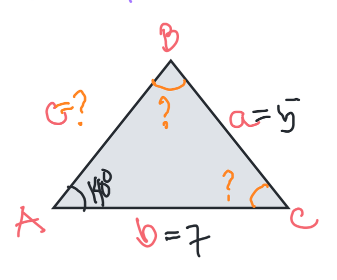
Law of Sine – Ambiguous Case
(SSA Case)

0, 1, 2 Δs

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

A = 140°
a = 5
b = 7



$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 140°}{5} \times \frac{\sin B}{7}$

$\frac{7 \sin 140°}{5} = \frac{5 \sin B}{5}$

$\frac{7 \sin 140°}{5} = \sin B$

$\sin^{-1}\left(\frac{7 \sin 140}{5}\right) = B$

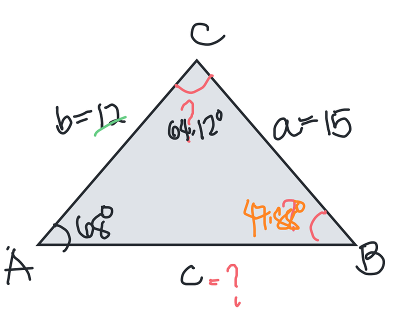
$64.15° = B$

$\angle A + \angle B = 140° + 64.15°$
$= 204.15°$

∴ no triangle will exists

(2) A = 68°
a = 15
b = 12

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 68°}{15} \times \frac{\sin B}{12}$

$\sin B = \frac{12 \sin 68°}{15}$

$B = \sin^{-1}\left(\frac{12 \sin 68°}{15}\right)$

$B = 47.88°$

∴ 1 Δ only

$\angle C = 180 - (68 + 47.88°)$
$= 64.12°$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 68°}{15} \times \frac{\sin 64.12°}{c}$

$c = \frac{15 \sin 64.12°}{\sin 68°} = 14.75$

Will there be 2 Δs?

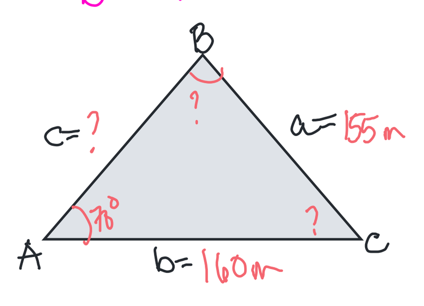
Get the supplement of ∠B
$180° - 47.88° = 132.12°$

∠A + supplement of ∠B
$68° + 132.12° = 200.12°$

Side notes
If the supplement of the computed angle is added to the original given angle, and they sums up to more than 180°, then there will be no 2nd Δ.

(3) A = 70°
a = 155 m
b = 160 m

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 70°}{155} \times \frac{\sin B}{160}$

$B = \sin^{-1}\left(\frac{160 \sin 70°}{155}\right)$

$B = 75.93°$

Will there be 2 Δs?

To know if there will be a 2nd Δ, just subtract the 2nd angle to its supplement (180°).

That is,

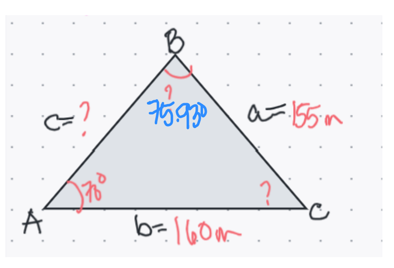
$180° - 75.93° = 104.07°$

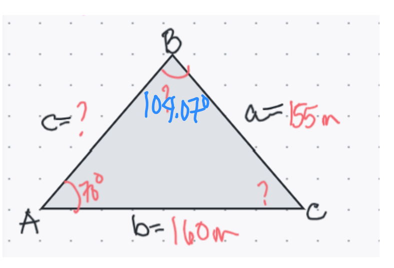
Add the resulting value to the given angle.

$104.07° + 70° = 174.07°$

If the resulting value is less than 180°, then you have a 2nd Δ. Otherwise, only 1 Δ can be formed.

Therefore, since 174.07° is less than 180°, we have a 2nd Δ.





$\angle C = 180° - (70° + 75.93°)$
$= 34.07°$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 70°}{155} \times \frac{\sin 34.07°}{c}$

$c = \frac{155 \sin 34.07°}{\sin 70°}$

$= 92.40$ m

$\angle C = 180° - (104.07° + 70°)$
$= 5.93°$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 70°}{155} \times \frac{\sin 5.93°}{c}$

$c = \frac{155 \sin 5.93°}{\sin 70°}$

$= 17.04$ m