Lesson 6: Introducing double number line diagrams

Goals

- Compare and contrast (orally and in writing) discrete diagrams and double number line diagrams representing the same situation.
- Explain (orally) how to use a double number line diagram to find equivalent ratios.
- Label and interpret a double number line diagram that represents a familiar context.

Learning Targets

- I can label a double number line diagram to represent batches of a recipe or colour mixture.
- When I have a double number line that represents a situation, I can explain what it means.

Lesson Narrative

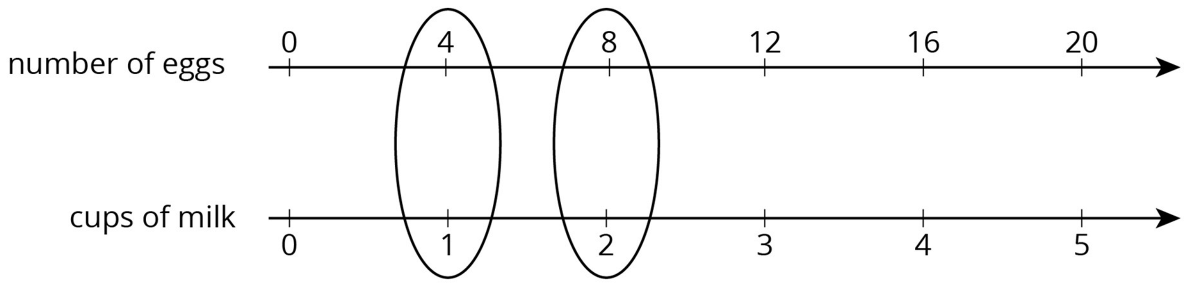
This lesson introduces the **double number line diagram**, a useful, efficient, and sophisticated tool for reasoning about equivalent ratios.

The lines in a double number line diagram are similar to the number lines students have seen in earlier grades in that:

- Numbers correspond to distances on the line (so that the distance between, say, 0 and 12 is three times the distance between 0 and 4);
- We can choose what scale to use (i.e., whether each interval represents 1 unit, 2 units, 5 units, etc.);
- The lines can be extended as needed.

In a double number line diagram we use two parallel number lines—one line for each quantity in the ratio—and choose a scale on each line so equivalent ratios line up vertically.

For example, if the ratio of number of eggs to cups of milk in a recipe is 4 to 1, we can draw a number line for the number of eggs and one for the cups of milk. On the number lines, the quantity of 4 for the number of eggs and the 1 for cups of milk would line up vertically, as would 8 eggs and 2 cups of milk, and so on.



Because they represent quantities with length on a number line rather than with counts of objects, double number lines are both more abstract and more general than discrete diagrams. Later in this unit, students will learn an even more abstract representation of equivalent ratios—the table of values. Connecting the concrete to the abstract helps students connect quantitative reasoning to abstract reasoning. Though some activities are designed to hone students' facility with particular representations, students should continue to have autonomy in choosing representations to solve problems, as long as they can explain their meaning.

Building On

- Number and Operations in Base Ten

Addressing

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Discussion Supports
- Number Talk
- Think Pair Share

Required Materials

Rulers

Student Learning Goals

Let's use number lines to represent equivalent ratios.

6.1 Number Talk: Adjusting Another Factor

Warm Up: 10 minutes

This Number Talk encourages students to think about the numbers in calculation problems and rely on what they know about structure, patterns, whole-number multiplication, and properties of operations to mentally solve a problem.

While many strategies may emerge, the focus of this string of problems is for students to see how adjusting a factor impacts the product, and how this insight can be used to reason about other problems. Four problems are given, however, given limited time, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem. Each problem was chosen to elicit slightly different reasoning, so as students explain their strategies, ask how the factors impacted their product.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find the value of each product mentally.

$(4.5) \times 4$

$(4.5) \times 8$

$\dfrac{1}{10} \times 65$

$\dfrac{2}{10} \times 65$

Student Response

- $(4.5) \times 4 = 18$. Possible strategies: $(4 \times 4) + [(0.5) \times 4]$, double and halve 9×2.
- $(4.5) \times 8 = 36$. Possible strategies: double the product from the first question because a factor doubled; $(8 \times 4) + [(0.5) \times 8]$.
- $\frac{1}{10} \times 65 = 6.5$. Possible strategies: $65 \div 10$, $65 \times (0.1)$.

