Lesson 11: Representing ratios with tables

Goals

- Comprehend the words "row" and "column" (in written and spoken language) as they are used to describe a table of equivalent ratios.
- Explain (orally and in writing) how to find a missing value in a table of equivalent ratios.
- Interpret a table of equivalent ratios that represents different sized batches of a recipe.

Learning Targets

- If I am looking at a table of values, I know where the rows are and where the columns are.
- When I see a table representing a set of equivalent ratios, I can come up with numbers to make a new row.
- When I see a table representing a set of equivalent ratios, I can explain what the numbers mean.

Lesson Narrative

Over the course of this unit, students learn to work with ratios using different representations. They began by using discrete diagrams to represent ratios and to identify equivalent ratios. Later, they reasoned more efficiently about ratios using double number lines. Here, they encounter situations in which using a double number line poses challenges and for which a different representation would be helpful. Students learn to organise a set of equivalent ratios in a **table**, which is a more abstract but also a more flexible tool for solving problems.

Although different representations are encouraged at different points in the unit, allowing students to use any representation that accurately represents a situation and encouraging them to compare the efficiency of different methods will develop their ability to make strategic choices about representations. Whatever choices they make, they should be encouraged to explain how their method works in solving a problem.

Building On

- Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Addressing

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate grid. Use tables to compare ratios.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Compare and Connect
- Think Pair Share

Student Learning Goals

Let's use tables to represent equivalent ratios.

11.1 How Is It Growing?

Warm Up: 10 minutes

This warm-up encourages students to look for regularity in how the tiles in the image are growing. Students may use each colour to reason about the total, while others may reason about the way the tiles in total increase each time. Emphasise both insights as students share their strategies.

Instructional Routines

- Think Pair Share

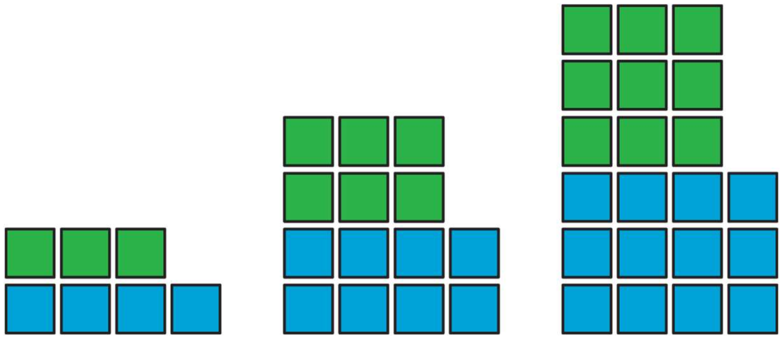
Launch

Arrange students in groups of 2. Display the image for all to see and tell students that the collection of images of green and blue tiles is growing. Ask how many tiles in total will be in the 4th, 5th and 10th image if it keeps growing in the same way. Tell students to give a signal when they have an answer and strategy. Give students 3 minutes of quiet think time, and then time to discuss their responses and reasoning with their partner.

Student Task Statement

Look for a pattern in the diagrams.

1. How many tiles in total will be in:
    a. the 4th diagram?
    b. the 5th diagram?
    c. the 10th diagram?
2. How do you see it growing?



Student Response

1. Answers vary. Sample response: 4th image: 28 tiles, 5th image: 35 tiles, 10th image: 70 tiles
2. Answers vary. Sample response: I see green increasing by 3 each time and blue increasing by 4.

Activity Synthesis

Invite students to share their responses and reasoning. Record and display the different ways of thinking for all to see. If possible, record the relevant reasoning on or near the images themselves. After each explanation, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to what is happening in the images each time.

11.2 A Huge Amount of Sparkling Orange Juice

15 minutes (there is a digital version of this activity)

Here, students are asked to find missing values for significantly scaled-up ratios. The activity serves several purposes:

- To uncover a limitation of a double number line (i.e., that it is not always practical to extend it to find significantly scaled-up equivalent ratios),
- To reinforce the multiplicative reasoning needed to find equivalent ratios (especially in cases when drawing diagrams or skip counting is inefficient), and
- To introduce a table as a way to represent equivalent ratios.

