when that particular statement would not be true for possible values for a and b. As students discuss their responses with a partner, monitor for students who correctly answered each question to share during the whole-class discussion.

Launch

Arrange students in groups of 2.

Ask students, "If we know for sure that $a + b = 180$, what are some possible values of a and b?" Give students 30 seconds of quiet think time, and then ask several students to share their responses. Some examples are $a = 90$ and $b = 90$, $a = 0$ and $b = 180$, and $a = 10$ and $b = 170$. Tell students that in this activity, we know for sure that $a + b = 180$, but we don't know the exact values of a and b.

Give students 2 minutes of quiet work time followed by 1 minute to discuss their responses with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may assume a and b both have a value of 90. Explain that this *may* be true, but that it is also possible that a and b are not equal to each other.

Student Task Statement

Given a and b are numbers, and $a + b = 180$, which statements also must be true?

$a = 180 - b$

$a - 180 = b$

$360 = 2a + 2b$

$a = 90$ and $b = 90$

Student Response

1. True. Students might reason inductively using several examples or draw a diagram showing the relationship between a, b, and 180.

2. Not always true ($a = 90$ and $b = 90$, for example), but could be true ($a = 180$ and $b = 0$).

3. True. The equation can be rewritten $2 \times 180 = 2(a + b)$. Since twice 180 and twice $a + b$ are equal, it must be true that 180 and $a + b$ are equal.

4. Not always true ($a = 100$ and $b = 80$ for example), but could be true.

Activity Synthesis

Select previously identified students to explain their reasoning for each statement. Poll the class if they agree or disagree after each student shares. If students disagree, allow students to discuss until they come to an agreement. Consider asking some of the following questions while students discuss:

- "Do you have an example that might support this statement being true (or untrue)?"
- "What evidence do you have to support that statement being true (or untrue)?"
- "What other values of a and b might work?"
- "What was done to the equation to make the statement true (or untrue)?"

If any of the answers the students decide upon are incorrect, give an example of when the statement would not be true.

3.2 Polygon Angles

10 minutes

In this activity, students see that angles do not need to be adjacent to each other in order to be considered complementary or supplementary. Students are given two different polygons and are asked to find complementary and supplementary angles, using any tools in their geometry toolkit. The most likely approaches are:

- measure each angle with a protractor and look for any that sum to 180 or 90 degrees.
- trace the lines making the angle with tracing paper and align its vertex and one side with another angle to see if the two angles, when adjacent, form a straight angle or a right angle.

As students work, monitor for students who use either approach listed or some other strategy. Also, encourage students to use precise vocabulary and language that they learned in previous activities and lessons.

Instructional Routine

- Discussion Supports

Launch

Remind students that in the previous lesson they learned the meaning of *complementary* and *supplementary* when describing angles. Invite students to share their definitions of the words and consider displaying the meanings for all to see through the remainder of the class. If students include in their definitions the idea that the angles need to be adjacent (for example, that they "make a straight line" or "make a right angle"), point out that while that was true for all the examples they have seen so far, that was *not* a part of the definition. Explain that angles do not need to be adjacent to one another to be complementary or supplementary. They just have to sum to 90 or 180 degrees.

Keep students in the same groups. Provide access to geometry toolkits. Give students 3–4 minutes of quiet work time, followed by partner and whole-class discussions.

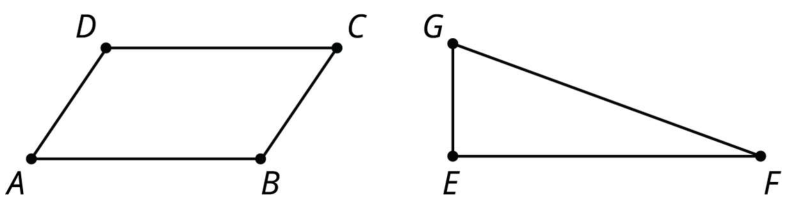
Action and Expression: Internalise Executive Functions. Provide students with a printed enlarged version of the shapes in the student task statement.
Supports accessibility for: Language; Organisation

Anticipated Misconceptions

Some students may struggle to use a protractor to measure angles when the rays are not drawn long enough to reach the edge of the protractor. Prompt them to extend the sides of the angle using a straightedge.

