Lesson 12: Percentages and bar models

Goals

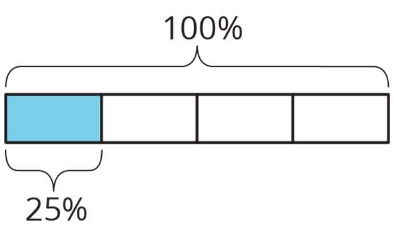
- Choose and create diagrams to solve problems such as A% of B is ? and A% of ? is C.
- Draw and label a bar model to represent a situation involving percentages.
- Interpret bar models that represent multiplicative comparisons and express such comparisons using fractions and percentages.

Learning Targets

- I can use bar models to solve different problems like “What is 40% of 60?” or “60 is 40% of what number?”

Lesson Narrative

In previous lessons students used double number lines to reason about percentages. Double number lines show different percentages when a given amount is identified as 100%, and emphasise that percentages are a rate per 100. In this lesson they use bar models. Bar models are useful for seeing the connection between percentages and fractions. For example, this bar model shows that 25% of a whole is the same as $\frac{1}{4}$ of that whole by showing that 25% of the whole is one part when 100% of the whole is divided into four equal parts.



Bar models are also useful in solving problems of the form A is B% of C when you are given two of the numbers and must find the third. When reasoning about percentages, it is important to indicate the whole as 100%, just as it is important to indicate the whole when working with fractions.

Addressing

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percentage.

Building Towards

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percentage.

Instructional Routines

- Stronger and Clearer Each Time
- Clarify, Critique, Correct
- Discussion Supports
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let's use bar models to understand percentages.

12.1 Notice and Wonder: Bar models

Warm Up: 5 minutes

The purpose of this warm-up is to elicit the idea that bar models can be used to think about fractions of a whole as percentages of the whole, which will be useful when students interpret and draw bar models in a later activity. While students may notice and wonder many things about these images, the important discussion points are that there are two rectangles of the same length, one of the rectangles is divided into four pieces of equal length, and a percentage is indicated.

Instructional Routines

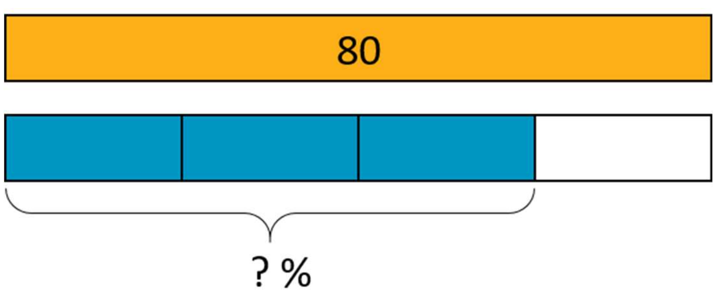
- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- It looks like a bar model.
- There are two rectangles that are the same length.
- One rectangle is yellow and the other is blue and white.
- One of the rectangles is divided into four pieces of equal length.
- One of the rectangles is labelled 80.

Things students may wonder:

- What do bar models have to do with percentages?
- What do the different colours mean?
- What situation does this represent?
- What percentage should be used in place of the question mark?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the four pieces of equal length do not come up during the conversation, ask students to discuss this idea. It is not necessary to decide what should be used in place of the question mark.

12.2 Revisiting Jada's Puppy

15 minutes

The purpose of this activity is for students to study and make sense of bar models that can be used to see benchmark percentages in terms of fractions. The first question shows a percentage as a part of the whole, and the second shows a comparison between two quantities. Both situations can be described in terms of fractions or percentages. The second situation is important for making connections between percentages greater than 100 and fractions greater than 1.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