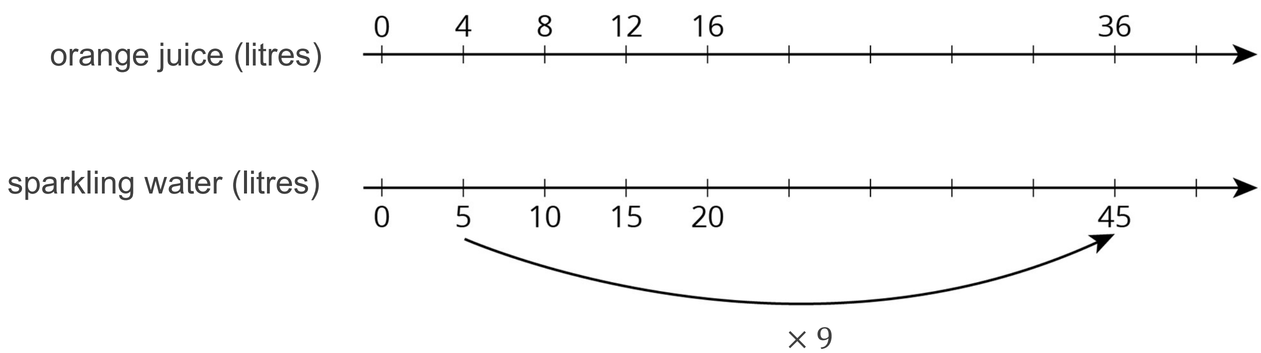
To find equivalent ratios involving large values, some students may simply try to squeeze numbers on the extreme right side of the paper, ignoring the previously equal intervals. Others may use multiplication (or division) and write expressions or equations to capture the given scenarios. Notice students' reasoning processes, especially any struggles with the double number line (e.g., the lines not being long enough, requiring much marking and writing, the numbers being too large, etc.), as these can motivate a need for a more efficient strategy.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Compare and Connect

Launch

Give students 2–3 minutes to work on the first two questions and then ask them to pause. As a class, discuss the two approaches students are likely to take: counting multiples of 4 and 5 up to 36 and 45; and multiplicative reasoning (asking "What number times 4 equals 36?"). Also discuss how a double number line like the one below might be used to support reasoning.



Reiterate the multiplicative relationship between equivalent ratios before students move on.

Representation: Internalise Comprehension. Activate or supply background knowledge about equivalent ratios describing situations that happen at the same rate. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

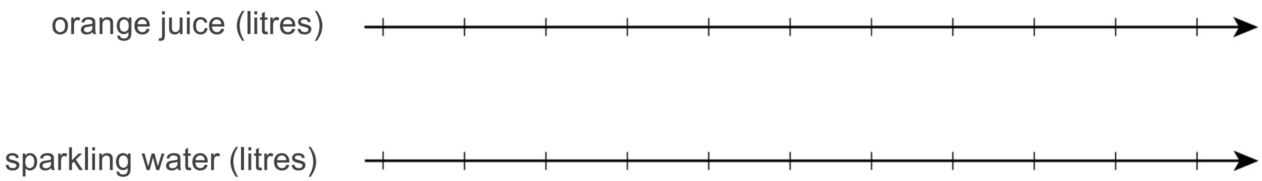
Anticipated Misconceptions

Students may become frustrated when they "run out of number line," but remind them of what they know about how to find ratios equivalent to 4 : 5 (they need to multiply both 4 and 5 by the same number). Consider directing their attention to a definition of equivalent ratios displayed in your room or in a previous lesson, or suggesting they re-examine some of the simpler cases (e.g., the relationship between 4 : 5 and 36 : 45). Be on the lookout for students trying to tape on more paper to extend their number lines.