Student Task Statement

Use any useful tools in the geometry toolkit to identify any pairs of angles in these shapes that are complementary or supplementary.



Student Response

- In the quadrilateral, there are four pairs of supplementary angles:
  - Angle BAD is supplementary with angle ABC.
  - Angle BAD is also supplementary with angle ADC.

- Angle BCD is supplementary with angle ABC.
- Angle BCD is also supplementary with angle ADC.

- In the triangle, there is one pair of complementary angles: angle EFG is complementary with angle EGF.

Activity Synthesis

Select previously identified students to share their answers and reasoning. If possible, have a student demonstrate each method for finding pairs of angles: measuring with a protractor or using tracing paper. Ensure that correct use of a protractor to find the size of an angle is clearly and carefully demonstrated. This will help all students prepare for the next activity where everyone will be using a protractor.

Speaking: Discussion Supports. Use this routine to support whole-class discussion. After each student demonstrates a method for finding pairs of angles (measuring with a protractor or using tracing paper), invite students to turn to a partner to restate what they heard, using precise mathematical language. Select one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their method. This will provide more students with an opportunity to describe methods for finding pairs of angles using mathematical language.
Design Principle(s): Support sense-making

3.3 Vertically opposite angles

15 minutes

The purpose of this activity is for students to learn about **vertically opposite angles.** Each student draws two intersecting lines and measures the four resulting angles. Then, students examine multiple examples to come up with a conjecture for any relationships they noticed. Because of the focus of the previous activity, students will likely notice that there are adjacent supplementary angles in their drawings. Encourage them to look for any *other* patterns they can find.

Instructional Routines

- Collect and Display

Launch

Arrange students in groups of 2–4. Provide access to geometry toolkits.

Ask students to read the task statement quietly to themselves. Then, ask them to read it again and underline any words that they are uncertain about. Invite students to share any words they underlined and record them for all to see. Before students start working, explain the meaning of any word they identified, which may include:

- *intersecting*—Some students may think that "intersecting" means "perpendicular." In this activity, it is important for students to examine some examples of vertically opposite angles made by intersecting lines that are *not* perpendicular. Consider holding up two meter sticks to demonstrate several examples of two lines that intersect versus two lines that do not intersect. Clarify that intersecting lines do not have to be perpendicular.

- *conjecture*—Explain that a conjecture is a statement we *think* is true but aren't certain about. It is more than just a guess. A conjecture could be a guess that is based on some evidence.

Give students 2–3 minutes to draw and measure the shape. Remind them to draw arcs to label the sizes of their angles in degrees. Follow with small-group and whole-class discussions.

Representation: Develop Language and Symbols. Create and maintain charts to display definitions and examples of vertically opposite angles, intersecting lines, and conjectures. *Supports accessibility for: Conceptual processing; Memory Speaking: Collect and Display.* As students work, listen for and collect vocabulary, gestures, and phrases that students use to describe the relationships they notice between angles (e.g., equivalent, adjacent, right, straight, complementary and supplementary angles, etc.). Organise the responses onto a visual display. Throughout the remainder of the lesson, continue to update the display and remind students to use the display as a resource if needed. During the lesson synthesis, after the term "vertically opposite angles" is introduced, ask students to identify any language on the display that could be used to describe vertically opposite angles. *Design Principle(s): Optimise output (for explanation); Maximise meta-awareness*

Anticipated Misconceptions

Some students may label the sizes of the angles toward the end of the rays, where they read the number from the protractor. This is not precise enough, because two different angles share each ray. Remind students about drawing arcs to clarify which angle they measured.

Student Task Statement

Use a straightedge to draw two intersecting lines. Use a protractor to measure all four angles whose vertex is located at the intersection.

Compare your drawing and measurements to the people in your group. Make a conjecture about the relationships between the sizes of the angles at an intersection.

