# Lesson 3: Representing proportional relationships

## Goals

- Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes.
- Determine what information is needed to create graphs that represent proportional relationships. Ask questions to elicit that information.

## Learning Targets

- I can scale and label coordinate axes in order to graph a proportional relationship.

## Lesson Narrative

Now that students have considered the scale from several perspectives, in this lesson they label and choose a scale for empty pairs of axes as part of graphing proportional relationships. In the first activity, students create representations of proportional relationships when given two to start from. For each representation, they identify key features such as the constant of proportionality and relate how they know that each representation is for the same situation. In the second activity, students use the info gap structure. The student with the problem card needs to graph a proportional relationship on an empty pair of axes that includes a specific point. In order to do so, they need to request information about the proportional relationship as well as calculate the specific point. The focus here is on the graphs students create and their decisions on how to scale the axes in an appropriate manner for the situation.

### Building On

- Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.
- Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. In general, $\left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{ad}{bc}$. How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{3}{4}$ cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ miles and area $\frac{1}{2}$ square miles?

- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

## Addressing

- Understand the connections between proportional relationships, lines, and linear equations.
- Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

## Instructional Routines

- Stronger and Clearer Each Time
- Collect and Display
- Information Gap Cards
- Discussion Supports
- Number Talk

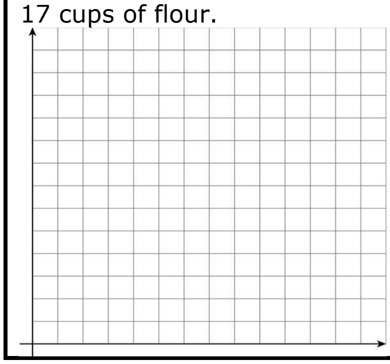
## Required Materials

**Graph paper**
**Pre-printed cards, cut from copies of the blackline master**

Info gap: Proportional Relationships

**Problem card 1**

Sketch a graph that shows the relationship between grams of honey and cups of flour needed for a bakery recipe.
Show on the graph how much honey is needed for 17 cups of flour.



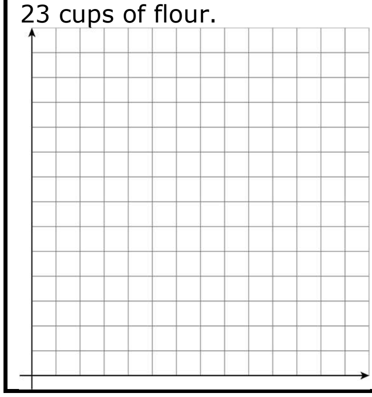
Info gap: Proportional Relationships

**Data card 1**

| salt (g) | honey (g) | flour (cups) |
|---|---|---|
| 15 | 27 | 6 |
| 25 | 45 | 10 |

Info gap: Proportional Relationships
**Problem card 2**

Sketch a graph that shows the relationship between grams of salt and cups of flour needed for a bakery recipe.
Show on the graph how much salt is needed for 23 cups of flour.



Info gap: Proportional Relationships
**Data card 2**

| salt (g) | honey (g) | flour (cups) |
|---|---|---|
| 10 | 14 | 4 |
| 25 | 35 | 10 |

**Straightedges**
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

### Required Preparation

Print and cut up cards from the Info Gap: Proportional Relationships blackline master. Prepare 1 set of cards for every 2 students. Provide all students with access to straightedges and graph paper.

### Student Learning Goals

Let's graph proportional relationships.

## 3.1 Number Talk: Multiplication

**Warm Up: 5 minutes**

This Number Talk encourages students to think about the numbers in a computation problem and rely on what they know about structure, patterns, multiplication, fractions, and decimals to mentally solve a problem. Only one problem is presented to allow students to share a variety of strategies for multiplication. Notice how students handle the multiplication by a decimal. Some students may use fraction equivalencies while others may use the decimal given in the problem. For each of those choices, ask students why they made that decision.

### Instructional Routines

- Collect and Display

- Discussion Supports
- Number Talk

### Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion. Be sure to elicit as many strategies as possible.

*Representation: Internalise Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.
*Supports accessibility for: Memory; Organisation*

### Student Task Statement

Find the value of each product mentally.