Student Task Statement

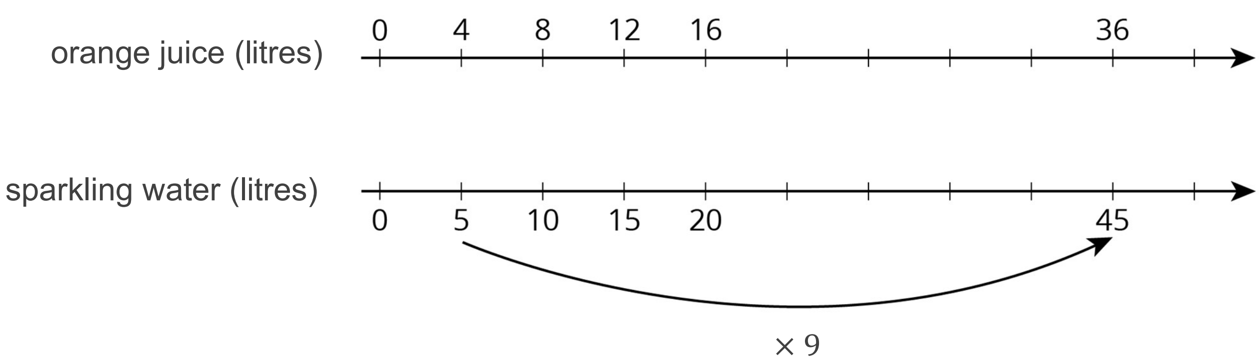
Noah's recipe for one batch of sparkling orange juice uses 4 litres of orange juice and 5 litres of sparkling water.

1. Use the double number line to show how many litres of each ingredient to use for different-sized batches of sparkling orange juice.



2. If someone mixes 36 litres of orange juice and 45 litres of sparkling water, how many batches would they make?
3. If someone uses 400 litres of orange juice, how much sparkling water would they need?
4. If someone uses 455 litres of sparkling water, how much orange juice would they need?
5. Explain the trouble with using a double number line diagram to answer the last two questions.

Student Response



1. Amounts for the first four batch sizes are shown.
2. 9 batches. $4 \times 9 = 36$ and $5 \times 9 = 45$.
3. 500 litres of sparkling water. $4 \times 100 = 400$ and $5 \times 100 = 500$.
4. 364 litres of orange juice. $455 = 5 \times 91$ and $4 \times 91 = 364$.
5. The numbers I needed to find were too big to fit on the number lines.

Activity Synthesis

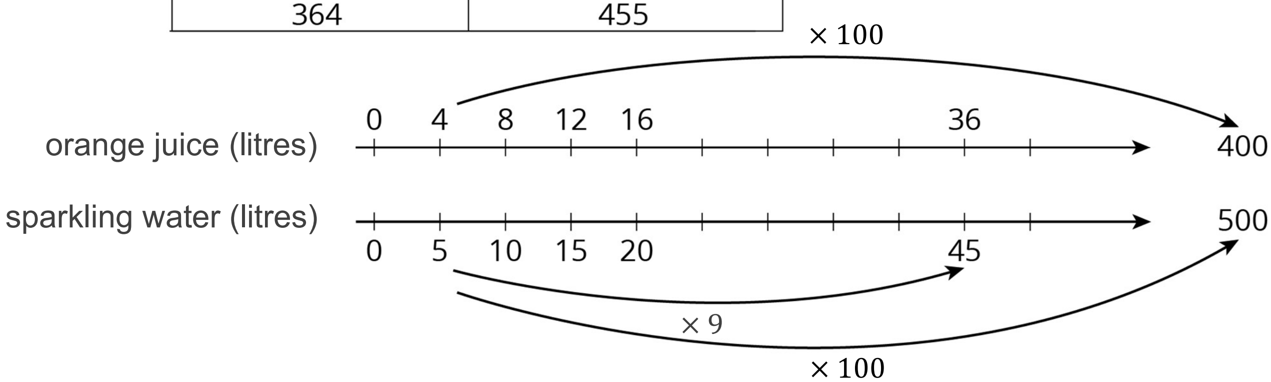
After students have a chance to share with a partner, select a few to share their reasoning with the class for the last few questions. Start with students who tried to extend the double number line (if anyone did so). Discuss any challenges of using the double number line and merits of alternative methods students might have come up with.