Student Response

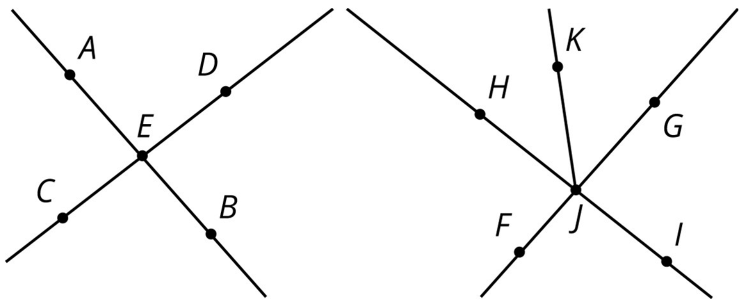
Answers vary. Possible conjecture: "The angles across from each other will always be equal."

Activity Synthesis

The goal of this discussion is for students to see that two intersecting lines form vertically opposite angles, and the angles across from each other are congruent. Select students to share their conjectures. If there are students who can use supplementary angles to explain why vertically opposite angles are equal, put them last in the sequence.

Define **vertically opposite angles** as a pair of angles, formed by two intersecting lines, that are opposite each other.

Display the image and ask students to identify four pairs of vertically opposite angles. In particular, students may have trouble seeing that angles FJI and HJG are vertically opposite angles.



Although students don't need to know a proof that vertically opposite angles are always the same, it may be helpful to show one way to understand why they are. In the image. . .

- Angles AED and AEC are supplementary, so the sum of their sizes in degrees is 180 degrees.
- Angles AEC and CEB are also supplementary, so the sum of their sizes in degrees is also 180 degrees.
- If we take angle AEC away from the straight angles, we see that angles AED and CEB must be the same size.

3.4 Row Game: Angles

Optional: 10 minutes

This activity gives students an opportunity to practise recognising complementary, supplementary, and vertically opposite angles and using what they know about those types of angles to find the size of unknown angles. Some students may feel comfortable writing equations to show their reasoning, but it is not important that all students use this strategy at this point, as it will be the focus of future lessons. Encourage students to continue using the new vocabulary.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

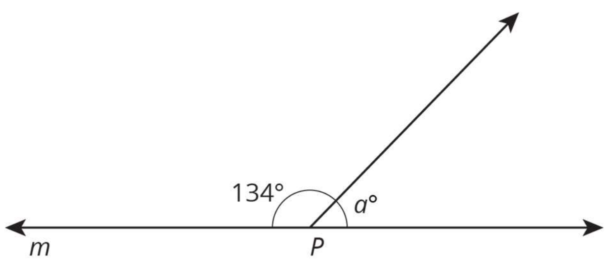
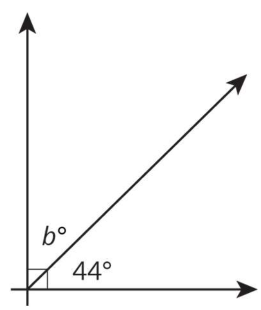
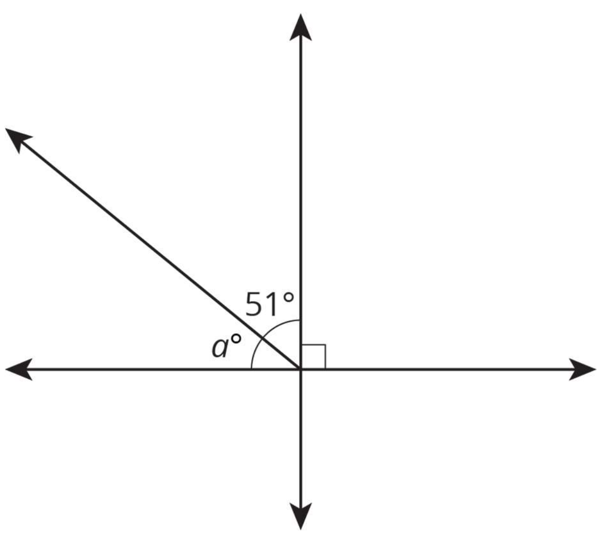
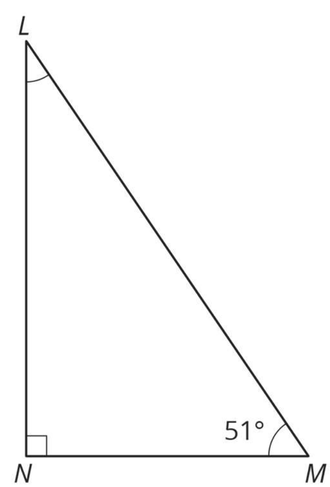
Arrange students in groups of 2. Make sure students know how to play a row game. Give students 5–6 minutes of partner work time followed by a whole-class discussion.