$15 \times 2$

$15 \times 0.5$

$15 \times 0.25$

$15 \times (2.25)$

### Student Response

- 30, 7.5, 3.75, 33.75. Explanations vary. Sample explanation:
- $15 \times (2.25) = 33.75$

Possible strategies:

- Distributive property: $(15 \times 2) + (15 \times (0.25))$ or $(15 \times 2) + \left(15 \times \frac{1}{4}\right)$
- Distributive property: $\left(16 \times 2\frac{1}{4}\right) - 2.25$
- Representing 2.25 as a fraction: $15 \times \frac{9}{4}$

### Activity Synthesis

Invite students to share their strategies. Use *Collect and Display* to record and display student explanations for all to see. Ask students to explain their choice of either using a decimal or fraction for 2.25 in their solution path. To involve more students in the conversation, consider asking:

- "Who can restate ____'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"

- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

At the end of discussion, if time permits, ask a few students to share a story problem or context that $15 \times (2.25)$ would represent.

*Speaking: Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
*Design Principle(s): Optimise output (for explanation)*

## 3.2 Representations of Proportional Relationships

**15 minutes**

The purpose of this activity is for students to graph a proportional relationship when given a blank pair of axes. They will need to label and scale the axes appropriately before adding the line representing the given relationship. In each problem, students are given two representations and asked to create two more representations so that each relationship has a description, graph, table, and equation. Then, they explain how they know these are different representations of the same situation. In the next lesson, students will use these skills to compare two proportional relationships represented in different ways.

Identify students making particularly clear graphs and using situation-appropriate scales for their axes. For example, since the second problem is about a car wash, the scale for the axis showing the number of cars does not need to extend into the thousands.

### Instructional Routines

- Stronger and Clearer Each Time

### Launch

Arrange students in groups of 2. Provide access to straightedges. Give 3 minutes of quiet work time for students to begin the first problem and then tell students to check in with their partners to compare tables and how they are labelling and scaling the axes. Ask students to pause their work and select a few students to share what scale they are using for the axes and why they chose it. It is important to note that the scale chosen should be reasonable based on the context. For example, using a very small scale for steps taken does not make sense.

Give partners time to finish the remaining problems and follow with a whole-class discussion.

*Action and Expression: Internalise Executive Functions.* Provide students with grid or graph paper to organise their work with creating a table and graph for the situation.
*Supports accessibility for: Language; Organisation*

## Student Task Statement

1. Here are two ways to represent a situation.

   Description:

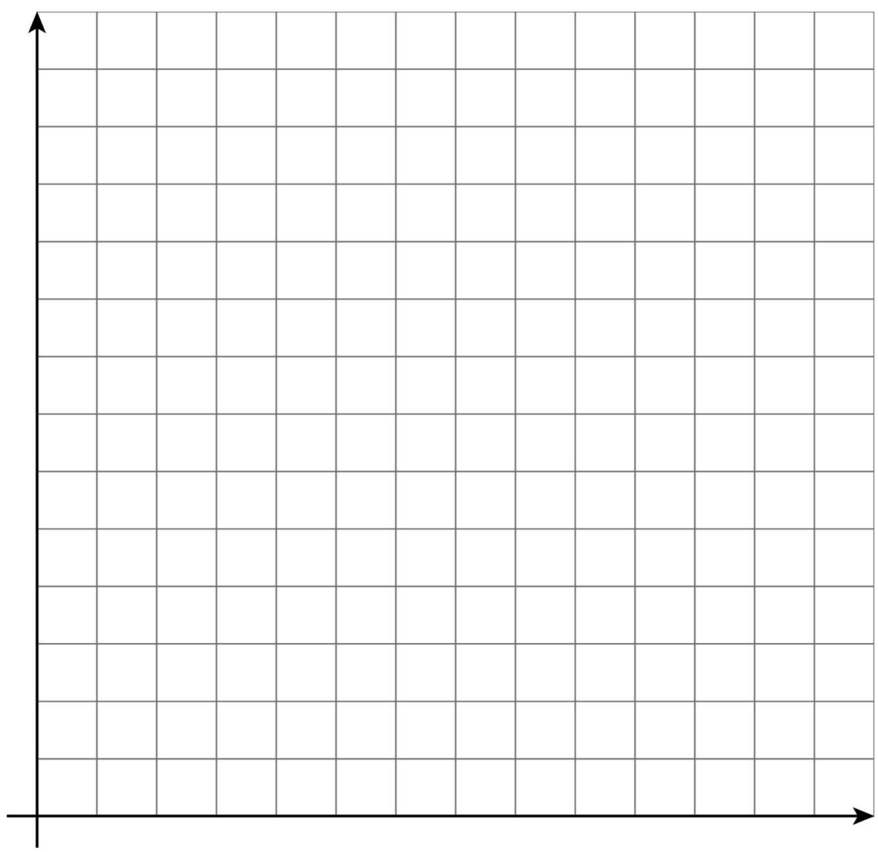
   Jada and Noah counted the number of steps they took to walk a set distance. To walk the same distance, Jada took 8 steps while Noah took 10 steps. Then they found that when Noah took 15 steps, Jada took 12 steps.

