Lesson 16: Two related quantities (Part 1)

Goals

- Compare and contrast (orally) graphs and equations that represent a relationship between the same quantities but have the independent and dependent variables switched.
- Comprehend the terms "independent variable" and "dependent variable" (in spoken and written language).
- Create a table, graph, and equation to represent the relationship between quantities in a set of equivalent ratios.

Learning Targets

- I can create tables and graphs that show the relationship between two amounts in a given ratio.
- I can write an equation with variables that shows the relationship between two amounts in a given ratio.

Lesson Narrative

This lesson is the first of two that apply new understanding of algebraic expressions and equations to represent relationships between two quantities. Students use and make connections between tables, graphs, and equations that represent these relationships.

In this lesson, students revisit and extend their understanding of equivalent ratios. A familiar scenario of mixing paints in a given ratio provides the context for writing equations that represents the relationship between two quantities. Students then create a table of values that shows how changes in one quantity affect changes in the other, and graph the points from the table in the coordinate plane. They are invited to notice that these points lie on a line. Students will study proportional relationships in more depth later in KS3.

Students learn that relationships between two quantities can be described by two different but related equations with one quantity, the **dependent variable**, affected by changes in the other quantity, the **independent variable**. When people engage in mathematical modelling, which variable is considered independent and which is considered dependent is often the choice of the modeller (though sometimes the situation suggests choosing one way over the other). The context in this lesson was intentionally chosen because the context does not suggest a preference about which quantity is chosen as the independent variable.

Alignments

Addressing

- Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyse the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2: 1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

- Discussion Supports

- Think Pair Share

Student Learning Goals

Let's use equations and graphs to describe relationships with ratios.

16.1 Which One Would You Choose?

Warm Up: 5 minutes

The purpose of this warm-up is for students to remember that unit price can be used to figure out which price option is a better deal and also how to compute unit price. When students explain their reasoning, they may engage in constructing arguments and critiquing the reasoning of their classmates. The question, "Which one would you choose?" is purposefully asked because there is not one correct answer. While there is a choice that is a better deal, that is not the question. In defending their reasoning, students may have other reasons for their choice based on how they make sense of the context. For example, students might reason that a 5-gallon container is easier to store, or that 3 1-gallon containers are easier to share, or they might reject both options because they don't like honey.

As students work, listen for reasoning about the problem in different ways to share in the whole-class discussion. Choose at least one student who reasoned by finding the unit cost.

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give 1 minute of quiet think time followed by 1 minute to discuss their responses with a partner. Follow with a whole-class discussion.

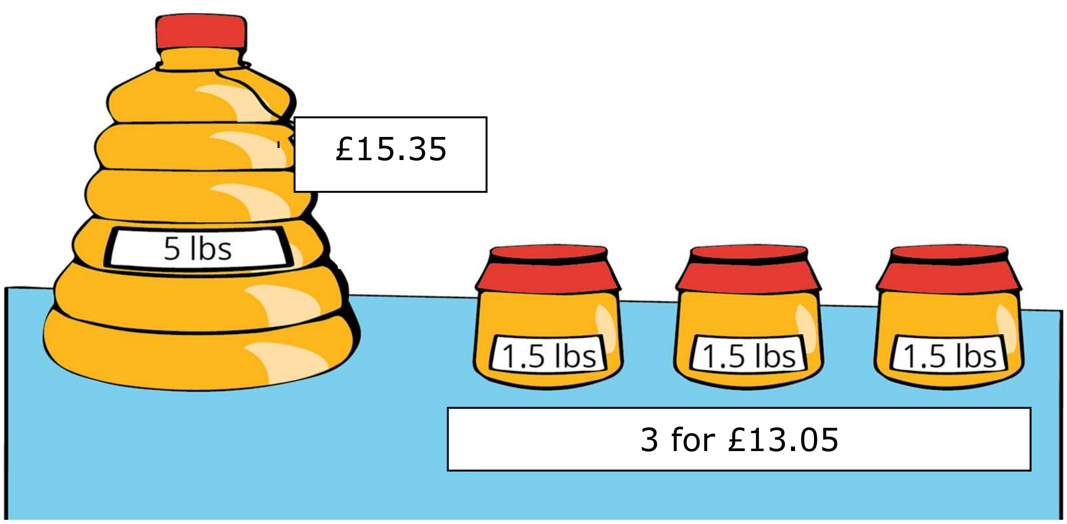
Anticipated Misconceptions

Students may choose the 5 pound jug because they assume that larger quantities are a better deal. Ask if they can be more precise in their reasoning and defend it mathematically.