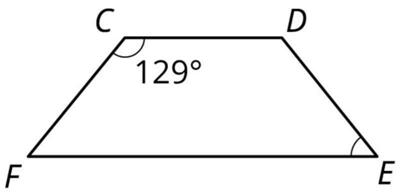
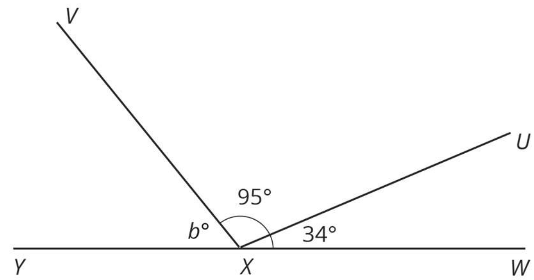
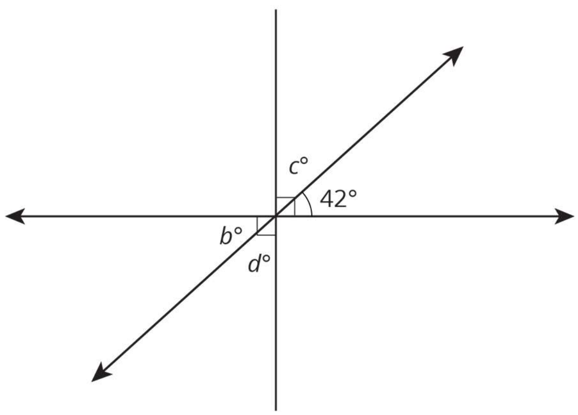
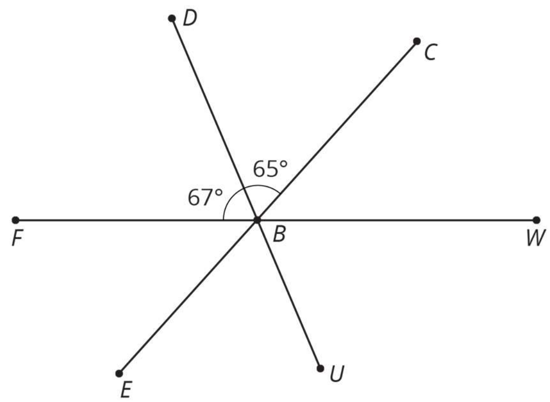
Anticipated Misconceptions

If students struggle to see relationships of angles in shapes, prompt students to look for complementary, supplementary, or vertically opposite angles.

Student Task Statement

Find the sizes of the angles in one column. Your partner will work on the other column. Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error and correct it.

Column A	Column B
P is on line m. Find the value of a. 	Find the value of b. 
Find the value of a. 	In right triangle LMN, angles L and M are complementary. Find the size of angle L. 

Angle C and angle E are supplementary. Find the size of angle E. 	X is on line WY. Find the value of b. 
Find the value of c. 	B is on line FW. Find angle CBW. 
Two angles are complementary. One angle measures 37 degrees. Find the other angle.	Two angles are supplementary. One angle measures 127 degrees. Find the other angle.

Student Response

1. 46 degrees
2. 39 degrees
3. 51 degrees
4. 48 degrees
5. 53 degrees

Activity Synthesis

Ask students, “Were there any rows that you and your partner did not get the same answer?” Invite students to share how they came to an agreement on the final answer for the problems in those rows.

Consider asking some of the following questions:

- "Did you and your partner use the same strategy for each row?"
- "What was the same and different about both of your strategies?"
- "Did you learn a new strategy from your partner?"
- "Did you try a new strategy while working on these questions?"

