5) $\int_0^{\pi}\int_0^{1-\text{sen}\theta} r^2\cos\theta\, dr\, d\theta$

$$\int_0^{1-\text{sen}\theta} r^2\cos\theta\, dr = \frac{r^3\cos\theta}{3}\Big|_0^{1-\text{sen}\theta}$$

$$\Rightarrow \frac{(1-\text{sen}\theta)^3}{3}\cdot\cos\theta$$

$$\frac{1}{3}\int_0^{\pi}(1-\text{sen}\theta)^3\cdot\cos\theta\, d\theta = -\frac{1}{3}\int u^3\, du =$$

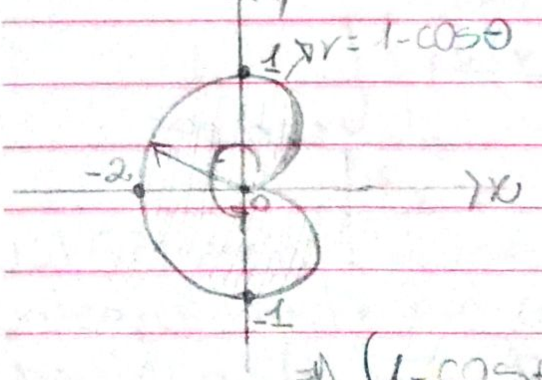
$u = 1-\text{sen}\theta$
$du = -\cos\theta\, d\theta$
$-du = \cos\theta\, d\theta$

$$\Rightarrow -\frac{1}{3}\cdot\frac{u^4}{4} = -\frac{(1-\text{sen}\theta)^4}{12}\Big|_0^{\pi}$$

$$\Rightarrow -\frac{(1-\text{sen}\pi)^4}{12} + \frac{(1-\text{sen}0)^4}{12} = 0$$

7) Use a integral dupla p/ calcular

7. A região compreendida pela ~~cardioide~~ cardioide $r = 1-\cos\theta$.



$$2\cdot\int_0^{\pi}\int_0^{1-\cos\theta} r\, dr\, d\theta$$

$$2\int_0^{\pi}\frac{r^2}{2}\Big|_0^{1-\cos\theta}$$

$$\Rightarrow \frac{(1-\cos\theta)^2}{2} - 0 = \frac{1-\cos^2\theta}{2}$$

$$\cancel{2}\int_0^{\pi}\frac{(1-\cos\theta)^2}{\cancel{2}}\, d\theta = \int_0^{\pi} 1-2\cos\theta+\cos^2\theta\, d\theta$$

$$= \int_0^{\pi} 1 - 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\, d\theta =$$

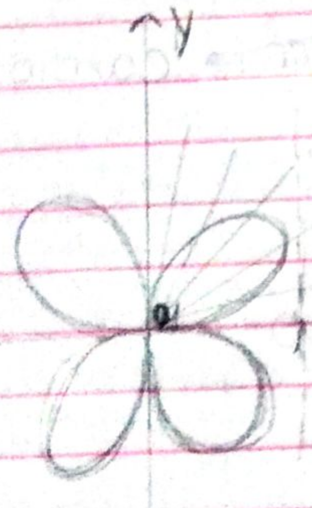
$$\int_0^{\pi} 1 - 2\cos(\theta) + \frac{1}{2} + \frac{\cos(2\theta)}{2}\, d\theta$$

$$\int_0^{\pi} \frac{3}{2} - 2\cos(\theta) + \frac{\cos(2\theta)}{2}\, d\theta$$

$$\frac{3\theta}{2} - 2\,\mathrm{sen}(\theta) + \frac{1}{2}\,\mathrm{sen}(2\theta)\,\frac{1}{2}\Big|_0^{\pi}$$

$$\left(\frac{3\pi}{2} - 2\,\cancel{\mathrm{sen}(\pi)}^{0} + \frac{1}{2}\,\cancel{\mathrm{sen}(2\pi)}\cdot\frac{1}{2}\right) = \frac{3\pi}{2}$$

9) A região do primeiro quadrante limitada por $r=1$ e $r = \mathrm{sen}\,2\theta$, com $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$



$$\int_{\pi/4}^{\pi/2}\int_{1}^{\mathrm{sen}\,2\theta} r\cdot dr\cdot d\theta$$

$$\int_1^{\mathrm{sen}\,2\theta} r\,dr = \frac{r^2}{2}\Big|_1^{\mathrm{sen}\,2\theta}$$

$$\Rightarrow \frac{(\mathrm{sen}\,2\theta)^2}{2} - \frac{1^2}{2} = \frac{\mathrm{sen}^2\theta}{2} - \frac{1}{2}$$

$$\frac{1}{2}\int_{\pi/4}^{\pi/2} \mathrm{sen}^2\theta - \frac{1}{2}\, d\theta =$$