   Equation:

   Let $x$ represent the number of steps Jada takes and let $y$ represent the number of steps Noah takes. $y = \frac{5}{4}x$

   a. Create a table that represents this situation with at least 3 pairs of values.

   b. Graph this relationship and label the axes.



   c. How can you see or calculate the constant of proportionality in each representation? What does it mean?

d. Explain how you can tell that the equation, description, graph, and table all represent the same situation.

2. Here are two ways to represent a situation.

Description:

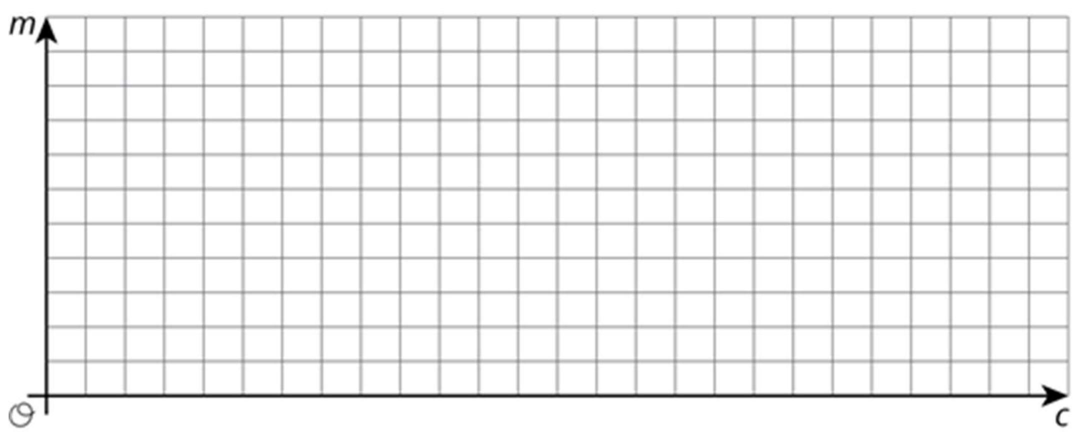
The Origami Club is doing a car wash fundraiser to raise money for a trip. They charge the same price for every car. After 11 cars, they raised a total of £93.50. After 23 cars, they raised a total of £195.50.

Table:

| number of cars | amount raised in pounds |
|---|---|
| 11 | 93.50 |
| 23 | 195.50 |

a. Write an equation that represents this situation. (Use $c$ to represent number of cars and use $m$ to represent amount raised in pounds.)

b. Create a graph that represents this situation.




c. How can you see or calculate the constant of proportionality in each representation? What does it mean?

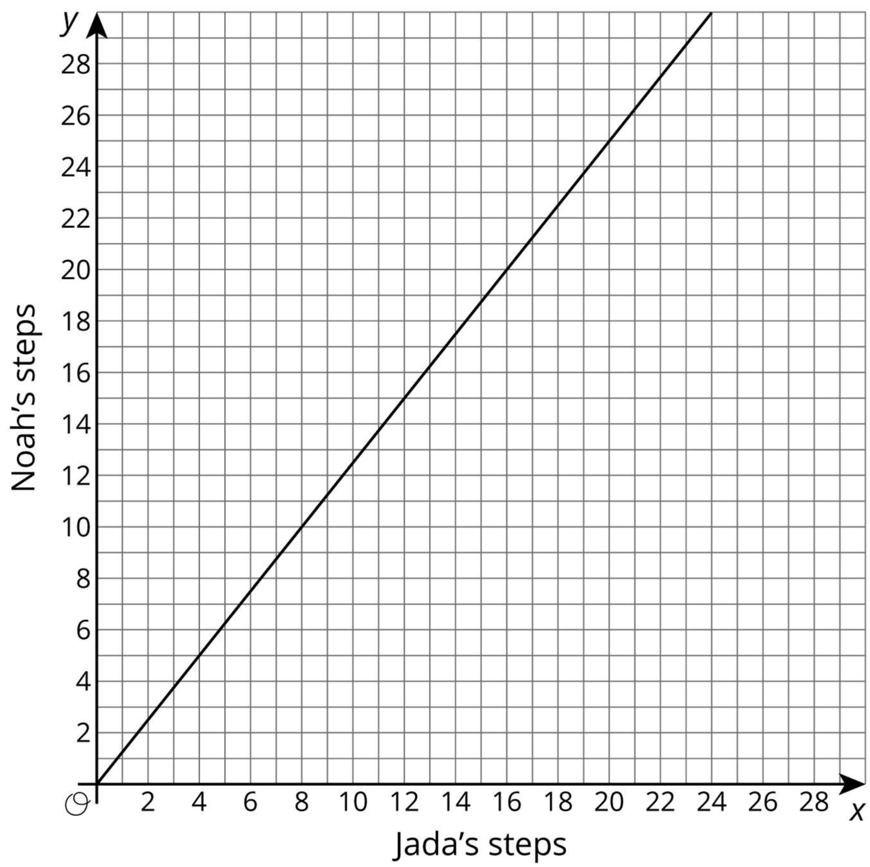
d. Explain how you can tell that the equation, description, graph, and table all represent the same situation.

