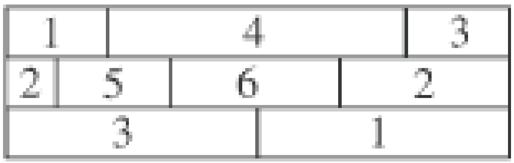
**Grade / Age: 10-14**
**Topic:** Geometry, arts, technology, STEAM
**Subject area:** spatial and plane geometry, architecture, visual arts, music
**Keywords:** Möbius strip (Möbius band), non-orientable surfaces , map colouring
**Single/ team work:** both
**Language:** (English or Local) English
**Duration:** 1,5 hours

**Description of the Task:**

Search on the Internet buildings and sculptures in the shape of a Möbius strip. The painter M. C. Escher also made interesting pictures of the Möbius strip, look for them, study the properties of the Möbius strip, try to define what a Möbius strip could be.
Who is the Möbius strip named after?
Cut out 3 strips of A4 paper, each 3 cm wide. Draw the centre line of one of them and the thirds line of the other.
You can glue the two shorter sides of the strip of paper together in two ways, as a cylinder surface or by turning one end of the strip 180° as a Möbius strip.
Start by painting one of the strips with a thick brush, dragging the line until it meets the starting point of the line. What do you find?
Cut the other strip along its centre line. Guess what you will get? Who hit it?
Cut the third strip along the third line. Guess what you will get? Who hit it?
You can also make multiple twisted bands, open the following GeoGebra file:
https://www.geogebra.org/m/sNBUUBE1
You can change on the d slider the number of times the strip should be twisted and on the b slider the width of the rectangle.
Map colouring:
What is the minimum number of colours needed to colour a map lying on a plane so that neighbouring countries (with a common edge) are of different colours? The answer to this question is 4. On the Möbius strip, the situation is different.




Draw the rectangle above and colour it so that the ranges marked with the same numbers are the same colour and the ranges marked with different numbers are different colours. Glue the rectangle together as a Möbius strip. Check that each range is adjacent to all the others.
Incredibly, the Möbius strip can also be associated with music.
You can watch and listen to J. S. Bach's Cancer Canon on Möbius strip here:
https://www.youtube.com/watch?v=xUHQ2ybTejU

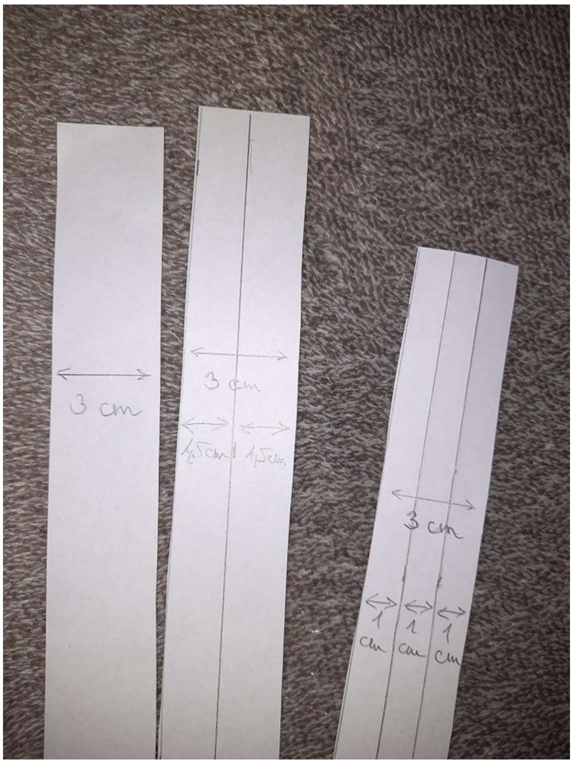
How does this music relate to the Möbius strip? Those of you who play an instrument, try to play the main theme.

**Solutions of the Task:**

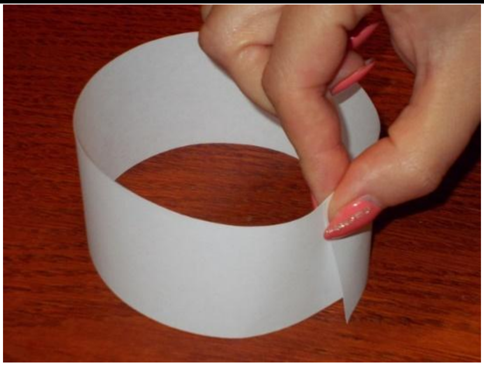
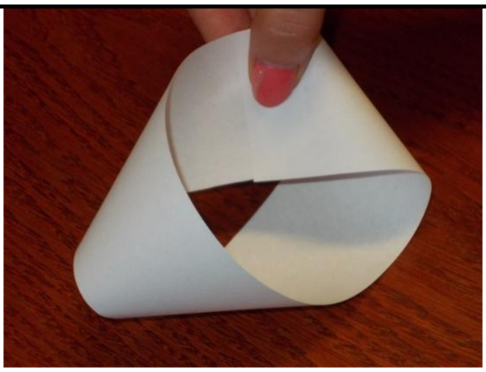
Möbius strip-shaped building in Taiwan:
http://architizer.com/blog/mobius-strip-building-brings-loops-and-bling-to-taiwan/

Sculptures: https://artpool.hu/K53/2020/Mobius5.html

Escher pictures: https://www.nga.gov/collection/art-object-page.61283.html
https://www.wikiart.org/en/m-c-escher/moebius-strip-ii




The Möbius strip is the best known one-sided, or if we translate the English name -- non-orientable, surface. What does it mean that a surface is non-orientable? There are several ways of explaining this. If you take a relatively long rectangle, the two shorter sides can be glued together in two ways, as a cylindrical surface, or with one end of the strip turned 180° as a Möbius strip.

With a wide brush, you can paint the entire Möbius strip without having to step over the edge of the strip. This does not work on the roller surface, the roller surface is orientable. The Möbius strip does not have an inner and outer side like the cylinder surface.

To colour the Möbius strip you need exactly 6 colours. It is interesting to note that the map colouring of the Möbius strip was proved before the theorem of planar map colouring. The problem in the plane was proved in 1976 by Kenneth Appel and Wolfgang Haken, who reduced the infinite number of possible maps to a finite arrangement, and then checked these cases by computer.

**Prior knowledge:**

**Comments:**
When searching the Internet, there can be many different results, and it is worth discussing these together and collecting the links.
Tools needed: paper, ruler, glue, scissors, coloured pencils

**Connection to other subjects/topics/areas:**
art, technique (making and colouring the strips)