Explain that there is a more appropriate tool—a **table**—that can be used to represent equivalent ratios. Display for all to see the double number line from the activity above and a table of equivalent ratios. Explain that even though the table is oriented vertically and the double number line is oriented horizontally, the two representations represent the same ratios. Explain what we mean by **row** and **column** and demonstrate the use of these words. Fill in the table using the values from the orange-sparkling ratios and, along the way, compare and contrast how the two representations work. A few other key insights to convey:

- Just as it was important to label the double number line, it is important to label the columns of the table to indicate what the values represent.
- Each row of a table shows a pair of values from a collection of equivalent ratios. Unlike a number line, distances between values do not matter.
- On each line of a double number line, numbers are shown in order. In each column of a table, order is not important, i.e., pairs of values can be placed in any order that is convenient. When complete, the display should look something like this:

orange juice (litres)	sparkling water (litres)
4	5
8	10
12	15
16	20
36	45
400	500
364	455

$\times 100$ $\times 100$



Representing: Compare and Connect. Use this routine to help students make connections between specific features of tables and double number lines. Ask students to describe to a partner how multiplication appears in each representation, and then invite listeners to restate or revoice what they heard, back to their partner, using mathematical language (e.g., product, row, column, table, equivalent ratio, etc.). After students have a chance to share with a partner, select a few to share their reasoning with the class.
Design Principle(s): Maximise meta-awareness

11.3 Batches of Trail Mix

10 minutes

This task gets students to interact with a table in a way that discourages skip counting. Numbers within each column are deliberately out of order. This is intended to encourage students to multiply the pairs of values from a given ratio by the same number and to emphasise that the order in which pairs of values appear is not a necessary part of the structure of a table. (Order within rows, however, is necessary.) The last question reinforces the definition of equivalent ratios.

Students may use the given values (7 and 5) as the basis for every calculation (e.g., for every row, they think “7 times what . . . ” or “5 times what . . .”). They may also reason with values from another row (e.g., they may see 250 as 10×25 rather than as 5×50). As students work, notice different approaches.

Launch

Explain that a **table** is just a list of equivalent ratios. In this case, one **column** contains amounts of almonds, and the other column contains corresponding amounts of raisins. Each **row** shows the amount of each ingredients in a particular batch.

Reiterate that multiplying both parts of a ratio by the same non-zero number always creates a ratio that is equivalent to the original ratio.

Anticipated Misconceptions

Students may make patterns that do not yield equivalent ratios. For example, they may think “7 minus 2 is 5, so for the next row, 28 minus 2 is 26.” Or they may think “7 plus 21 is 28, so then 5 plus 21 is 26.” If so, consider:

- Appealing to what students know about batches of recipes. “The second row represents how many batches of trail mix?” (4, because 28 is 7×4.) “Okay, so to make 4 batches of trail mix, how will we figure out how many raisins?” (Also multiply the 5 by 4.)
- Refreshing what students learned about equivalent ratios. “We need a ratio that is equivalent to the ratio represented in row 1. So what do we need to do to the 7 and the 5?” (Multiply them by the same number.)

Students may be unsure about how to find the missing value in the row with 3.5. Encourage them to reason about it the same way they reasoned about the other rows. "We need a ratio that is equivalent to the ratio represented in row 1. So what do we need to do to the 7 and the 5?" They may have to get there by way of division. 7 divided by 2 is 3.5, so 7 times $\frac{1}{2}$ is 3.5; this means multiplying 5 by $\frac{1}{2}$ as well.