$$\frac{1}{2}\int_{\pi/4}^{\pi/2} \frac{sen^2\theta + 1}{2}\, d\theta$$

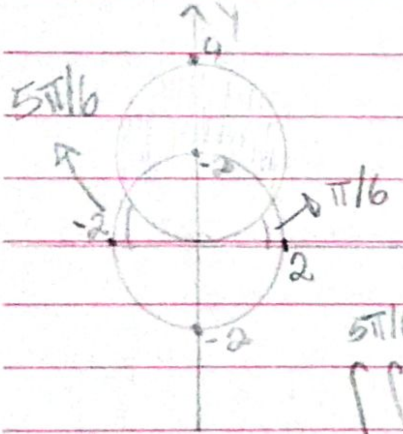
$$\frac{1}{2}\int_{\pi/4}^{\pi/2} \frac{1}{2}\left(\frac{1}{2}\cos 2\theta - \frac{1}{2}\right) d\theta$$

$$\frac{1}{2}\left(\frac{1\theta}{2} - \frac{1}{2} sen\, 2\theta \cdot \frac{1}{2} - \frac{1\theta}{2}\right)\Bigg|_{\pi/4}^{\pi/2}$$

$$-\frac{sen\, 2\theta}{8}\Bigg|_{\pi/4}^{\pi/2} = \frac{\left(-sen\frac{2\pi}{2}\right)}{8} - \frac{\left(-sen\frac{2\pi}{4}\right)}{8} = 0{,}125$$

11-12) Seja R a região descrita. Esboce a região R e preecha os extremos.

11) a região no interior do círculo r=4senθ e fora do círculo r = 2.



$4sen\theta = 2$

$sen\theta = \frac{1}{2}$

$sen\theta = \frac{\pi}{6}$

$$\int_{\pi/6}^{5\pi/6}\int_{2}^{4sen\theta} f(r,\theta)\, r\, dr\, d\theta$$

$z = \sqrt{9 - r^2}$

$r^2 + z^2 = 9$

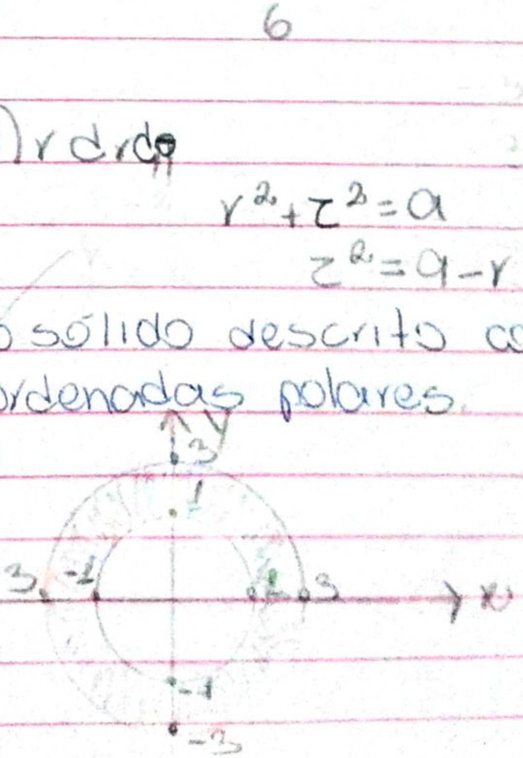
$z^2 = 9 - r^2$

13) Expresse o volume do sólido descrito como uma integral dupla em coordenadas polares.

Dentro de $x^2 + y^2 + z^2 = 9$

Fora de $x^2 + y^2 = 1$

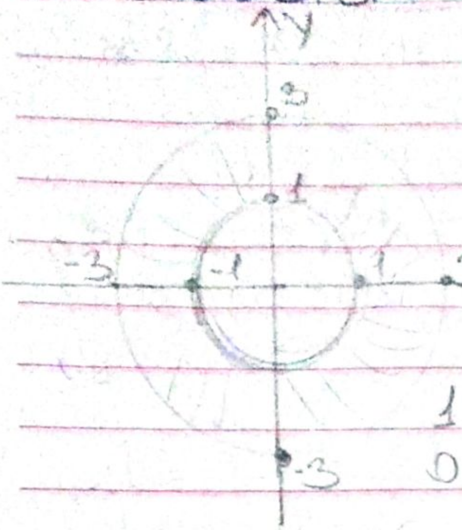
$$V = \int_0^{2\pi}\int_1^3 \sqrt{9 - r^2}\, r\, dr\, d\theta$$



credeal

17-20 Encontre o volume do sólido descrito no exercício indicado.

17) Exercício 13.