Speaking: Discussion Supports. Use this routine to support students as they describe their strategies for calculating the unknown angle to their partner. Provide sentence frames such as, "I noticed that _____, so I . . ." or "First, I _____ because . . . ." Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language. Listen for students who refer to the relationships between angles and those who identify complementary, supplementary, or vertically opposite angles.
Design Principle(s): Optimise output (for explanation)

Lesson Synthesis

- Do supplementary or complementary angles need to be next to one another? (No.) Think of examples where they are not.
- What are vertically opposite angles? (A pair of angles across from one another where two lines cross.)
- What is true about the sizes of vertically opposite angles? (The sizes are always the same.)

Display diagrams and definitions of new vocabulary somewhere in the classroom so that students can refer back to them during subsequent lessons. "Vertically opposite angles" is new vocabulary; you might consider also adding "intersecting lines" and "conjecture." As the unit progresses, new terms can be added.

3.5 Finding Angle Pairs

Cool Down: 5 minutes

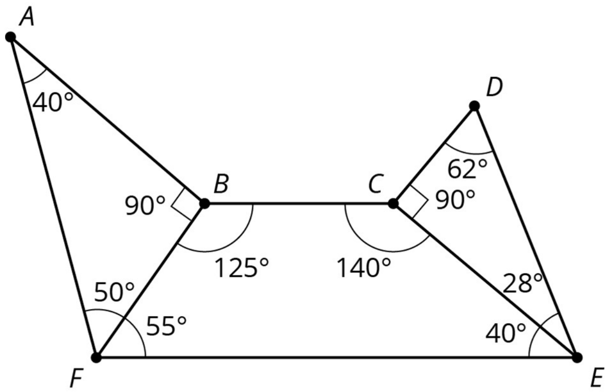
Launch

Make sure students realise that when a question says "two pairs" it is referring to a total of 4 angles.

Student Task Statement

1. Name *two pairs* of complementary angles in the diagram.
2. Name *two pairs* of supplementary angles in the diagram.

3. Draw another angle to make a pair of vertically opposite angles. Label your new angle with its size in degrees.



Student Response

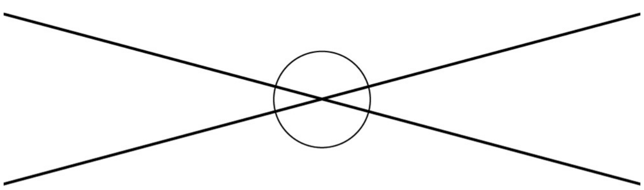
1. Any two of: Angles BAF and AFB, angles CDE and DEC, or angles CEF and AFB.

2. Any two of: Angles ABF and DCE, angles FBC and BFE, angles BCE and CEF, or angles BCE and BAF.

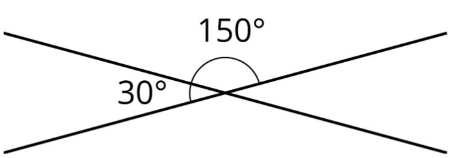
3. Answers vary. Extending any two intersecting segments will create a vertically opposite angle that is the same as the angle before extending.

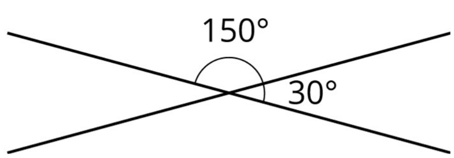
Student Lesson Summary

When two lines cross, they form two pairs of **vertically opposite angles**. Vertically opposite angles are across the intersection point from each other.

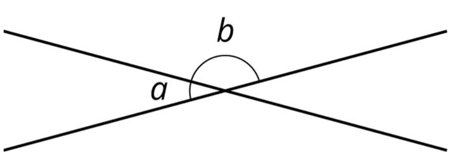


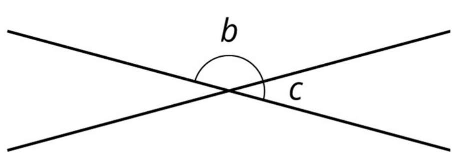
Vertically opposite angles are always equal. We can see this because they are always supplementary with the same angle. For example:





This is always true!