- $\frac{2}{10} \times 65 = 13$. Possible strategies: double the product from the previous question because a factor doubled; $65 \div 10 \times 2$.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. Ask students if or how the factors in the problem impacted the strategy choice. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

6.2 Drink Mix on a Double Number Line

15 minutes

In this activity, a double number line, a new representation, is presented and interpreted alongside the more familiar discrete diagrams and in the familiar context of recipes.

Students learn that, just like discrete diagrams, double number lines represent equivalent ratios. They see that alignment between the numbers of the two lines matters; that it is through the alignment that the association of two quantities are shown. Students notice pairs of numbers that "line up" vertically are equivalent ratios.

Because double number lines are quicker to draw and can be extended easily to show many more equivalent ratios, they are more efficient than discrete diagrams, especially for dealing with larger quantities.

As students work, monitor for those who contrast the two representations in terms of using graphic symbols versus numbers, and those who think about equivalent ratios in terms of the alignment of numbers in the double number line diagram.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Ask students to recall the mixture of powdered drink mix and water from a previous lesson. Ask: "How much drink mix and water was in one batch?" (4 teaspoons of drink mix and 1 cup of water.) "What would you need to mix a double batch?" (8 teaspoons of drink mix and 2 cups of water.) Explain that they are going to show batches of a mixture using a **double number line diagram**.

Give students 5 minutes of quiet think time to make sense of the new representation and answer the questions, followed by time to share their response with a partner and a whole-class discussion afterwards.

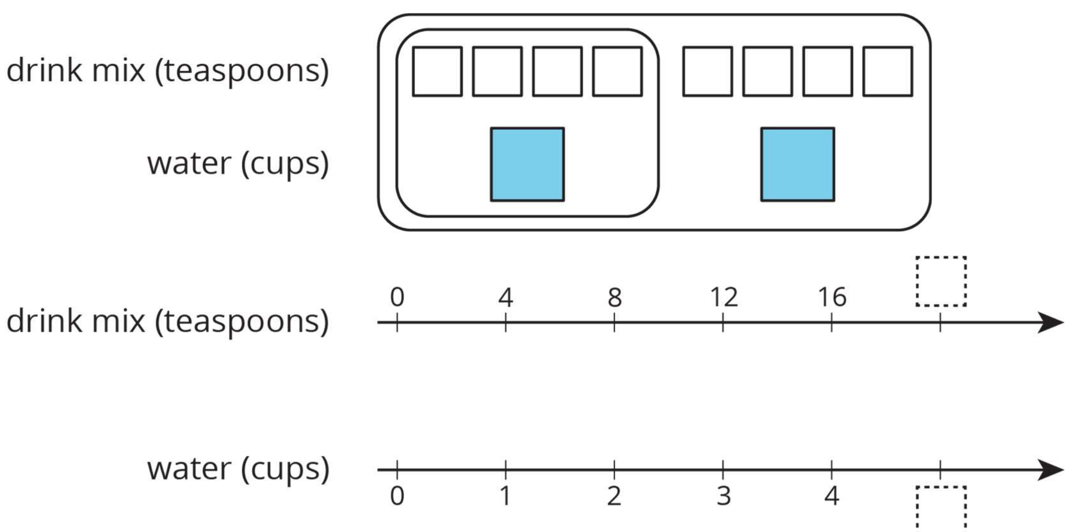
Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "The representations are the same/different because ..."
Supports accessibility for: Language; Social-emotional skills Reading, Writing: Discussion Supports. If necessary, remind students of the meaning of these terms; recipe, batch, mixture, and diagram. This will support student understanding of the context so that they can make sense of the double number line.
Design Principle(s): Support sense-making; Optimise output (for explanation)

Anticipated Misconceptions

While the double number line diagram is given here, some students may not feel comfortable with seeing the same numbers (the 4's) in different positions. Remind students that each number line represents a different quantity, and that the two 4's have different meanings.