Dentro: $x^2+y^2+z^2=9$
fora de: $x^2+y^2=1$

$\to z = +\sqrt{9-r^2}$

$z = -\sqrt{9-r^2}$

$1 \leq r \leq 3$
$0 \leq \theta \leq 2\pi$

$$2\int_0^{2\pi}\int_1^3 \sqrt{9-r^2}\, r\,dr\,d\theta$$

$$\int_1^3 \underbrace{\sqrt{9-r^2}}_{u}\, r\,dr = \int_1^3 \sqrt{u}\,\frac{du}{-2} = -\frac{1}{2}\int_1^3 u^{1/2}\,du$$

$u = 9-r^2$

$du = -2r\,dr$

$\frac{du}{-2} = r\,dr$

$$-\frac{1}{2}\frac{u^{3/2}}{3/2} = \quad r=3 \to u = 9-3^2 = 0$$
$$r=1 \to u = 9-1^2 = 8$$

$$V = 2\int_0^{2\pi}\int_8^0 (u)^{1/2}\left(\frac{du}{-2}\right)$$

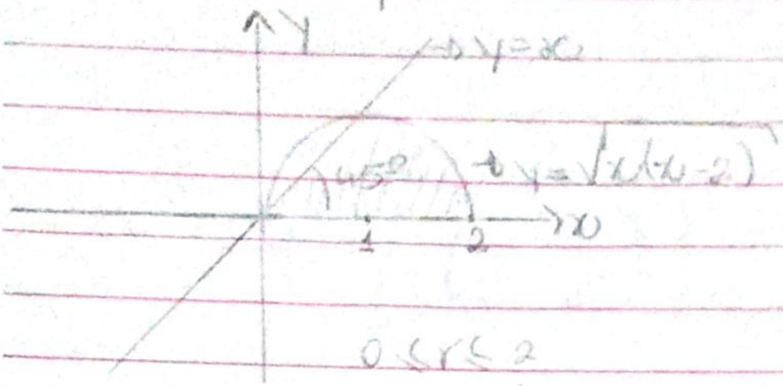
$$2 \cdot 2\pi\left(-\frac{1}{2}\right)\cdot\frac{u^{3/2}}{3/2}\Bigg|_8^0$$

$$= \frac{4\pi}{3}\left(0^{3/2} - 8^{3/2}\right)$$

$$V = \frac{4\pi}{3}\sqrt{8^3} = \frac{4\pi \cdot 8\sqrt{8}}{3} = \frac{32\pi}{3}\sqrt{8}$$

credeal

26) $\iint_R 2y\,dA$, onde R é a região do primeiro quadrante limitada por $(x-1)^2+y^2=1$ e abaixo de $y=x$.



$x=\sqrt{x(x-2)}$

$x^2=x(x-2)$

$x^2=-x^2+2x$

$2x^2-2x=0$

$2x(x-1)=0$

$x=0$ e $x=1$

$\frac{\pi}{4}$

$0\leq r\leq 2$

$0\leq\theta\leq\pi/4$

$$\int_0^{\pi/4}\int_0^2 2\cdot r\,\text{sen}\,\theta\,dr\,d\theta$$

$$2\int_0^2 r\,\text{sen}\,\theta\,dr = \cancel{2}\cdot\frac{r^2}{\cancel{2}}\text{sen}\,\theta\Big|_0^2 = r^2\,\text{sen}\,\theta\Big|_0^2$$

$$\Rightarrow 2^2\,\text{sen}\,\theta - 0^2\,\text{sen}\,\theta = 4\,\text{sen}\,\theta$$

$$\int_0^{\pi/4} 4\,\text{sen}\,\theta\,d\theta = 4(-\cos\theta)\Big|_0^{\pi/4} = -4\cos\left(\frac{\pi}{4}\right)+4\cos(0)$$

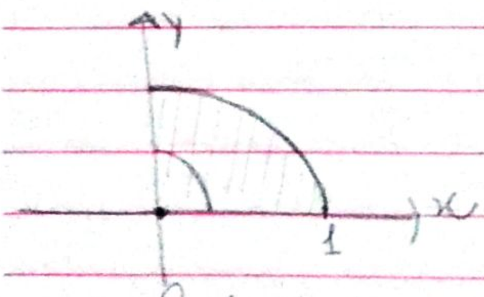
$$= -\frac{\sqrt{2}}{2}+4 = 4-\frac{\sqrt{2}}{2}$$

27) Calcule a integral iterada convertendo p/ coordenadas polares

27. $\int_0^1\int_0^{\sqrt{1-x^2}}\underbrace{(x^2+y^2)}_{r^2}\,dy\,dx$

$0\leq y\leq\sqrt{1-x^2}$

$0\leq x\leq 1$



$0\leq r\leq 1$

$0\leq\theta\leq\pi/2$

$$\int_0^{\pi/2}\int_0^1 r^2\,r\,dr\,d\theta = \int_0^{\pi/2}\int_0^1 r^3\,dr\,d\theta$$

$$\int_0^1 r^3\,dr = \frac{r^4}{4}\Big|_0^1 \Rightarrow \frac{1}{4}-0=\frac{1}{4}$$

$$\int_0^{\pi/2}\frac{1}{4}\,d\theta = \frac{\theta}{4}\Big|_0^{\pi/2} = \frac{\pi/2}{4} = \frac{\pi}{8}$$