$a + b = 180$ so $a = 180 - b$.

$c + b = 180$ so $c = 180 - b$.

That means $a = c$.

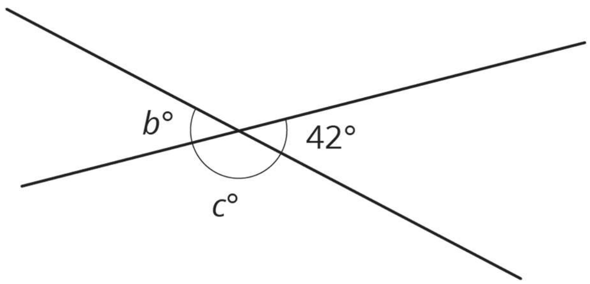
Glossary

- vertically opposite angles

Lesson 3 Practice Problems

1. **Problem 1 Statement**

   Two lines intersect. Find the value of b and c.

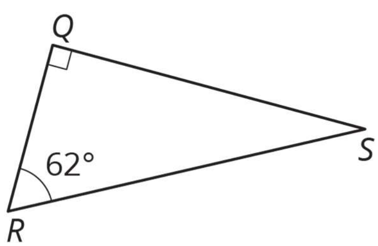


   Solution

   $c = 138, b = 42$

2. **Problem 2 Statement**

   In this shape, angles R and S are complementary. Find the size of angle S.



   Solution

   28°

3. **Problem 3 Statement**

   If two angles are both vertically opposite and supplementary, can we determine the angles? Is it possible to be both vertically opposite and complementary? If so, can you determine the angles? Explain how you know.

   Solution

   Yes, they are both possible. Vertically opposite and supplementary angles must be 90° each, because the two angles must be the same and sum to 180°. Vertically opposite and complementary angles must be 45°, because the two angles must be the same and sum to 90°.

4. **Problem 4 Statement**

Match each expression in the first list with an equivalent expression from the second list.

A. $5(x+1)-2x+11$

B. $2x+2+x+5$

C. $\frac{-3}{8}x-9+\frac{5}{8}x+1$

D. $2.06x-5.53+4.98-9.02$

E. $99x+44$

1. $\frac{1}{4}x-8$

2. $\frac{1}{2}(6x+14)$

3. $11(9x+4)$

4. $3x+16$

5. $2.06x+(-5.53)+4.98+(-9.02)$

Solution

- A: 4
- B: 2
- C: 1
- D: 5
- E: 3

5. **Problem 5 Statement**

Factorise each expression.

a. $15a-13a$

b. $-6x-18y$

c. $36abc+54ad$

Solution

a. $a(15 - 13)$

b. -6$(1x + 3y)$ (or 6(-$x - 3y$))

c. $9a(4bc + 6d)$

6. **Problem 6 Statement**

The directors of a dance show expect many students to participate but don't yet know how many students will come. The directors need 7 students to work on the technical crew. The rest of the students work on dance routines in groups of 9. For the show to work, they need at least 6 full groups working on dance routines.

a. Write and solve an inequality to represent this situation, and represent the solution on a number line.

b. Write a sentence to the directors about the number of students they need.

Solution

a. $\frac{x-7}{9} \geq 6$, $x \geq 61$. The number line should have a closed circle at $x = 61$. Some students may start at $x = 61$ and draw a line with an arrow extending to the right; others may draw dots on integers to the right of $x = 61$.

b. The directors need at least 61 students to show up. (Possibly, they may only be happy if they get 61, 70, 79, etc. students so they have even groups of nine.)

7. **Problem 7 Statement**

A small dog gets fed $\frac{3}{4}$ cup of dog food twice a day. Using d for the number of days and f for the amount of food in cups, write an equation relating the variables. Use the equation to find how many days a large bag of dog food will last if it contains 210 cups of food.

Solution

$f = 1.5 \times d$ or equivalent. The bag will last 140 days since $210 \div 1.5 = 140$.