Student Task Statement

Which one would you choose? Be prepared to explain your reasoning.

- A 5-pound jug of honey for £15.35
- Three 1.5-pound jars of honey for £13.05



Student Response

Answers vary. Unit costs are £3.07 per pound for the large jug and £2.90 per pound for the smaller jars.

Activity Synthesis

Poll the class to find who selected each of the options. Ask selected students to share their reasoning. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to _____’s reasoning?”
- “Do you agree or disagree? Why?”

16.2 Painting the Set

25 minutes (there is a digital version of this activity)

This activity helps students recall what they know about solving problems in a ratio context, and then extends their thinking to consider writing two equations that relate the two quantities in the ratio and representing them with graphs.

Instructional Routines

- Discussion Supports

Launch

Explain the meaning of independent variable and dependent variable. (This can be done before students start working, or you can have students pause after completing the table.) An example may help: Suppose Lin and her neighbor share the same birthday, but Lin is 3 years older. You can find the neighbor's age by taking Lin’s age and subtracting 3. If Lin’s age is represented by L, and her neighbor’s age is represented by n, then the equation $n = L - 3$ describes the relationship. In this equation, the value of n depends on the value of L, so we call n the **dependent variable** and L the **independent variable**.

Give students 5-8 minutes of quiet work time, followed by a whole-class discussion.

Some students may not be familiar with the word “set” in this context. Explain that it means the scenery and other props and objects that are used on stage during a play or production.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: dependent variable, independent variable. Invite students to suggest language or diagrams to include on the display that will support their understanding of these terms. *Supports accessibility for: Conceptual processing; Memory*

Anticipated Misconceptions

For students who struggle to write an equation that relates the two quantities, help them to represent the situation in a concrete way, like with a bar model or a discrete diagram. You can also draw their attention to the completed table: “What can you do to each number in the r column to get the number in the y column?” Expressing the relationship in words can be a helpful step to expressing it with an equation.

Student Task Statement

Lin needs to mix a specific shade of orange paint for the set of the school play. The colour uses 3 parts yellow for every 2 parts red.

1. Complete the table to show different combinations of red and yellow paint that will make the shade of orange Lin needs.

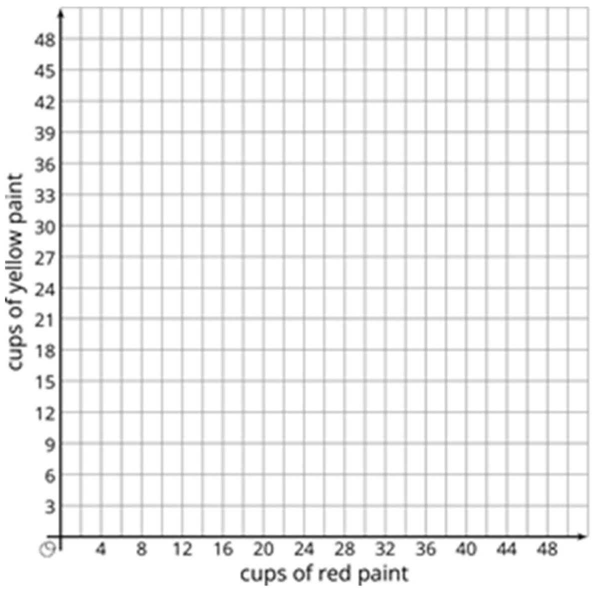
cups of red paint (r)	cups of yellow paint (y)	total cups of paint (t)
2	3	
6		
		20
	18	
14		
16		
		50
	42	