## Student Response

a. Answers vary. Sample response:

| $x$ | $y$ |
|---|---|
| 8 | 10 |
| 10 | 12.5 |
| 12 | 15 |

b. Answers vary. Sample response:




c. Table: Any $x$ and $y$ values have $\frac{y}{x} = \frac{5}{4}$. In the equation: $k = \frac{5}{4}$, Description: Noah's steps for each of Jada's steps $\left(\frac{5}{4}\right)$, Graph: every coordinate $(x, y)$ on the graph has the relationship that $\frac{y}{x} = \frac{5}{4}$. For every one step Noah takes, Jada takes 1 and a quarter steps.

d. Answers vary. Sample response: All four representations have a constant of proportionality equal to $\frac{5}{4}$.

a. Equation: $m = 8.50c$

b. Graph:


c. In the equation: $k = 8.50$, Table: Any $x$ and $y$ for which $\frac{y}{x} = 8.50$, Description: money raised in pounds per car (£), Graph: every coordinate $(x, y)$ on the graph has the relationship that $\frac{y}{x} = £8.50$ for each car washed.

d. Answers vary. Sample response: All four representations have a constant of proportionality of 8.5.

### Activity Synthesis

Ask previously identified students to share their graphs and how they chose the scales for their axes. If possible, display several graphs from each question for all to see as students share.

Ask students "Which representation makes it more difficult (and less difficult) to calculate the constant of proportionality? Why?" and give 1 minute of quiet think time. Invite several students to share their responses.

Tell students that the constant of proportionality can be thought of as the **rate of change** of one variable with respect to the other. In the case of Jada and Noah, the rate of change of $y$, the number of steps Noah takes, with respect to $x$, the number of steps Jada takes, is $\frac{5}{4}$ Noah steps per Jada steps. In the case of the Origami Club's car wash, the rate of change of $m$, the amount they raise in pounds, with respect to $c$, the number of cars they clean, is 8.50 pounds per car.

*Writing, Conversing: Stronger and Clearer Each Time.* Use this routine for students to respond in writing to the final question, "Explain how you can tell that the equation, description, graph, and table all represent the same situation." Give students time to meet with 2–3 partners, to share their responses and get feedback. Encourage the listener to ask clarifying questions such as, "How did you identify the same constant of proportionality?" or "Did the scales for the axes cause any confusion?" Have the students write a final draft based on their peer feedback. This will help students to generalise the process for identifying the same constant of proportionality.
*Design Principle(s): Optimise output; Cultivate conversation*

# 3.3 Info Gap: Proportional Relationships

**15 minutes**

This info gap activity gives students an opportunity to determine and request the information needed when working with proportional relationships. In order to graph the relationship and the requested information, students need to think carefully about how they can scale the axes.

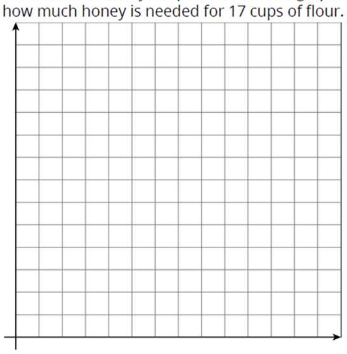
The info gap structure requires students to make sense of problems by determining what information is necessary, and then asking for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Here is the text of one of the cards for reference and planning:

Info Gap: Proportional Relationships

Problem Card 1

Sketch a graph that shows the relationship between grams of honey and cups of flour needed for a bakery recipe. Show on the graph how much honey is needed for 17 cups of flour.