Student Task Statement

A recipe for trail mix says: "Mix 7 ounces of almonds with 5 ounces of raisins." Here is a **table** that has been started to show how many ounces of almonds and raisins would be in different-sized batches of this trail mix.

almonds (oz)	raisins (oz)
7	5
28	
	10
3.5	
	250
56	

1. Complete the table so that ratios represented by each row are equivalent.
2. What methods did you use to fill in the table?
3. How do you know that each row shows a ratio that is equivalent to 7 : 5? Explain your reasoning.

Student Response

1. Here is the table:

almonds (oz)	raisins (oz)
7	5
28	20
14	10
3.5	2.5
350	250
56	40

2. Answers vary.
3. To find each row, multiply 7 and 5 by the same thing. This means that each row has values of a ratio equivalent to 7 : 5.

Are You Ready for More?

You have created a best-selling recipe for chocolate chip cookies. The ratio of sugar to flour is 2 : 5.

Create a table in which each entry represents amounts of sugar and flour that might be used at the same time in your recipe.

- One entry should have amounts where you have fewer than 25 cups of flour.
- One entry should have amounts where you have between 20–30 cups of sugar.
- One entry can have any amounts using more than 500 units of flour.

Student Response

Answers vary. Sample response:

sugar	flour
2	5
8	20
26	65
240	600

Activity Synthesis

Invite one or more students who used multiplicative approaches to share their reasoning with the class. Consider displaying the table and using it to facilitate gesturing and arrow-drawing while students explain. Highlight the strategy of multiplying the 7 and 5 values by the same number.

Lesson Synthesis

Sometimes it is easier to use a **table** rather than a double number line to represent equivalent ratios. Each **row** contains a ratio that is equivalent to all the other ratios, so if we know one row, we can multiply both of its values by the same number to find another row's values.

11.4 Batches of Cookies in a Table

Cool Down: 5 minutes

Student Task Statement

In previous lessons, we worked with a diagram and a double number line that represented this cookie recipe. Here is a table that represents the same situation.

flour (cups)	vanilla (teaspoons)
5	2
15	6
$2\frac{1}{2}$	1

1. Write a sentence that describes a ratio shown in the table.
2. What does the second row of numbers represent?
3. Complete the last row for a different batch size that hasn't been used so far in the table. Explain or show your reasoning.

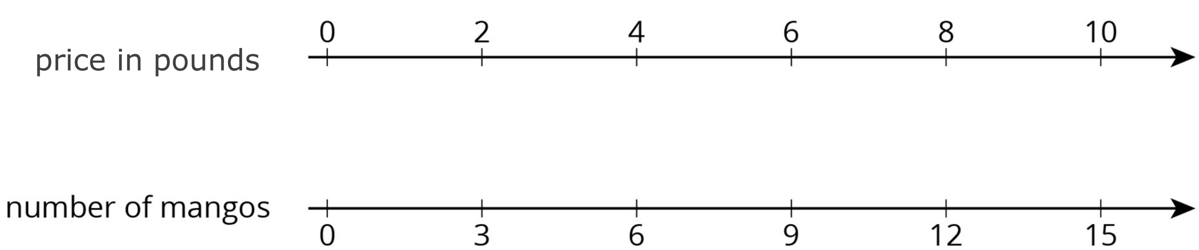
Student Response

1. Answers vary. Sample responses:
   - The ratio of cups of flour to teaspoons of vanilla is $5 : 2$.
   - This recipe uses 5 cups of flour for every 2 teaspoons of vanilla.
   - This recipe uses $2\frac{1}{2}$ cups of flour per teaspoon of vanilla.
2. For 15 cups of flour, you need 6 teaspoons of vanilla.
3. Answers vary. Sample response: 10 cups of flour and 4 teaspoons of vanilla.

Student Lesson Summary

A **table** is a way to organise information. Each horizontal set of entries is called a *row*, and each vertical set of entries is called a *column*. (The table shown has 2 columns and 5 rows.) A table can be used to represent a collection of equivalent ratios.

