Mathematics Education Innovation

Lesson 19: Expanding and factorising

Goals

- Apply the distributive property to expand or factorise an expression that includes negative coefficients, and explain (orally and using other representations) the reasoning.
- Comprehend the terms “expand” and “factorise” (in spoken and written language) in relation to the distributive property.

Learning Targets

- I can organise my work when I use the distributive property.
- I can use the distributive property to rewrite expressions with positive and negative numbers.
- I understand that factorising and expanding are words used to describe using the distributive property to write equivalent expressions.

Lesson Narrative

Earlier in KS3, students worked extensively with the distributive property involving both addition and subtraction, but only with positive coefficients. In the previous lesson, students learned to rewrite subtraction as "adding the opposite" to avoid common pitfalls. In this lesson, students practise using the distributive property to write equivalent expressions when there are rational coefficients. Some of the expressions they will work with are in preparation for understanding combining like terms in terms of the distributive property, coming up in the next lesson. (For example, $17a - 13a$ can be rewritten $a(17 - 13)$ using the distributive property, so it is equivalent to $a \times 4$ or $4a$.

Building On

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Addressing

- Apply properties of operations as strategies to add, subtract, factorise, and expand linear expressions with rational coefficients.

Building Towards

- Apply properties of operations as strategies to add, subtract, factorise, and expand linear expressions with rational coefficients.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports

- Number Talk

Student Learning Goals

Let's use the distributive property to write expressions in different ways.

19.1 Number Talk: Brackets

Warm Up: 10 minutes

The purpose of this number talk is to remind students that when we evaluate expressions, we multiply before we add or subtract. Brackets are used to indicate that the order should be different. Remembering how this works will be important for an activity in this lesson.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find the value of each expression mentally.

$2 + 3 \times 4$

$(2 + 3)(4)$

$2 - 3 \times 4$

$2 - (3 + 4)$

Student Response

Strategies vary. Possible responses:

- 14, because absent brackets, I know to evaluate multiplication before addition. So $2 + 3 \times 4 = 2 + 12 = 14$.
- 20, because brackets indicate their contents should be evaluated first, and next-to means multiply. So $(2 + 3)(4) = 5 \times 4 = 20$

- -10, because multiply before subtract, so $2 - 3 \times 4 = 2 - 12 =$ -10.
- -5, because brackets first, so $2 - (3 + 4) = 2 - 7 =$ -5.

Activity Synthesis

The second question is an opportunity to remind students that "next to" implies "multiply." The second expression could be rewritten $(2 + 3) \times 4$. Point out that we can also know that the second expression is 20 by using the distributive property: $2 \times 4 + 3 \times 4$.

The fourth expression could also be rewritten $2 - 3 - 4$.

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

19.2 Factorising and Expanding with Negative Numbers

20 minutes

This activity is an opportunity for students to practise rewriting expressions using the distributive property. It is a step up from the same type of work earlier in KS3 because arithmetic with directed numbers is required.

The row with $6a - 2b$ is designed to allow students to figure out how to factorise with reasoning based on structure they already understand, instead of learning how to factorise based on a procedure that a teacher demonstrates first. The rows with $k(4 - 17)$ and $10a - 13a$ are designed to prepare students for combining like terms in the next lesson.

Instructional Routines

- Clarify, Critique, Correct

Launch

Draw students' attention to the organisers that appear above the table, and tell them that these correspond to the first three rows in the table. Let them know that they are encouraged to draw more organisers like this for other rows as needed.

Arrange students in groups of 2. Instruct them to take turns writing an equivalent expression for each row. One partner writes the equivalent expression and explains their reasoning while the other listens. If the partner disagrees, they work to resolve the discrepancy before moving to the next row.

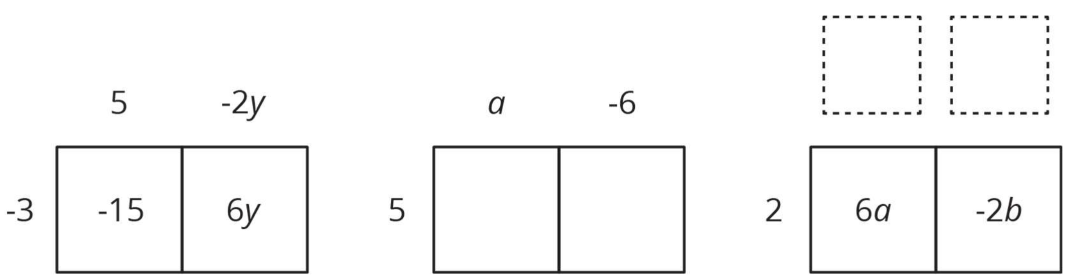
Action and Expression: Internalise Executive Functions. Provide students with blank diagrams to organise their work for the factorised and expanded expressions.
Include example diagrams labelled "expanded" and "factorised."
Supports accessibility for: Language; Organisation

Anticipated Misconceptions

If students are unsure how to proceed, remind them of tools and understandings they have seen recently that would be helpful. For example, "Draw an organiser and think about how the organiser represents terms in the expression." Also, "Rewrite subtraction as adding the opposite."

