Lesson 4: Square roots on the number line

Goals

- Calculate an approximate value of a square root to the nearest tenth, and represent the square root as a point on the number line.
- Determine the exact length of a line segment on a coordinate grid and express the length (in writing) using square root notation.
- Explain (orally) how to verify that a value is a close approximation of a square root.

Learning Targets

- I can find a decimal approximation for square roots.
- I can plot square roots on the number line.

Lesson Narrative

In this lesson, students begin to transition from understanding square roots simply as side lengths to recognising that all square roots are specific points on the number line. This understanding takes time to develop because students have previously only worked with rational numbers, which can be found by dividing the segment between two numbers into equal intervals. In the first activity, they still find $\sqrt{10}$ by relating it to the side length of a square of area 10 square units, but then are asked to approximate the value of $\sqrt{10}$ to the nearest tenth. In the second activity, students find a decimal approximation for $\sqrt{3}$ by looking at areas and also computing squares of numbers. This lesson shows students that irrational numbers are *numbers*—specific points on the number line—and we can find them by rotating a tilted square until it is sitting "flat." This is the conceptual foundation for the approximation work in the next lesson.

Addressing

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time

- Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Compasses
Four-function calculators
Tracing paper

Student Learning Goals

Let's explore square roots.

4.1 Notice and Wonder: Diagonals

Warm Up: 5 minutes

This warm-up transitions from work in previous lessons and prepares students to locate square roots on a number line in this lesson. Students must use the structure of the circle to relate the length of the segment to a point on the number line.

Instructional Routines

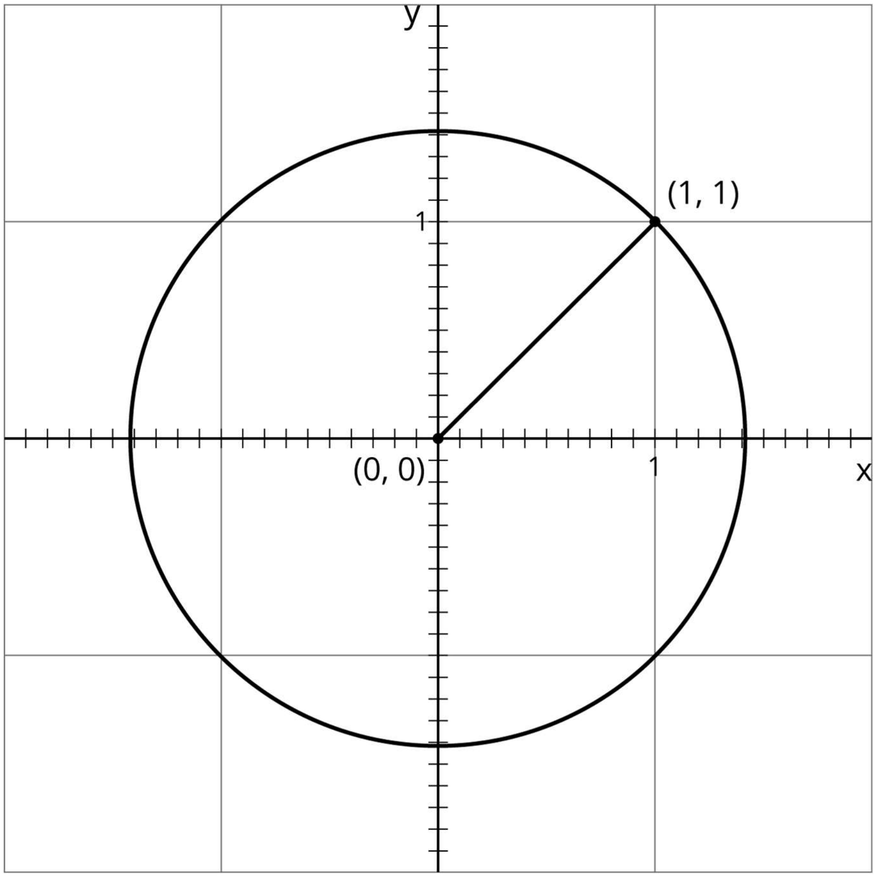
- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- The centre of the circle is at (0,0).
- There is a point labelled at (1,1) on the circle.
- There are many tick lines between 0 and 1.

Things students may wonder:

- How to find the distance across the circle?
- Where exactly does the circle land on the x and y axis?

Activity Synthesis

Ask students what the exact length is (it should be familiar to them from earlier lessons). The focus of the discussion is how you can see the decimal approximation from the diagram by looking at where the circle intersects an axis.

4.2 Squaring Lines

10 minutes

In this activity, students determine the length of a "diagonal" line segment on a grid. Students can give an exact value for the length of the line segment by finding the area of a square and writing the side length using square root notation. The goal of this activity is for students to connect values expressed using square roots with values expressed in decimal form—a form they are more familiar with.