Info Gap: Proportional Relationships

Data Card 1

| salt (g) | honey (g) | flour (c) |
|---|---|---|
| 15 | 27 | 6 |
| 25 | 45 | 10 |

Listen for how students request (and supply) information about the relationship between the two ingredients. Identify students using different scales for their graphs that show clearly the requested information to share during the discussion.

### Instructional Routines

- Information Gap Cards

### Launch

Tell students that they will continue their work graphing proportional relationships. Explain the Info Gap and consider demonstrating the protocol if students are unfamiliar with it. Arrange students in groups of 2. Provide access to straightedges. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for the second problem and instruct them to switch roles.

*Representation: Provide Access for Perception.* Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting

students to rephrase directions in their own words. Consider keeping the display of directions visible throughout the activity.
*Supports accessibility for: Language; Memory Conversing*: This activity uses *Information Gap* to give students a purpose for discussing information necessary to choose a scale and graph a proportional relationship. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”
*Design Principle(s): Cultivate Conversation*

### Anticipated Misconceptions

Some students may be unsure how large to make their scale before they answer the question on the card. Encourage these students to answer the question on their card and then think about how to scale their graph.

### Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *“What specific information do you need?”* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.
3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.

5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

### Student Response

Problem 1: 76.5 grams of honey are needed for 17 cups of flour. Graphs vary. Possible scale: 0–28 on the cups of flour axis, 0–140 on the grams of honey axis.

Problem 2: 57.5 grams of salt are needed for 23 cups of flour. Graphs vary. Possible scale: 0–28 on the cups of flour axis, 0–70 on the grams of salt axis.

### Are You Ready for More?

Ten people can dig five holes in three hours. If $n$ people digging at the same rate dig $m$ holes in $d$ hours:

1. Is $n$ proportional to $m$ when $d = 3$?
2. Is $n$ proportional to $d$ when $m = 5$?
3. Is $m$ proportional to $d$ when $n = 10$?

### Student Response

1. Yes. If 10 people can dig 5 holes in 3 hours, then 2 people can dig 1 hole in 3 hours, so $n = 2m$.
2. No. If you double the number of people then you half the time it takes to dig the holes. So $n$ is not a constant times $d$.
3. Yes. If 10 people can dig 5 holes in 3 hours, then they can dig $\frac{5}{3}$ holes in 1 hour, so $m = \frac{5}{3}d$.

### Activity Synthesis

After students have completed their work, ask previously identified students to share their graphs and explain how they chose their axis. Some guiding questions:

- "Other than the answer, what information would have been nice to have?"
- "How did you decide what to label the two axes?"
- "How did you decide to scale the horizontal axis? The vertical?"
- "What was the rate of change of grams of honey per cups of flour? Where can you see this on the graph you made?" (4.5 grams of honey per cup of flour.)
- "What was the rate of change of grams of salt per cups of flour? Where can you see this on the graph you made?" (2.5 grams of salt per cups of flour)

## Lesson Synthesis

Consider asking some of the following questions.

- "The proportional relationship $y = 5.5x$ includes the point (18,99) on its graph. How could you choose a scale for a pair of axes with a 10 by 10 grid to show this point?" (Have each grid line represent 10 or 20 units.)
- "What are some things you learned about graphing today that you are going to try to remember for later?"