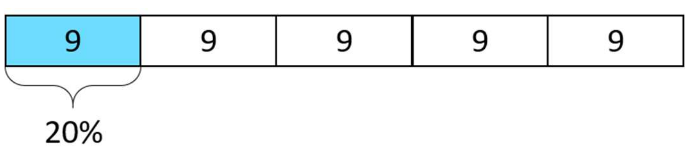
Give students 1 minute of quiet think time, and then have them turn to a partner to discuss the first question. Poll the class to be sure that everyone can see that the puppy is $\frac{1}{5}$ of its adult weight. Ask the students what 100% is in this situation, and label the diagram with 100%. Give students 1 minute of quiet think time, and then have them discuss the second question with a partner.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. Check in with students after the first 2-3 minutes of work time. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organisation; Attention Writing, Speaking, Listening: Stronger and Clearer Each Time. After students have had the opportunity to think about the first question, ask students to write a brief explanation for how the puppy's current weight as a fraction of its adult weight is represented on the bar model. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "Can you explain how…", "You should expand on...", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other ways to interpret the bar model.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Student Task Statement

Jada has a new puppy that weighs 9 pounds. It is now at about 20% of its adult weight.

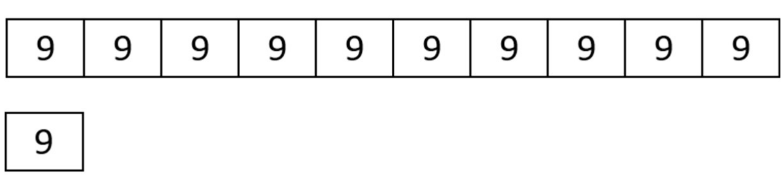
1. Here is a diagram that Jada drew about the weight of her puppy.



   a. The adult weight of the puppy will be 45 pounds. How can you see that in the diagram?

   b. What fraction of its adult weight is the puppy now? How can you see that in the diagram?

2. Jada's friend has a dog that weighs 90 pounds. Here is a diagram Jada drew that represents the weight of her friend's dog and the weight of her puppy.



a. How many times greater is the dog’s weight than the puppy’s?

b. Compare the weight of the puppy and the dog using fractions.

c. Compare the weight of the puppy and the dog using percentages.

Student Response

1. For the diagram showing 20%.

   a. The adult weight will be 45 pounds because there are 5 9's and $5 \times 9 = 45$.

   b. The puppy is $\frac{1}{5}$ of its adult weight because the whole is divided into 5 pieces of equal length.

2. For the diagram showing ten 9's compared with one 9.

   a. The dog's weight is 10 times greater than the weight of the puppy.

   b. The puppy's weight is $\frac{1}{10}$ the weight of the dog.

   c. The puppy's weight is 10% the weight of the dog.

Activity Synthesis

Display the second diagram for all to see.

9	9	9	9	9	9	9	9	9	9

9

Label the dog’s weight with a 1, and ask the students how we should label the puppy's weight if we are comparing using fractions.

9	9	9	9	9	9	9	9	9	9

9

Display another copy of the second diagram. Ask students, “When we compare the puppy’s weight to the dog’s weight, what represents 100%? How should we label the diagram to show it? What should we label the puppy’s weight?”

12.3 5 Pounds

15 minutes

The purpose of this activity is for students to describe multiplicative comparison problems given in terms of percentages using fractions.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Give students 3 minutes of quiet work time. Have them turn to a partner to discuss their answer to the first question. Then give them 3 minutes of quiet think time for the second question, followed by a whole-class discussion.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations in a problem. For example, ask students to use the same colour to represent the amount of money Elena has compared to Noah in a bar model, written as a value, and represented as a fraction.
Supports accessibility for: Visual-spatial processing Speaking: Discussion Supports. After students have had enough time to work on the first question, and to share their bar models for the first question with a partner, bring the whole class back together. During the discussion, press for details in students' explanations by asking where they see Elena's £2 and Noah's £5 represented in the diagram. Use a visual display of the bar models to annotate (or mark) student responses. Since Elena has 40% or $\frac{2}{5}$ as much money as Noah, ask students where they see 40% or $\frac{2}{5}$ represented in the bar model. As an additional challenge, since Noah has 250% or $\frac{5}{2}$ as much money as Elena, ask students where they see 250% or $\frac{5}{2}$ represented in the bar model. This will help students make sense of bar models and see the relationship between percentages and fractions.
Design Principle(s): Support sense-making

Student Task Statement

Noah has £5.

a. Elena has 40% as much as Noah. How much does Elena have?

b. Compare Elena's and Noah's money using fractions. Draw a diagram to illustrate.

a. Diego has 150% as much as Noah. How much does Diego have?

b. Compare Diego's and Noah's money using fractions. Draw a diagram to illustrate.