Student Task Statement

In each row, write the equivalent expression. If you get stuck, use a diagram to organise your work. The first row is provided as an example. Diagrams are provided for the first three rows.



factorised	expanded
-3$(5-2y)$	-15 $+ 6y$
$5(a-6)$	
	$6a-2b$
-4$(2w-5z)$	
-$(2x-3y)$	
	$20x-10y+15z$

$k(4-17)$	
	$10a-13a$
-$2x(3y-z)$	
	$ab-bc-3bd$
-$x(3y-z+4w)$	

Student Response

Expressions equivalent to these are also acceptable. For example, instead of $a(10-13)$, one could write $(10-13)\times a$ or -$3a$.

factorised	expanded
-$3(5-2y)$	-$15+6y$
$5(a-6)$	$5a-30$
$2(3a-b)$	$6a-2b$
-$4(2w-5z)$	-$8w+20z$
-$(2x-3y)$	-$2x+3y$
$5(4x-2y+3z)$	$20x-10y+15z$
$k(4-17)$	$4k-17k$
$a(10-13)$	$10a-13a$
-$2x(3y-z)$	-$6xy+2xz$
$b(a-c-3d)$	$ab-bc-3bd$
-$x(3y-z+4w)$	-$3xy+xz-4xw$

Are You Ready for More?

Expand to create an equivalent expression that uses the fewest number of terms:

$$\left(\left(\left((x+1)\frac{1}{2}\right)+1\right)\frac{1}{2}\right)+1.$$

If we wrote a new expression following the same pattern so that there were 20 sets of brackets, how could it be expanded into an equivalent expression that uses the fewest number of terms?

Student Response

$\frac{1}{4}(x+7)$, $\frac{1}{2^{10}}(x+2^{11}-1)$

Activity Synthesis

Much of the discussion will take place in small groups. Display the correct equivalent expressions and work to resolve any discrepancies. Expanding the term -$(2x-3y)$ may require particular care. One way to interpret it is to rewrite as -1 × $(2x-3y)$. If any confusion about handling subtraction arises, encourage students to employ the strategy of rewriting subtraction as adding the opposite.

To wrap up the activity, ask:

- "Which rows did you and your partner disagree about? How did you resolve the disagreement?"
- "Which rows are you the most unsure about?"
- "Describe a process or procedure for taking a factorised expression and writing its corresponding expanded expression."
- "Describe a process or procedure for taking an expanded expression and writing its corresponding factorised expression."

Writing: Clarify, Critique, Correct. Present an incorrect response for one of the expanded expressions in the table. For example, "An equivalent expression for $6a-2b$ is $4(a-b)$ because $6-2$ is 4 and $(a-b)$ is left on its own." Prompt students to clarify any of the language and reasoning in the incorrect response and then to identify the error(s). Invite students to work with a partner to write a correct response. This helps students evaluate and improve on the written mathematical arguments of others.
Design Principle(s): Maximise meta-awareness

Lesson Synthesis

- To write an equivalent expression by factorising means to use the distributive property to write a sum as a product.
- To write an equivalent expression by expanding means to use the distributive property to write a product as a sum.

Ask students to give an example of each.

19.3 Equivalent Expressions

Cool Down: 5 minutes

Student Task Statement

1. Expand to write an equivalent expression: -$\frac{1}{2}$(-$2x+4y$)

2. Factorise to write an equivalent expression: $26a - 10$

If you get stuck, use a diagram to organise your work.

Student Response

1. $x - 2y$

2. $2(13a - 5)$

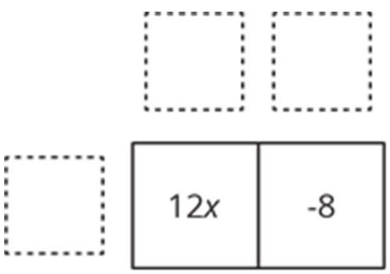
Expressions equivalent to these are also acceptable, like $(13a - 5) \times 2$.

Student Lesson Summary

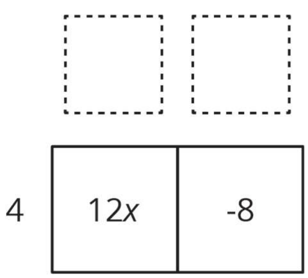
We can use properties of operations in different ways to rewrite expressions and create equivalent expressions. We have already seen that we can use the distributive property to **expand** an expression, for example $3(x + 5) = 3x + 15$. We can also use the distributive property in the other direction and **factorise** an expression, for example $8x + 12 = 4(2x + 3)$.

