Grades 9-12 (AS)

Duration: 30 min

Tools: one Logifaces Set / 2-3 pairs or 4-6 students

Individual / Pair work

Keywords: Cosine, Formula, Proof

537 - Ratio of Heights

MATHS / TRIGONOMETRY

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

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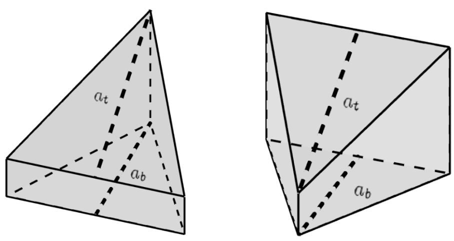
DESCRIPTION

In the 9 pcs or 16 pcs Set students choose those blocks that have two vertical edges with the same height and one with different height. These are blocks 112, 113, 122, 133, 223 and 233.
They denote the altitude of the base triangle by a_1 and the altitude of the top triangle starting from the vertex of the different height by a_t. The following connection holds between the angle of the planes of the top and base triangles (α) and the altitudes a_b and a_t: $cos(\alpha) = \frac{a_b}{a_t}$.



LEVEL 1 Students use this formula to complete the table below. The two altitudes can be measured or calculated (see exercise 411 - Area of Triangles for the calculated values), the angle in the last column (with grey background) can be found using the formula above .

Block	a_b	a_t	α
112			
113			
122			
133			
223			
233			

LEVEL 2 Students prove the formula $cos(\alpha) = \frac{a_b}{a_t}$.

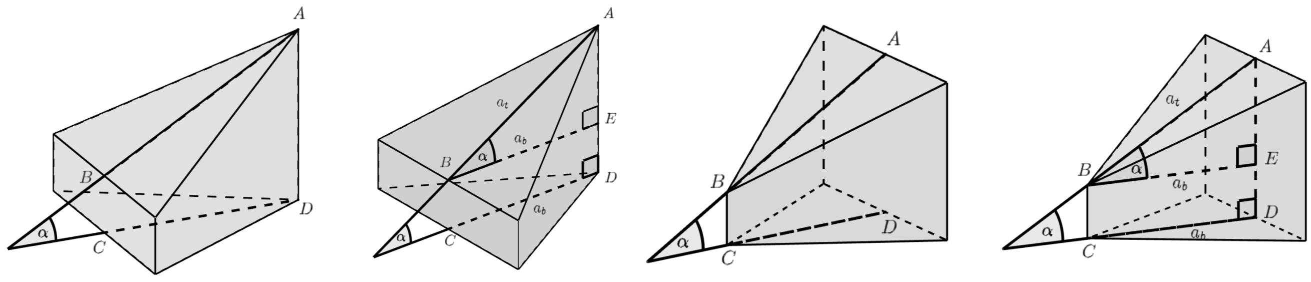
Hint: Use the fact that both triangles have an edge that is parallel to the common line of the two planes and the heights a_b and a_t are perpendicular to that edge. In fact, the proof works for any triangle with this property.

SOLUTIONS / EXAMPLES

LEVEL 1 The solutions are given in standard units, but can also be calculated using the real lengths. The results in the last column of the table are the same in both cases.

Block	a_b	a_t	α
112	$2\sqrt{3}$	$\sqrt{13}$	$\alpha \approx 16°$
113	$2\sqrt{3}$	4	$\alpha = 30°$
122	$2\sqrt{3}$	$\sqrt{13}$	$\alpha \approx 16°$
133	$2\sqrt{3}$	4	$\alpha = 30°$
223	$2\sqrt{3}$	$\sqrt{13}$	$\alpha \approx 16°$
233	$2\sqrt{3}$	$\sqrt{13}$	$\alpha \approx 16°$

LEVEL 2 Let $|CD| = a_b$ and $|AB| = a_t$ be the two altitudes given in the exercise, as seen in the figure. First note that the intersection of the planes of the base and top triangles is perpendicular to the lines through AB and CD, hence the angle α in the figure is the angle between the planes.
Let E be a point on the line AD such that BE is parallel to CD. The triangle ABE is a right-angled triangle, with hypotenuse AB of length a_t and angle α adjacent to the leg BE of length a_b. By definition, the cosine of the angle α in the triangle ABE is given by the formula $cos(\alpha) = \frac{a_b}{a_t}$.



PRIOR KNOWLEDGE

Angle between two planes, Altitude of triangles, Trigonometric ratios, Angles in parallel lines

RECOMMENDATIONS / COMMENT

The exercise is suitable for differentiation, as proving the formula is more difficult than applying it.
Exercise 411 - Area of Triangles is recommended before this exercise to calculate the lengths of the altitudes and the areas of the triangles.

Exercise 536 - Different Slopes is recommended before this exercise in order to clarify the concept of the angle between two planes and the difficulty of measuring this angle.
The calculations can be verified using GeoGebra, see exercise 528 - Read the Results in GeoGebra.