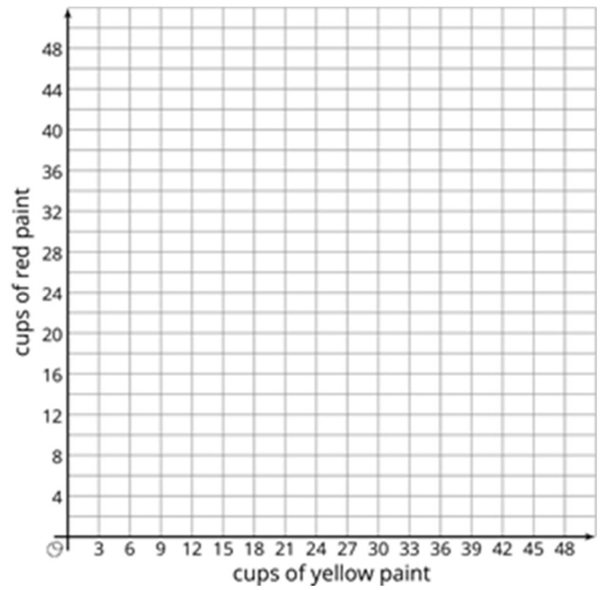
2. Lin notices that the number of cups of red paint is always $\frac{2}{5}$ of the total number of cups. She writes the equation $r = \frac{2}{5}t$ to describe the relationship. Which is the **independent variable**? Which is the **dependent variable**? Explain how you know.

3. Write an equation that describes the relationship between r and y where y is the independent variable.

4. Write an equation that describes the relationship between y and r where r is the independent variable.

5. Use the points in the table to create two graphs that show the relationship between r and y. Match each relationship to one of the equations you wrote.

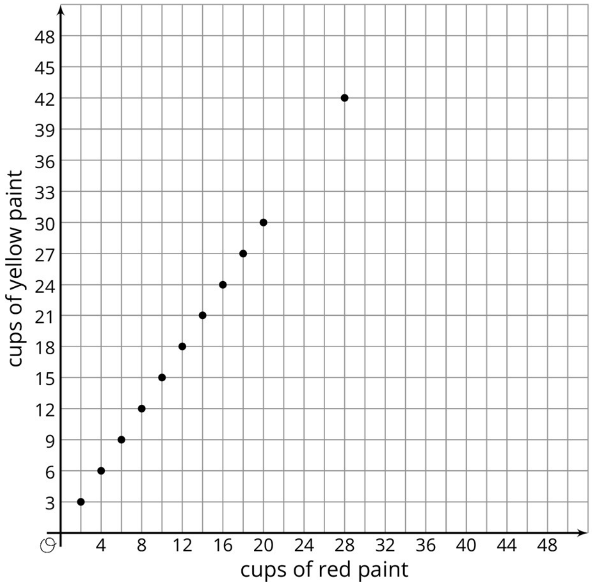


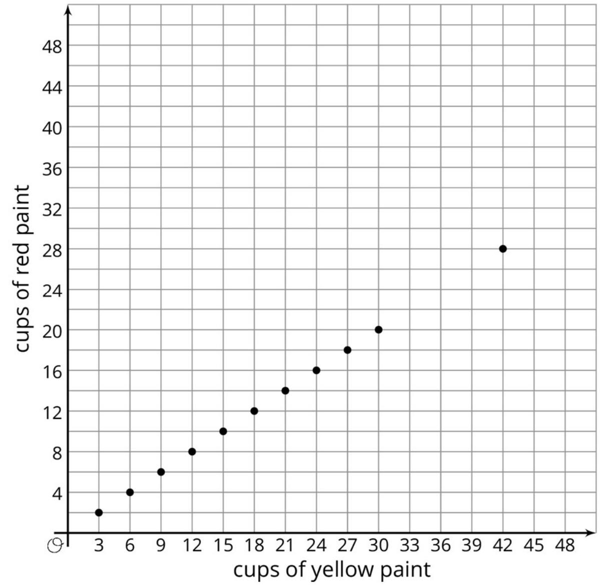


Student Response

cups of red paint (r)	cups of yellow paint (y)	total cups of paint (t)
2	3	5
6	9	15
8	12	20
12	18	30
14	21	35
16	24	40
20	30	50
28	42	70