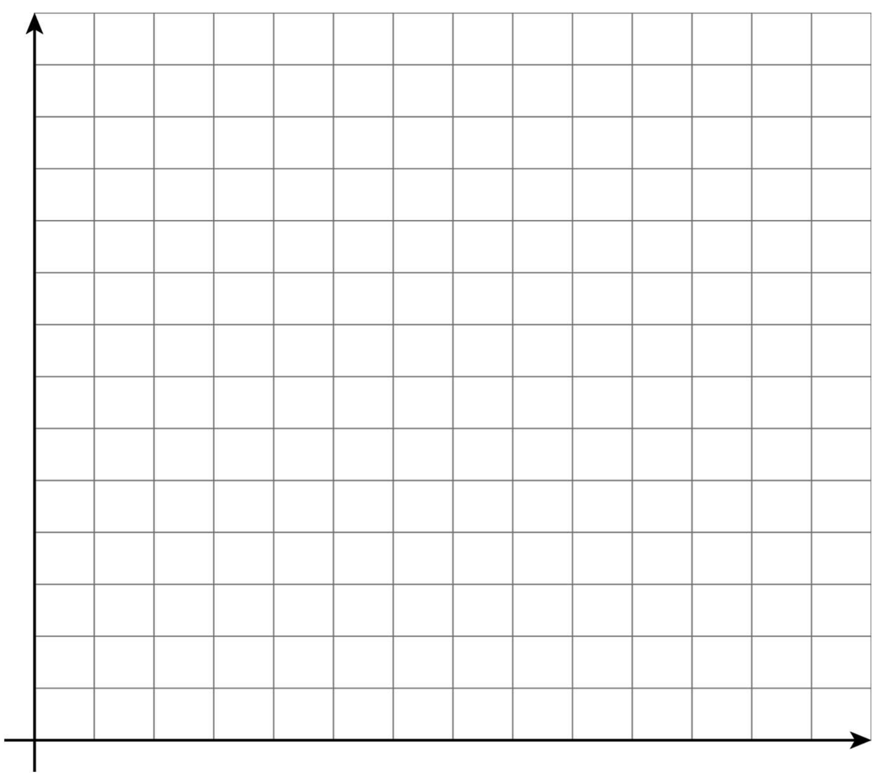
# 3.4 Graph the Relationship

**Cool Down: 5 minutes**

### Student Task Statement

Sketch a graph that shows the relationship between grams of honey and grams of salt needed for a bakery recipe. Show on the graph how much honey is needed for 70 grams of salt.

| salt (g) | honey (g) | flour (c) |
|---|---|---|
| 10 | 14 | 4 |
| 25 | 35 | 10 |



**Student Response**

Answers vary. Possible graph: Label each axis from 0 to 140. For grams of salt on the horizontal axis and grams of honey on the vertical, the points (0,0), (10,14), and (70,98).

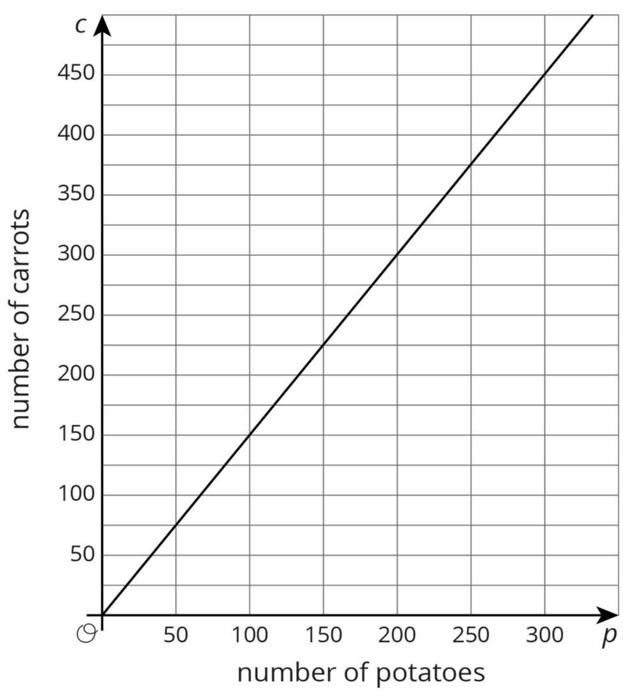
## Student Lesson Summary

Proportional relationships can be represented in multiple ways. Which representation we choose depends on the purpose. And when we create representations we can choose helpful values by paying attention to the context. For example, a stew recipe calls for 3 carrots for every 2 potatoes. One way to represent this is using an equation. If there are $p$ potatoes and $c$ carrots, then $c = \frac{3}{2}p$.

Suppose we want to make a large batch of this recipe for a family gathering, using 150 potatoes. To find the number of carrots we could just use the equation: $\frac{3}{2} \times 150 = 225$ carrots.

Now suppose the recipe is used in a restaurant that makes the stew in large batches of different sizes depending on how busy a day it is, using up to 300 potatoes at a time.

Then we might make a graph to show how many carrots are needed for different amounts of potatoes. We set up a pair of coordinate axes with a scale from 0 to 300 along the horizontal axis and 0 to 450 on the vertical axis, because $450 = \frac{3}{2} \times 300$. Then we can read how many carrots are needed for any number of potatoes up to 300.