Student Response

a. Elena has £2.

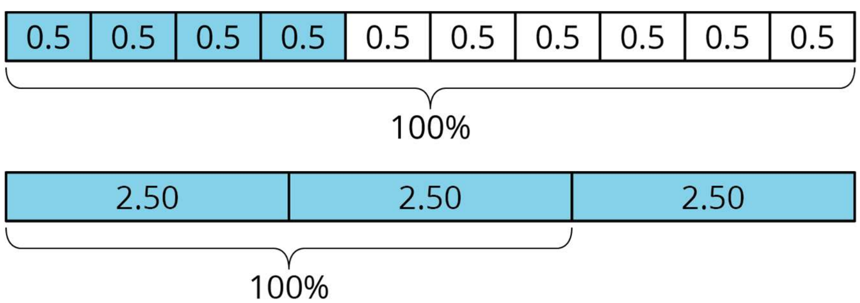
b. Answers vary. Sample responses: Elena has $\frac{2}{5}$ as much money as Noah; Noah has $\frac{5}{2}$ as much money as Elena.

a. Diego has £7.50.

b. Answers vary. Sample responses: Diego has $1\frac{1}{2}$ as much money as Noah; Noah has $\frac{2}{3}$ as much money as Diego.

Activity Synthesis

Have students show and explain their diagrams. Then show these if no one has something equivalent:



12.4 Staying Hydrated

Optional: 10 minutes

In this activity, students explore percentages that describe parts of a whole. They find both B and C, where $A\%$ of B is C, in the context of available and consumed water on a hike.

Students who use a double number line may notice that the value of B is the same in both questions, so the same double number line can be used to solve both parts of the problem. To solve the second question, however, the diagram needs to be partitioned with more tick marks.

As in the previous task, students may solve using other strategies, including by simply multiplying or dividing, i.e., $(1.5) \times 2 = 3.0$ and $\frac{80}{100} \times (3.0) = 2.4$. Encourage them to also explain their reasoning with a double number line, table, or bar model. Monitor for at least one student using each of these representations.

Instructional Routines

- Clarify, Critique, Correct

- Think Pair Share

Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

After students create a double number line diagram with tick marks at 50% and 100%, some may struggle to know how to fit 80% in between. Encourage them to draw and label tick marks at 10% increments or work with a table instead. Some students may think that the second question is asking for the amount of water Andre drank on the second part of the hike. Clarify that it is asking for his total water consumption on the entire hike.

Student Task Statement

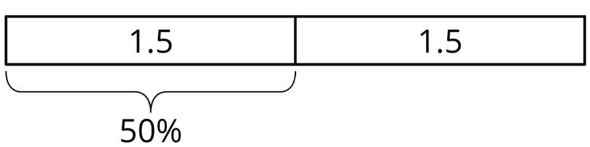
During the first part of a hike, Andre drank 1.5 litres of the water he brought.

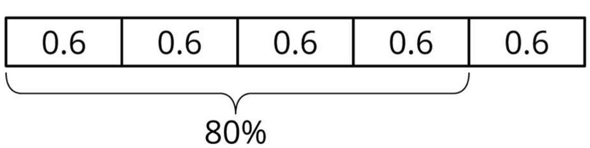
1. If this is 50% of the water he brought, how much water did he bring?
2. If he drank 80% of his water on his entire hike, how much did he drink?