Student Task Statement

The other day, we made drink mixtures by mixing 4 teaspoons of powdered drink mix for every cup of water. Here are two ways to represent multiple batches of this recipe:



1. How can we tell that 4 : 1 and 12 : 3 are equivalent ratios?

2. How are these representations the same? How are these representations different?

3. How many teaspoons of drink mix should be used with 3 cups of water?

4. How many cups of water should be used with 16 teaspoons of drink mix?

5. What numbers should go in the empty boxes on the **double number line diagram**? What do these numbers mean?

Student Response

1. 12 and 3 are 3 times 4 and 1, respectively. On the number line diagram, you can see that 4 and 1 line up vertically, as do 12 and 3.

2. Same: Each representation shows the amount of drink mix and water for one batch and two batches. They each show teaspoons of drink mix along the top and cups of water along the bottom.
   Different: The first diagram uses squares to represent each teaspoon of drink mix and cup of water, but the number line diagram has these amounts written with numbers. The first diagram shows only two batches and the number line diagram shows 0, 1, 2, 3, and 4 batches (with space for 5 batches).

3. 12 teaspoons

4. 4 cups

5. The numbers 20 and 5 should go in the missing places. These numbers mean that the result of mixing 20 teaspoons of drink mix with 5 cups of water would taste the same as the other mixtures, or that these amounts would make 5 batches of the recipe.

Are You Ready for More?

Recall that a *perfect square* is a number of objects that can be arranged into a square. For example, 9 is a perfect square because 9 objects can be arranged into 3 rows of 3. 16 is also a perfect square, because 16 objects can be arranged into 4 rows of 4. In contrast, 12 is not a perfect square because you can’t arrange 12 objects into a square.

1. How many whole numbers starting with 1 and ending with 100 are perfect squares?

2. What about whole numbers starting with 1 and ending with 1,000?

Student Response

1. There are 10 perfect squares between 1 and 100, because $1^2 = 1$ and $10^2 = 100$.

2. There are 31 perfect squares between 1 and 1,000, because $31^2 = 961$, but $32^2 = 1\,024$.

Activity Synthesis

Select students to share their observations about how the two representations are alike and how they differ. As students discuss solutions to the questions, circle pairs of associated quantities on the double number line. Help students connect information as it is represented in the different diagrams.

On the last question, ask students how they knew that 20 was the next number on the line representing teaspoons of drink mix? (Skip counting by 4; multiply the next number of cups of water by 4.)

Ask students to think more generally for a minute about the representations at hand:

- What is a **double number line diagram**? What do they do? What do the numbers on the tick marks represent and how should they be scaled?
- What might be some benefits of using double number lines instead of diagrams? (We can use them to show many more batches; they are quicker to draw.)

6.3 Blue Paint on a Double Number Line

15 minutes

The purpose of this activity is for students to practise labelling the tick marks on a double number line diagram with equivalent ratios. This activity revisits a familiar context from a previous lesson so students can apply reasoning about different sized batches of a recipe to help them understand the more abstract representation of a double number line diagram.

Some students may interpret the diagram as showing 3 tablespoons of blue paint for every 1 cup of white paint and may choose to label the top line of the double number line diagram counting by 1s instead of 2s and the bottom line counting by 3s instead of 6s. This is also an acceptable correct answer. There are two reasons to monitor for students using this alternate representation. First, when students are comparing their diagrams with their partner if one partner counter by 2s and the other partner counted by 1s, they may need guidance in determining that these are both correct answers. Second, during the whole-class discussion, consider selecting a student with the less common representation to share their solution at the end.

Instructional Routines

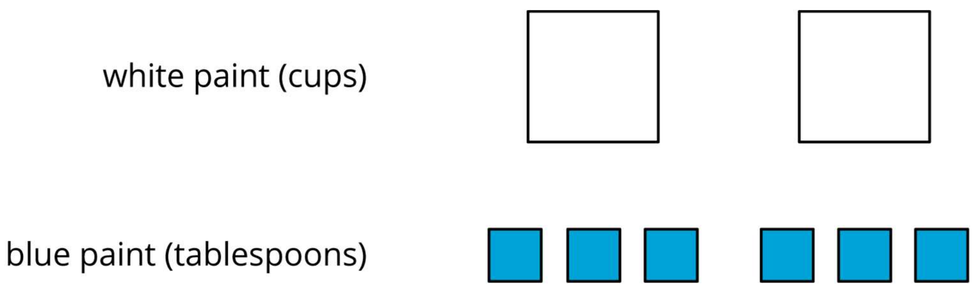
- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time