Here is a double number line diagram and a table that both represent the situation: "The price is £2 for every 3 mangos."



	column	column
	price in pounds	number of mangos
row	2	3
row	4	6
row	6	9
row	8	12
row	10	15

Glossary

- table

Lesson 11 Practice Problems

Problem 1 Statement

Complete the table to show the amounts of yellow and red paint needed for different-sized batches of the same shade of orange paint.

yellow paint (quarts)	red paint (quarts)
5	6

Explain how you know that these amounts of yellow paint and red paint will make the same shade of orange as the mixture in the first row of the table.

Solution

Answers vary. Sample response:

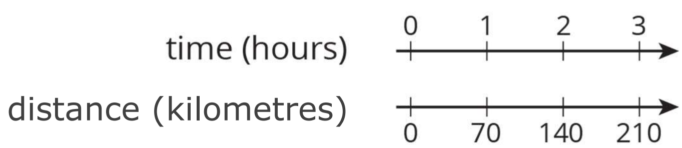
yellow paint (quarts)	red paint (quarts)
5	6

$\frac{5}{4}$	$\frac{3}{2}$ or equivalent
$\frac{5}{2}$	3 or equivalent
$\frac{15}{4}$	$\frac{9}{2}$ or equivalent

Each row is a multiple of the first row.

Problem 2 Statement

A car travels at a constant speed, as shown on the double number line.



How far does the car travel in 14 hours? Explain or show your reasoning.

Solution

980 kilometres. Possible strategy: Make a table because there isn't enough room to continue the double number line that far.

time (hours)	distance (kilometres)
1	70
2	140
3	210
14	980

Problem 3 Statement

The olive trees in an orchard produce 3 000 pounds of olives a year. It takes 20 pounds of olives to make 3 litres of olive oil. How many litres of olive oil can this orchard produce in a year? If you get stuck, consider using the table.

olives (pounds)	olive oil (litres)
20	3
100	
3 000	

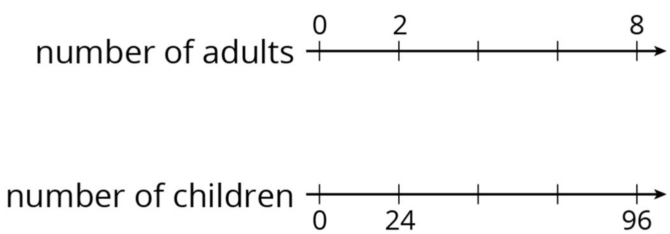
Solution

The orchard produces 450 litres of olive oil per year. Possible strategy:

olives (pounds)	olive oil (litres)
20	3
100	15
3 000	450

Problem 4 Statement

At a school breaktime, there needs to be a ratio of 2 adults for every 24 children on the playground. The double number line represents the number of adults and children on the playground at breaktime.



a. Label each remaining tick mark with its value.

b. How many adults are needed if there are 72 children? Circle your answer on the double number line.

Solution

a. Adults: 0, 2, 4, 6, 8. Children: 0, 24, 48, 72, 96.

b. 6 adults. The portion of the double number line at 6 adults and 72 children is circled.

Problem 5 Statement

While playing basketball, Jada's heart rate goes up to 160 beats per minute. While jogging, her heart beats 25 times in 10 seconds. Assuming her heart beats at a constant rate while jogging, which of these activities resulted in a higher heart rate? Explain your reasoning.

Solution

Playing basketball. Sample explanation: 25 times in 10 seconds means 150 heartbeats per minute ($25 \times 6 = 150$). 150 beats per minute is lower than 160 beats per minute, so Jada's heart rate is lower when she goes jogging than when she plays basketball.

Problem 6 Statement

A shopper bought the following items at the farmer's market:

a. 6 ears of corn for £1.80. What was the cost per ear?

b. 12 apples for £2.88. What was the cost per apple?

c. 5 tomatoes for £3.10. What was the cost per tomato?

Solution

a. £0.30

b. £0.24

c. £0.62