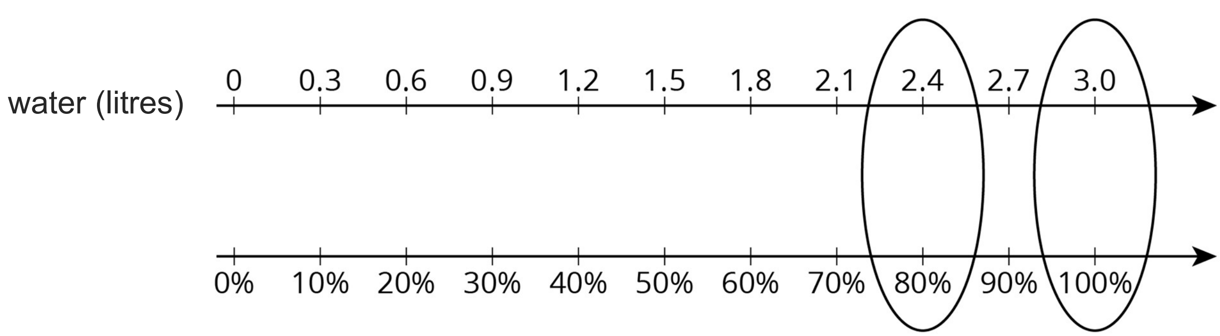
Student Response

1. Andre brought 3 litres of water on the hike.
2. Andre drank 2.4 litres of water.

Possible strategies:







water (litres)	percentage
1.5	50
3.0	100
0.6	20
2.4	80

Are You Ready for More?

Decide if each scenario is possible.

1. Andre plans to bring his dog on his next hike, along with 150% as much water as he brought on this hike.
2. Andre plans to drink 150% of the water he brought on his hike.

Student Response

1. This is possible because it means he will bring $1\frac{1}{2}$ times as much water next time.
2. This is not possible, because it means he will drink more water than he brought. He can only do so if he drinks someone else's water!

Activity Synthesis

Select 1–2 students who used a double number line, a table of equivalent ratios, and a bar model to share their strategies. As students explain, illustrate and display those representations for all to see.

Ask students how they knew what 100% means in the context. At this point it is not necessary for students to formally conceptualise the two ways percentages are used (to describe parts of whole, and to describe comparative relationships). Drawing their attention to concrete and contextualised examples of both, however, serves to build this understanding intuitively.

Reading, Writing, Speaking: Clarify, Critique, Correct. Before students share their answers for the second question, present an incorrect answer and explanation. For example, "If Andre drank 80% of his water on his entire hike, then he drank 1.2 litres of water because 0.8 times 1.5 litres is 1.2 litres." Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss in partners, listen for students who identify and clarify the assumption in the statement. For example, the author assumed that Andre brought 1.5 litres for his entire hike; however, the problem states that Andre drank 1.5 litres of the water he brought. This will remind students to carefully identify the

quantity they are finding a percentage of.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Lesson Synthesis

If you are comparing two quantities using percentages, you can also compare them using fractions. Drawing a bar model can sometimes help us see how to do this more easily.

Questions for discussion:

- If you have 50% of the money needed to buy a book, what fraction is that?
- If you run 125% of your goal for the week, what fraction is that?

Seeing percentages in terms of fractions can help us solve percentage problems.

12.5 Small and Large

Cool Down: 5 minutes

Student Task Statement

Complete the statement with a situation and a unit of your choice. Then answer the question and draw a diagram.

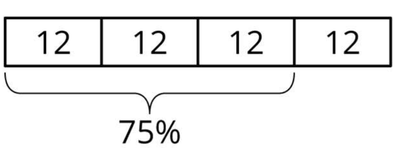
A small ______________ holds 75% as much as a large ______________.

1. If the small holds 36 units, how much does the large hold?
2. Draw a diagram to illustrate your answer.

Student Response

Answers vary. Sample response. A small scoop holds 75% as much as a large scoop.

1. 48 units
2. Answers vary. Sample diagrams:

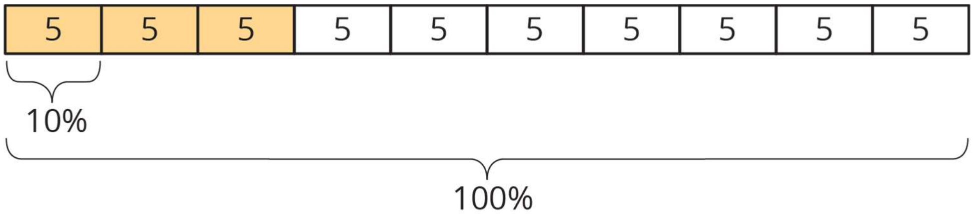


Student Lesson Summary

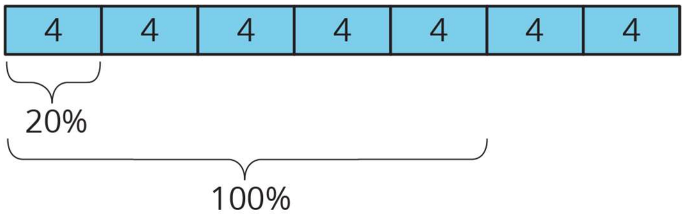
Bar models can help us make sense of percentages.

Consider two problems that we solved earlier using double number lines and tables: “What is 30% of 50 pounds?” and “What is 100% of a number if 140% of it is 28?”

Here is a bar model that shows that 30% of 50 pounds is 15 pounds.



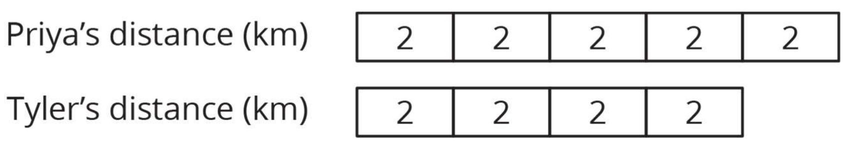
This diagram shows that if 140% of some number is 28, then that number must be 20.



Lesson 12 Practice Problems

Problem 1 Statement

Here is a bar model that shows how far two students walked.



a. What percentage of Priya's distance did Tyler walk?

b. What percentage of Tyler's distance did Priya walk?

Solution

a. 80%

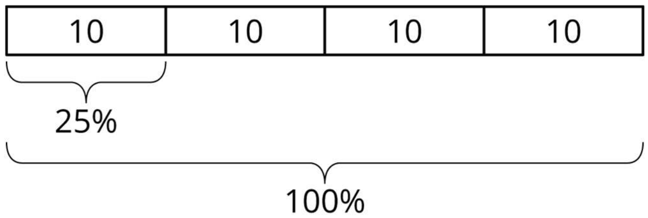
b. 125%

Problem 2 Statement

A bakery makes 40 different flavours of muffins. 25% of the flavours have chocolate as one of the ingredients. Draw a bar model to show how many flavours have chocolate and how many don't.

Solution

Each unit in the bar model represents 25%, so 10 have chocolate and 30 do not.



Problem 3 Statement

There are 70 students in the school band. 40% of them are year 7s, 20% are year 8s, and the rest are year 9s.

a. How many band members are in year 7?

b. How many band members are in year 8?

c. What percentage of the band members are in year 9? Explain your reasoning.

Solution

a. 28 ($70 \times 0.4 = 28$)

b. 14 ($70 \times 0.2 = 14$)

c. 40% because the other percentages add up to 60% and that leaves 40%, because $100 - 60 = 40$.

Problem 4 Statement

Jada has a monthly budget for her mobile phone bill. Last month she spent 120% of her budget, and the bill was £60. What is Jada’s monthly budget? Explain or show your reasoning.

Solution

£50. Strategies vary. Sample reasoning: If 120% is 60, then 20% is 10, which I get by multiplying each by $\frac{1}{6}$. If 20% is 10, then 100% is 50, which I get by multiplying each by 5.

Problem 5 Statement

Which is a better deal, 5 tickets for £12.50 or 8 tickets for £20.16? Explain your reasoning.

Solution

5 tickets for £12.50 is a better deal. 5 tickets for £12.50 equals a unit rate of £2.50 per ticket, ($12.50 \div 5 = 2.50$), and 8 tickets for £20.16 equals a unit rate of £2.52 per ticket, ($12.50 \div 8 = 2.52$).

Problem 6 Statement

An athlete runs 8 miles in 50 minutes on a treadmill. At this rate:

a. How long will it take the athlete to run 9 miles?

b. How far can the athlete run in 1 hour?

Solution

a. 56.25 minutes (or equivalent)

b. 9.6 miles (or equivalent)