Launch

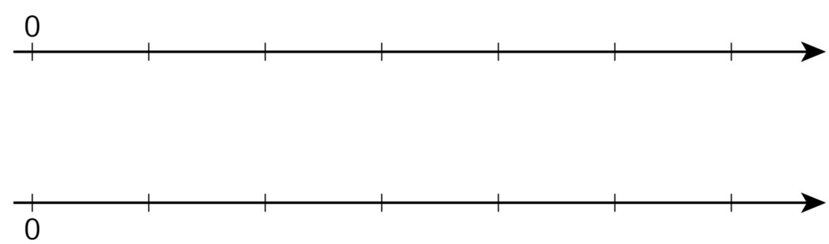
Give students 4 minutes of quiet work time, followed by time to share their response with a partner and then a whole-class discussion afterwards.

Student Task Statement

Here is a diagram showing Elena's recipe for light blue paint.

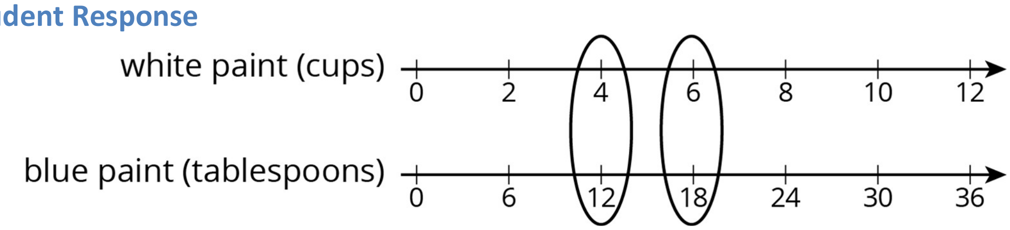


1. Complete the double number line diagram to show the amounts of white paint and blue paint in different-sized batches of light blue paint.



2. Compare your double number line diagram with your partner. Discuss your thinking. If needed, revise your diagram.
3. How many cups of white paint should Elena mix with 12 tablespoons of blue paint? How many batches would this make?
4. How many tablespoons of blue paint should Elena mix with 6 cups of white paint? How many batches would this make?
5. Use your double number line diagram to find another amount of white paint and blue paint that would make the same shade of light blue paint.
6. How do you know that these mixtures would make the same shade of light blue paint?

Student Response



1.

2. Answers vary.

3. 4 cups of white paint, because that is the number on the top line that lines up with the 12 on the bottom line. This would make 2 batches of paint.

4. 18 tablespoons of blue paint, because that is the number on the bottom line that lines up with the 6 on the top line. This would make 3 batches of paint.

5. Answers vary. Sample responses:

   - 8 cups of white paint and 24 tablespoons of blue paint
   - 10 cups of white paint and 30 tablespoons of blue paint
   - 3 cups of white paint and 9 tablespoons of blue paint
   - 1 cup of white paint and 3 tablespoons of blue paint

6. I know these mixtures would make the same shade of light blue paint because the ratios of the amounts of each paint colour are equivalent to the ratio in the original recipe.

Activity Synthesis

Select students to present their solutions. Help them connect the different ways in which the information is represented in the different diagrams. Emphasise the importance of labelling everything clearly so the interpretation is easy to make.

Writing, Conversing: Stronger and Clearer Each Time. Use this routine to prepare students for the whole-class discussion by providing them with multiple opportunities to clarify their reasoning through conversation. Before the whole-class discussion begins, give students time to meet with 2–3 partners to share their response to the final question. Display prompts for feedback such as, "Can you explain how you used your double number line diagram?" or "You should expand on . . . ." Invite listeners to press for details and mathematical language.
Design Principle(s): Optimise output (for justification); Cultivate conversation

Lesson Synthesis

The main ideas to draw out of this lesson are the reasons for using a **double number line diagram**.

- Double number lines easily display equivalent ratios, with the numbers in each equivalent ratio lining up vertically.
- A double number line diagram can be used when a discrete diagram would be cumbersome or even impossible.