Or if the recipe is used in a food factory that produces very large quantities and the potatoes come in bags of 150, we might just make a table of values showing the number of carrots needed for different multiples of 150.

| number of potatoes | number of carrots |
|---|---|
| 150 | 225 |
| 300 | 450 |
| 450 | 675 |
| 600 | 900 |

No matter the representation or the scale used, the constant of proportionality, $\frac{3}{2}$, is evident in each. In the equation it is the number we multiply $p$ by; in the graph, it is the slope; and in the table, it is the number we multiply values in the left column to get numbers in the right column. We can think of the constant of proportionality as a **rate of change** of $c$ with respect to $p$. In this case the rate of change is $\frac{3}{2}$ carrots per potato.

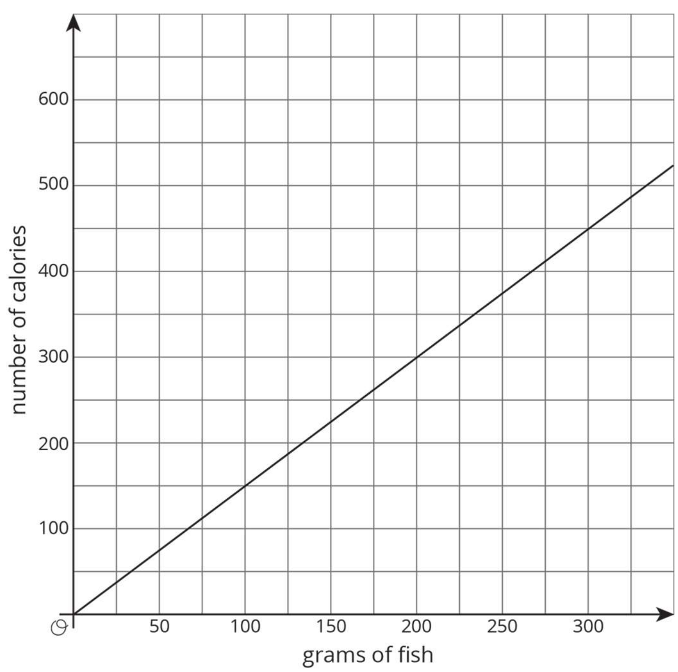
## Glossary

- rate of change

## Lesson 3 Practice Problems

1. **Problem 1 Statement**

   Here is a graph of the proportional relationship between calories and grams of fish:




   a. Write an equation that reflects this relationship using $x$ to represent the amount of fish in grams and $y$ to represent the number of calories.

b. Use your equation to complete the table:

| grams of fish | number of calories |
|---|---|
| 1 000 | |
| | 2 001 |
| 1 | |

**Solution**

a. $y = \frac{3}{2}x$

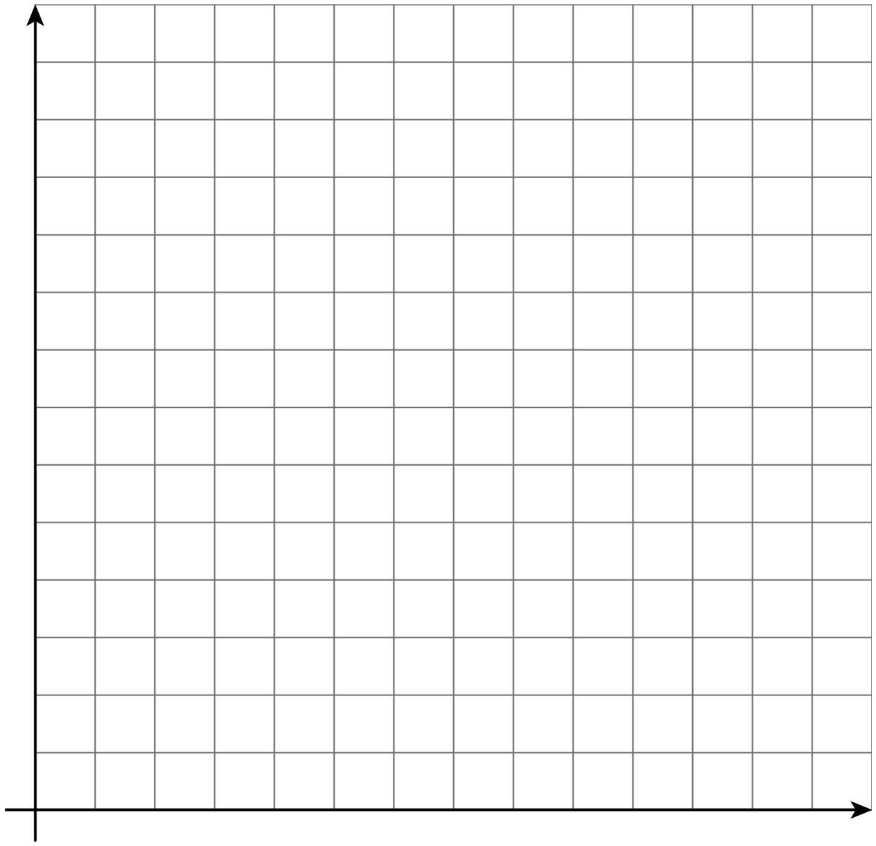
| grams of fish | number of calories |
|---|---|
| 1 000 | 1 500 |
| 1 334 | 2 001 |
| 1 | $\frac{3}{2}$ |