1. See table.
2. t is the independent variable and r is the dependent variable. The equation says that r is $\frac{2}{5}$ of t, so the value of r depends on the value of t.
3. $r = \frac{2}{3}y$
4. $y = \frac{3}{2}r$
5. First graph matches $y = \frac{3}{2}r$, second graph matches $r = \frac{2}{3}y$.





Are You Ready for More?

A fruit stand sells apples, peaches, and tomatoes. Today, they sold 4 apples for every 5 peaches. They sold 2 peaches for every 3 tomatoes. They sold 132 pieces of fruit in total. How many of each fruit did they sell?

Student Response

32 apples, 40 peaches, 60 tomatoes

Activity Synthesis

The discussion should focus on how all the representations (tables, equations, graphs) capture the same information but look at it in a different way.

Some guiding questions:

- “Why is it possible to write two different equations to describe the same situation?” (You can choose either quantity to be the dependent variable.)
- “What do you notice about the numbers that multiply by the independent variable in each equation?” (They are reciprocals, they represent the two unit rates for the situation.)
- “Are there other equations we could have written that describe this situation?” (Yes, for example, we can write equations to describe the relationship between the amount of yellow paint and the total number of cups: $y = \frac{3}{5}t$ and $t = \frac{5}{3}y$)
- “What do you notice about the similarities and differences between the graphs?” (Both have points that appear to lie on a line, both lines go up as you move to the right, the line for Lin’s equation slants up more, the coordinates of the points are reversed.)

Representing: Discussion Supports. Ask students to identify where the independent and dependent variables are represented in the table, graph, and equation. Use sentence frames such as “For the equation $r = \frac{2}{5}t$, r is the _____ variable because _____. t is the _____ variable because _____.” This will help students make comparisons of the relationship between the variables and how they are represented.
Design Principle(s): Optimise output (for comparison)

Lesson Synthesis

In this lesson we revisited equivalent ratios by writing equations that represent sets of equivalent ratios and graphing them. Some guiding questions for discussion:

- “We wrote two equations to represent the relationship between cups of red paint and cups of yellow paint. How were these the same and how were they different?”
- “Explain the meaning of **dependent** and **independent** variable.”
- “How do you know which quantity to choose as the independent variable when you write an equation to describe a situation?” (It depends on what you know and what you need to calculate.)
- “When we first studied equivalent ratios, we used double number lines and tables. How do graphs and equations add to your understanding of equivalent ratios? Do they help in solving problems? If so, how?”

16.3 Baking Brownies

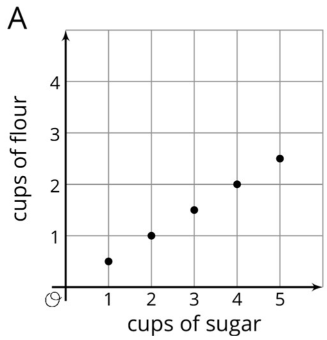
Cool Down: 5 minutes

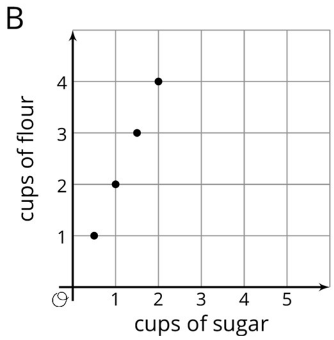
Launch

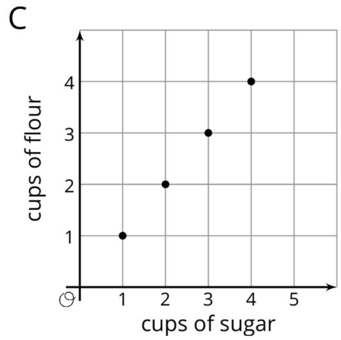
For English Language Learners, clarify that the phrase “calls for” means something that is needed or required in the situation.