The other major goal of this lesson is building up students' fluency in creating double number lines. Students will have further opportunities in upcoming lessons, but watch for common errors such as inconsistent labelling and failing to align the equivalent ratios.

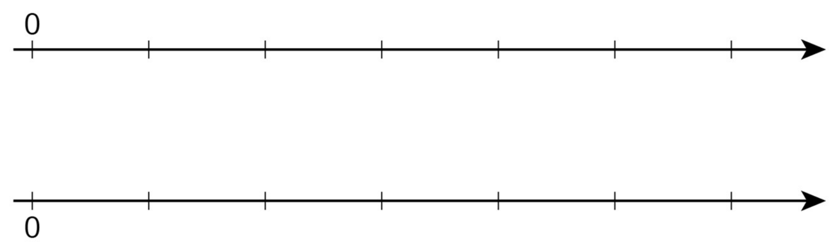
6.4 Batches of Cookies on a Double Number Line

Cool Down: 5 minutes

Student Task Statement

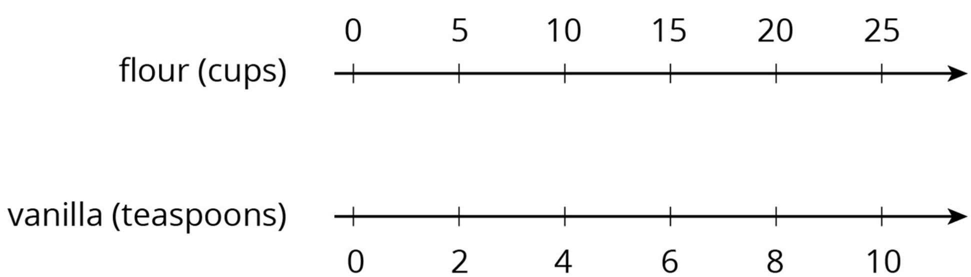
A recipe for one batch of cookies uses 5 cups of flour and 2 teaspoons of vanilla.

1. Complete the double number line diagram to show the amount of flour and vanilla needed for 1, 2, 3, 4, and 5 batches of cookies.



2. If you use 20 cups of flour, how many teaspoons of vanilla should you use?
3. If you use 6 teaspoons of vanilla, how many cups of flour should you use?

Student Response

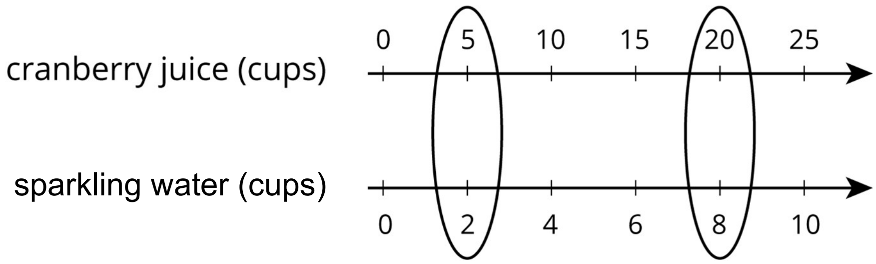


1. See diagram.
2. You should use 8 teaspoons of vanilla.
3. You should use 15 cups of flour.

Student Lesson Summary

You can use a **double number line diagram** to find many equivalent ratios. For example, a recipe for fizzy juice says, “Mix 5 cups of cranberry juice with 2 cups of sparkling water.”

The ratio of cranberry juice to sparkling water is 5 : 2. Multiplying both ingredients by the same number creates equivalent ratios.



This double number line shows that the ratio 20 : 8 is equivalent to 5 : 2. If you mix 20 cups of cranberry juice with 8 cups of sparkling water, it makes 4 times as much fizzy juice that tastes the same as the original recipe.