2. **Problem 2 Statement**

Students are selling raffle tickets for a school fundraiser. They collect £24 for every 10 raffle tickets they sell.

a. Suppose $M$ is the amount of money the students collect for selling $R$ raffle tickets. Write an equation that reflects the relationship between $M$ and $R$.

b. Label and scale the axes and graph this situation with $M$ on the vertical axis and $R$ on the horizontal axis. Make sure the scale is large enough to see how much they would raise if they sell 1 000 tickets.



**Solution**

a. $M = \frac{12}{5}R$ (or equivalent)

b. On coordinate axes with $R$ on the horizontal axis and $M$ on the vertical axis, a ray through (0,0) and (10,24) (or equivalent)

3. **Problem 3 Statement**

Describe how you can tell whether a line's slope is greater than 1, equal to 1, or less than 1.

**Solution**

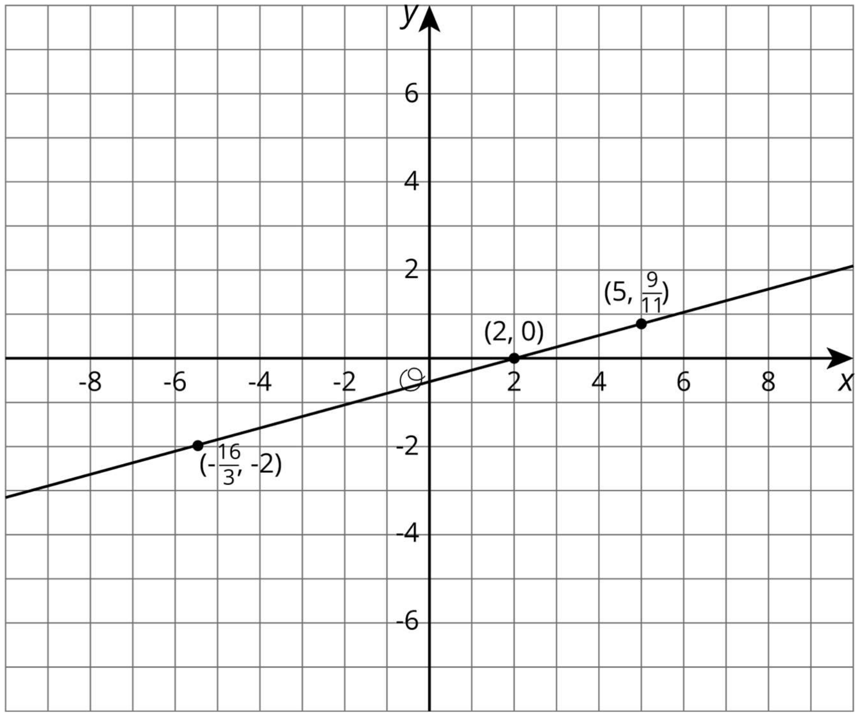
Answers vary. Sample response: Build a slope triangle. If its vertical length is greater than its horizontal length, the slope is greater than 1. If its vertical and horizontal lengths are equal, the slope is equal to 1. If the slope triangle's vertical length is less than its horizontal length, the slope is less than 1.

4. **Problem 4 Statement**

A line is represented by the equation $\frac{y}{x-2} = \frac{3}{11}$. What are the coordinates of some points that lie on the line? Graph the line on graph paper.

## Solution

Answers vary. Possible response:




(Note that there is one point on the line which has to be treated differently. The point (2,0) is on the sketched line but is not a pair that can be substituted into $\frac{y}{x-2} = \frac{3}{11}$, as it results in a denominator of 0. The point (2,0) does satisfy the equivalent equation $y = \frac{3}{11}(x - 2)$, however.)