Student Task Statement

A brownie recipe calls for 1 cup of sugar and $\frac{1}{2}$ cup of flour to make one batch of brownies. To make multiple batches, the equation $f = \frac{1}{2}s$ where f is the number of cups of flour and s is the number of cups of sugar represents the relationship. Which graph also represents the relationship? Explain how you know.







Student Response

Graph A because for every cup of sugar, there is a half cup of flour. So, at 1 cup of sugar, there is $\frac{1}{2}$ cup of flour. At 2 cups of sugar, there is 1 cup of flour. For each batch, the sugar goes up by 1 cup and the flour goes up by a half cup.

Student Lesson Summary

Equations are very useful for describing sets of equivalent ratios. Here is an example.

A pie recipe calls for 3 green apples for every 5 red apples. We can create a table to show some equivalent ratios.

We can see from the table that r is always $\frac{5}{3}$ as large as g and that g is always $\frac{3}{5}$ as large as r.

green apples (g)	red apples (r)
3	5
6	10
9	15
12	20

We can write equations to describe the relationship between g and r.

- When we know the number of green apples and want to find the number of red apples, we can write:

  $r = \frac{5}{3}g$

  In this equation, if g changes, r is affected by the change, so we refer to g as the **independent variable** and r as the **dependent variable**.

  We can use this equation with any value of g to find r. If 270 green apples are used, then $\frac{5}{3} \times (270)$ or 450 red apples are used.

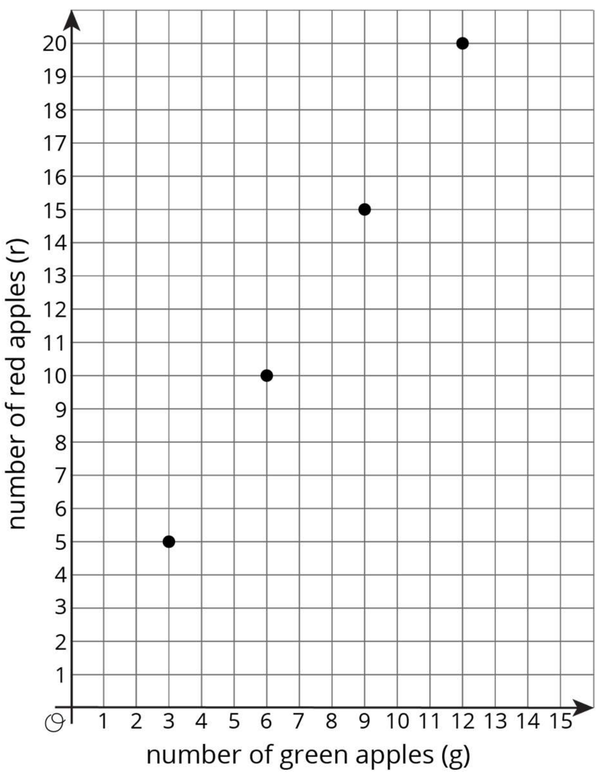
- When we know the number of red apples and want to find the number of green apples, we can write:

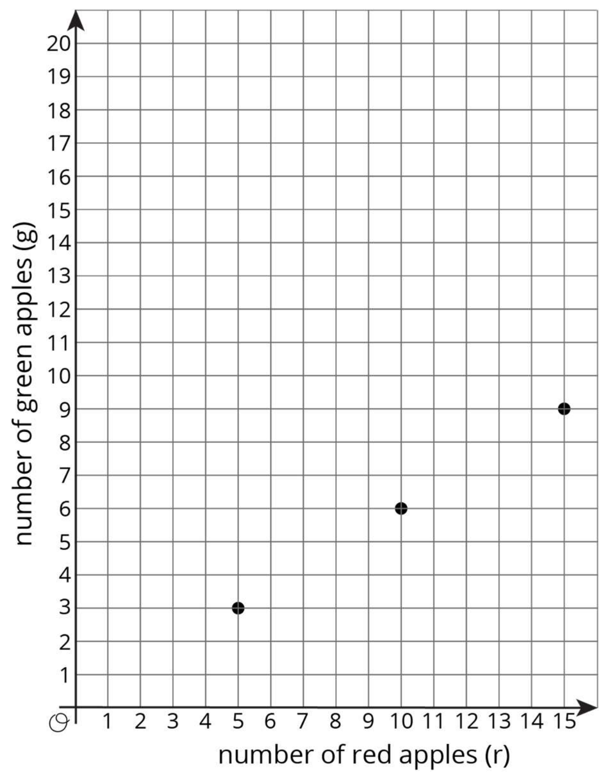
  $g = \frac{3}{5}r$

  In this equation, if r changes, g is affected by the change, so we refer to r as the independent variable and g as the dependent variable.

  We can use this equation with any value of r to find g. If 275 red apples are used, then $\frac{3}{5} \times (275)$ or 165 green apples are used.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities.





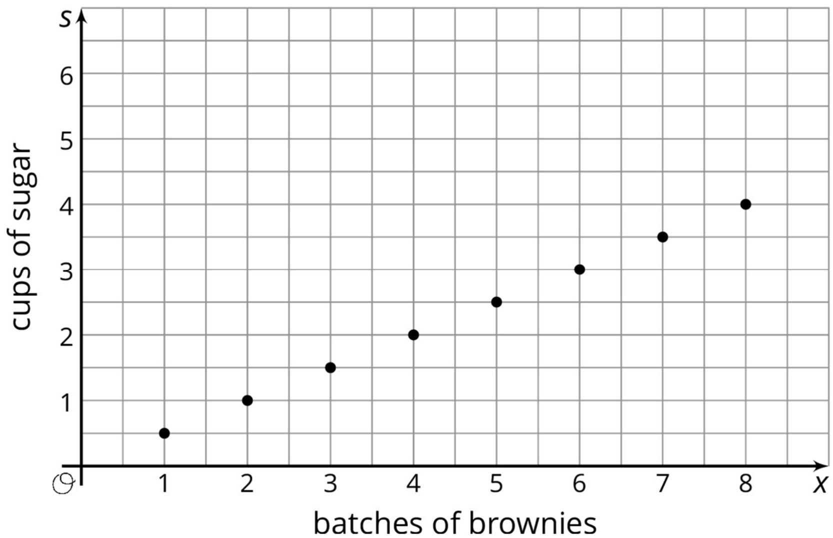
Glossary

- dependent variable
- independent variable

Lesson 16 Practice Problems

1. **Problem 1 Statement**

   Here is a graph that shows some values for the number of cups of sugar, s, required to make x batches of brownies.



a. Complete the table so that the pair of numbers in each column represents the coordinates of a point on the graph.

x	1	2	3	4	5	6	7	8
s								

b. What does the point (8,4) mean in terms of the amount of sugar and number of batches of brownies?

c. Write an equation that shows the amount of sugar in terms of the number of batches.

Solution

a.

x	1	2	3	4	5	6	7	8
s	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4

b. To make 8 batches of brownies, you need 4 cups of sugar.

c. $s = \frac{1}{2}x$

2. **Problem 2 Statement**

Each serving of a certain fruit snack contains 90 calories.

a. Han wants to know how many calories he gets from the fruit snacks. Write an equation that shows the number of calories, c, in terms of the number of servings, n.

b. Tyler needs some extra calories each day during his sports season. He wants to know how many servings he can have each day if all the extra calories come from the fruit snack. Write an equation that shows the number of servings, n, in terms of the number of calories, c.