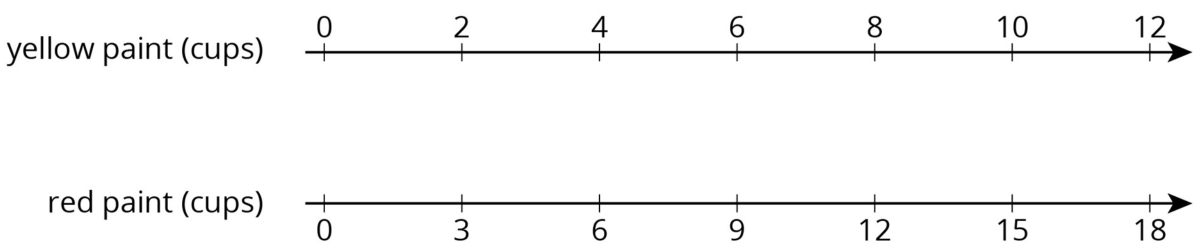
Glossary

- double number line diagram

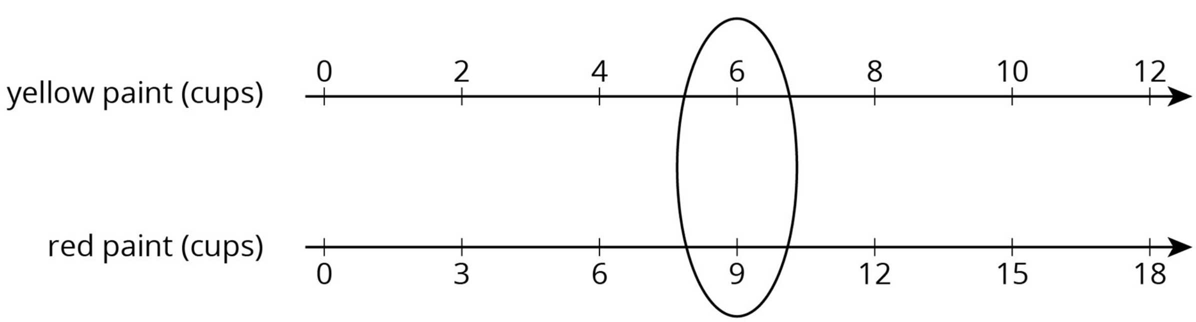
Lesson 6 Practice Problems

Problem 1 Statement

A particular shade of orange paint has 2 cups of yellow paint for every 3 cups of red paint. On the double number line, circle the numbers of cups of yellow and red paint needed for 3 batches of orange paint.

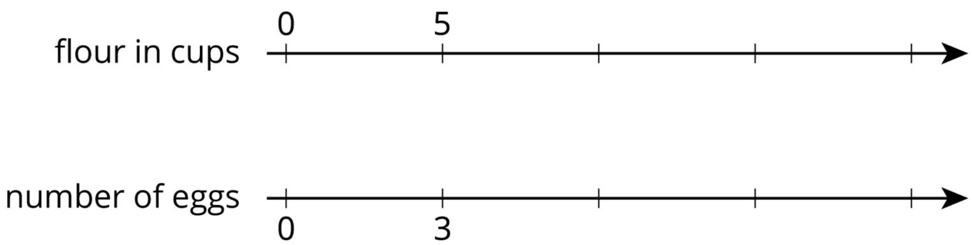


Solution



Problem 2 Statement

This double number line diagram shows the amount of flour and eggs needed for 1 batch of cookies.



a. Complete the diagram to show the amount of flour and eggs needed for 2, 3, and 4 batches of cookies.

b. What is the ratio of cups of flour to eggs?

c. How much flour and how many eggs are used in 4 batches of cookies?

a. How much flour is used with 6 eggs?

b. How many eggs are used with 15 cups of flour?

Solution

a. Flour in cups: 5, 10, 15, 20. Number of eggs: 3, 6, 9, 12.