We can organise the work of using the distributive property to rewrite the expression $12x - 8$. In this case we know the product and need to find the factors.

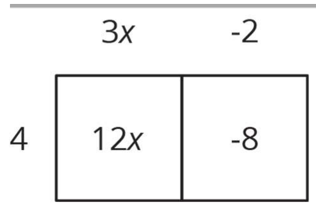
The terms of the product go inside:



We look at the expressions and think about a factor they have in common. $12x$ and -8 each have a factor of 4. We place the common factor on one side of the large rectangle:



Now we think: "4 times *what* is $12x$?" "4 times *what* is -8?" and write the other factors on the other side of the rectangle:



So, $12x - 8$ is equivalent to $4(3x - 2)$.

Glossary

- expand
- factorise (an expression)

Lesson 19 Practice Problems

1. **Problem 1 Statement**

    a. Expand to write an equivalent expression: $-\frac{1}{4}(\text{-}8x + 12y)$

    b. Factorise to write an equivalent expression: $36a - 16$

    Solution

    a. $2x - 3y$

    b. $4(9a - 4)$ (or $2(18a - 8)$)

2. **Problem 2 Statement**

    Lin missed her maths lesson on the day they worked on expanding and factorising. Kiran is helping Lin catch up.

    a. Lin understands that expanding is using the distributive property, but she doesn't understand what factorising is or why it works. How can Kiran explain factorising to Lin?

    b. Lin asks Kiran how the diagrams with boxes help with factorising. What should Kiran tell Lin about the boxes?

    c. Lin asks Kiran to help her factorise the expression $\text{-}4xy - 12xz + 20xw$. How can Kiran use this example to Lin understand factorising?

    Solution

    a. Answers vary. Sample response: Factorising is the distributive property in the other direction. Instead of expanding a product to a sum of terms, factorising takes a sum of terms and makes it into a product by looking for common factors in the terms that can be written outside the brackets.

b. Answers vary. Sample response: The expression in each box is the product of {the expression to the left of the big rectangle} and {the expression above the box}, just as the area of a rectangle is length times width. Together, the boxes form a long rectangle, so it is still true that {the expression to the left of the box} times {the expression above the long rectangle} equals the sum of all the terms in the boxes. If you want to factorise an expression, look for a common factor in each box, and place it to the left of the rectangle. To decide what to write above each box, think, "What times that common factor equals what is in the box?"

c. Answers vary and should describe the box or steps. Sample response: First, find the common factor, which is $4x$. Write "$4x(\dots)$." We are going to decide what needs to go in the brackets to make an expression equivalent to $-4xy - 12xz + 20xw$. To get $-4xy$, we need to multiply by $-y$. Using similar reasoning, we can fill in the rest: $4x(-y - 3z + 5w)$.

3. **Problem 3 Statement**

Complete the equation with numbers that makes the expression on the right side of the equals sign equivalent to the expression on the left side.

$75a + 25b = \_(\_a + b)$

Solution

$25(3a + b)$

4. **Problem 4 Statement**

Elena makes her favourite shade of purple paint by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ of a cup of white paint. Elena has $\frac{2}{3}$ of a cup of white paint.

a. Assuming she has enough red paint and blue paint, how much purple paint can Elena make?

b. How much blue paint and red paint will Elena need to use with the $\frac{2}{3}$ of a cup of white paint?

Solution

a. $\frac{20}{3}$ cups. One batch of purple paint makes 5 cups. Elena can make $\frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$ batches so that's $\frac{20}{3}$ cups.

b. 4 cups of blue paint and 2 cups of red paint.

5.

6. **Problem 5 Statement**

   Solve each equation.

   a. $\frac{-1}{8}d - 4 = \frac{-3}{8}$

   b. $\frac{-1}{4}m + 5 = 16$

   c. $10b + -45 = -43$

   d. $-8(y - 1.25) = 4$

   e. $3.2(s + 10) = 32$

   Solution

   a. $d = -29$

   b. $m = -44$

   c. $b = \frac{1}{5}$ (or equivalent)

   d. $y = 0.75$ (or equivalent)

   e. $s = 0$

7. **Problem 6 Statement**

   Select **all** the inequalities that have the same solutions as $-4x < 20$.

   a. $-x < 5$

   b. $4x > -20$

   c. $4x < -20$

   d. $x < -5$

   e. $x > 5$

   f. $x > -5$

   Solution ["A", "B", "F"]