Solution

a. $c = 90n$

b. $n = \frac{c}{90}$ or $n = c \div 90$

3. **Problem 3 Statement**

Kiran shops for books during a 20% off sale.

a. What percent of the original price of a book does Kiran pay during the sale?

b. Complete the table to show how much Kiran pays for books during the sale.

c. Write an equation that relates the sale price, s, to the original price p.

d. On graph paper, create a graph showing the relationship between the sale price and the original price by plotting the points from the table.

original price in pounds (p)	sale price in pounds (s)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

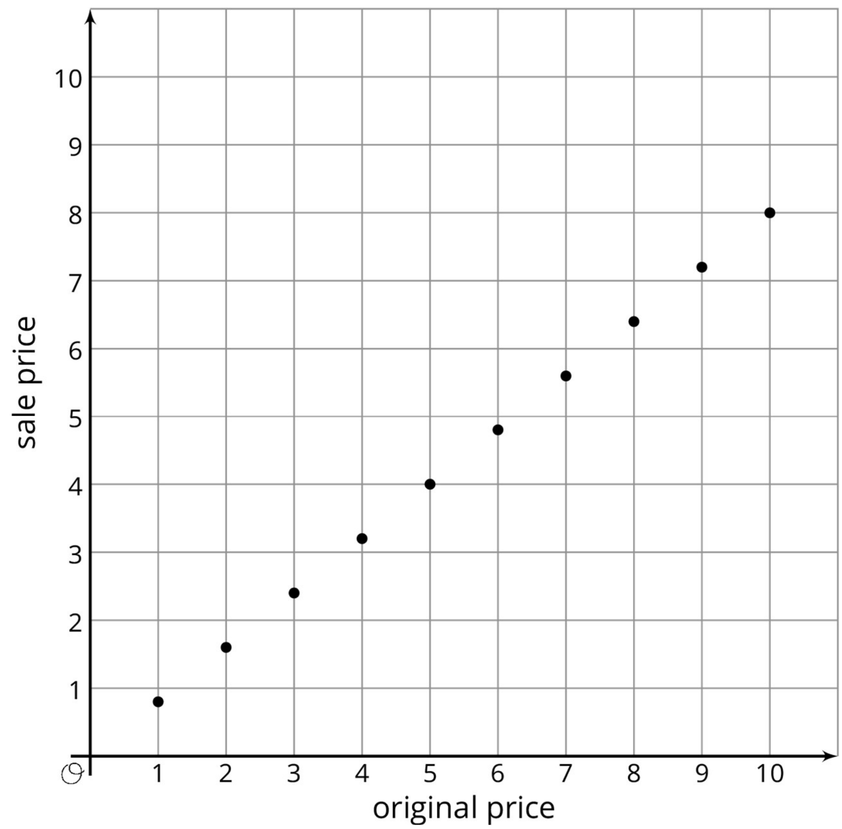
Solution

a. 80%

b. Sale prices: 0.80, 1.60, 2.40, 3.20, 4.00, 4.80, 5.60, 6.40, 7.20, 8.00

c. $s = 0.8p$

d.



4. **Problem 4 Statement**

Evaluate each expression when x is 4 and y is 6.

a. $(6 - x)^3 + y$

b. $2 + x^3$

c. $2^x - 2y$

d. $\left(\frac{1}{2}\right)^x$

e. $1^x + 2^x$

f. $\frac{2^x}{x^2}$

Solution

a. 14

b. 66

c. 4

d. $\frac{1}{16}$

e. 17

f. 1

5. **Problem 5 Statement**

Find $(12.34) \times (0.7)$. Show your reasoning.

Solution

8.638. Sample reasoning: $1\,234 \times 7 = 8\,638$ Because 12.34 is $\frac{1}{100}$ of 1 234 and 0.7 is $\frac{1}{10}$ of 7, the product 8 638 needs to be multiplied by $(\frac{1}{100} \times \frac{1}{10})$ or $\frac{1}{1\,000}$.

6. **Problem 6 Statement**

For each expression, write another division expression that has the same value and that can be used to help find the quotient. Then, find each quotient.

a. $302.1 \div 0.5$

b. $12.15 \div 0.02$

c. $1.375 \div 0.11$

Solution

Answers vary. Sample response:

a. $3,021 \div 5$. The quotient is 604.2.

b. $1,215 \div 2$. The quotient is 607.5.

c. $137.5 \div 11$. The quotient is 12.5