b. 5 : 3 or equivalent

c. 20 cups of flour and 12 eggs

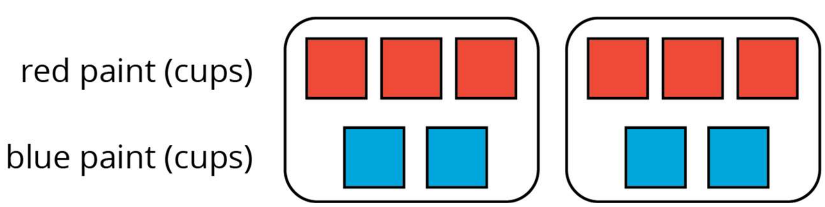
d. 10 cups

e. 9 eggs

Problem 3 Statement

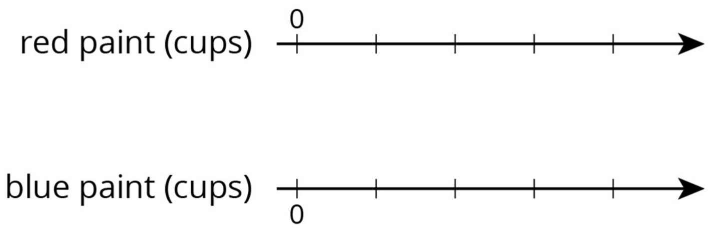
Here is a representation showing the amount of red and blue paint that make 2 batches of purple paint.

a. On the double number line, label the tick marks to represent amounts of red and blue paint used to make batches of this shade of purple paint.



a. How many batches are made with 12 cups of red paint?

b. How many batches are made with 6 cups of blue paint?



Solution

a. Red (cups): 0, 3, 6, 9, 12; Blue (cups): 0, 2, 4, 6, 8

b. 4 batches

c. 3 batches

Problem 4 Statement

Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select **all** the statements that express this ratio.

a. The ratio of kids to pizzas is 7 : 3.

b. The ratio of pizzas to kids is 3 to 7.

c. The ratio of kids to pizzas is 3 : 7.

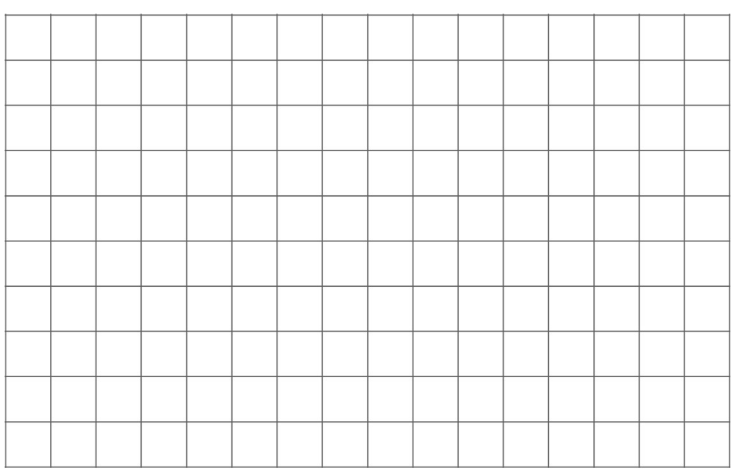
d. The ratio of pizzas to kids is 7 to 3.

e. For every 7 kids there need to be 3 pizzas.

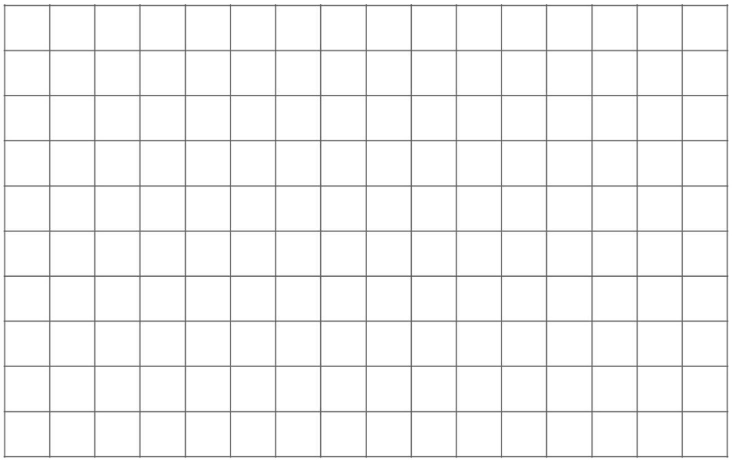
Solution ["A", "B", "E"]

Problem 5 Statement

a. Draw a parallelogram that is not a rectangle that has an area of 24 square units. Explain or show how you know the area is 24 square units.



b. Draw a triangle that has an area of 24 square units. Explain or show how you know the area is 24 square units.



Solution

Answers vary. There are many possible pairs of base and height lengths to make an area of 24 square units.