Monitor for students who draw a tilted square for the first problem during the first two minutes of work time. Then monitor for students who use the following strategies to find the length of the segment:

- drawing a square and finding the area
- using tracing paper
- using a compass to make a circle

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports

Launch

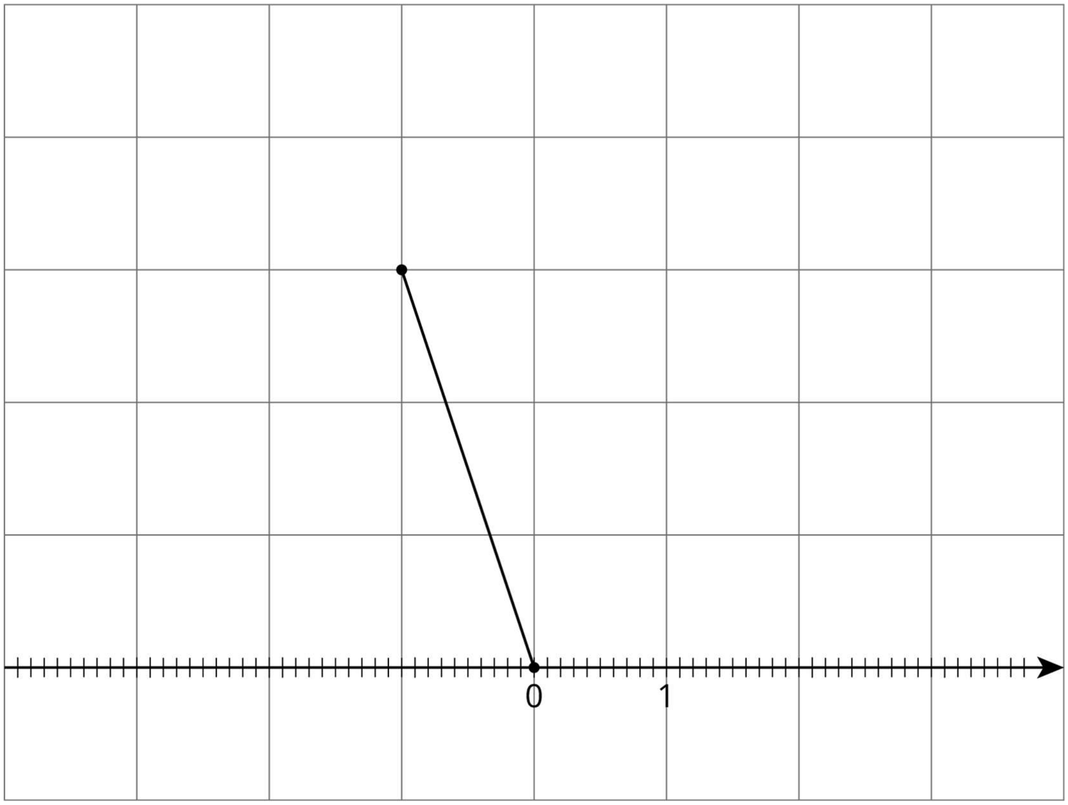
For this activity, it is best if students do not have access to a calculator with a square root button. If student calculators do have a square root button that students are familiar with, tell students that their explanations about their answers to the second problem need to dig deeper than pressing a button. In later lessons, however, they will be able to use it.

Before students begin, remind students that "exact length" means it can't just be an approximation, so if it is not a rational number, we should write it with square root notation. For example, a square with area 17 has a side length of exactly $\sqrt{17}$, which is a little larger than 4, since $4^2 = 16$.

Begin by displaying the diagram for all to see. Ask students how this diagram is similar and how it is different from the diagram in the warm-up. Then 2–3 minutes after students begin working, pause the class and select a previously identified student who drew a square on the grid to share what they did and why. Give 2–3 minutes work time to finish the problems followed by a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Provide students with access to tracing paper, straight edges, and compasses to support information processing in estimating the length of the line segment.
Supports accessibility for: Visual-spatial processing; Organisation

Student Task Statement



1. Estimate the length of the line segment to the nearest tenth of a unit (each grid square is 1 square unit).

2. Find the exact length of the segment.

Student Response

1. 3.1 units

2. $\sqrt{10}$ units

Activity Synthesis

Select students to present in this sequence:

- Someone who drew a square and used the area to find the exact side length.

- Someone who used tracing paper. This is essentially like the number line as a ruler. Ask students what this tells us about the exact value we found with the square. ($\sqrt{17}$ is about 3.1.)

- Someone who used a compass to find the approximate side length. This is a more formal geometric construction, but it is just another way to use the number line as a ruler.

Speaking, Listening: Discussion Supports. As students share their methods for finding the exact length of the line segment, press for details in students' reasoning by asking how they know the figure they drew in the coordinate plane is a square. Listen for and amplify the language students use to describe the important features of the square (e.g., opposite sides are equal, opposite sides are parallel, each angle is 90°). Then ask students to explain why the side length of the tilted square must be $\sqrt{10}$. This will support rich and inclusive discussion about strategies for finding the exact length of a line segment in the coordinate plane.
Design Principle(s): Support sense-making

4.3 Square Root of 3

10 minutes

In previous activities and lessons, students found the exact area of a square in order to find an approximation for the square root of an integer. In this activity, students start with a square root of an integer, and draw a square to verify that a given approximation of the square root is reasonable. This is the first time students have seen or drawn squares that do not have vertices at the intersection of grid lines, so it may take them a few minutes to make sense of the new orientation.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

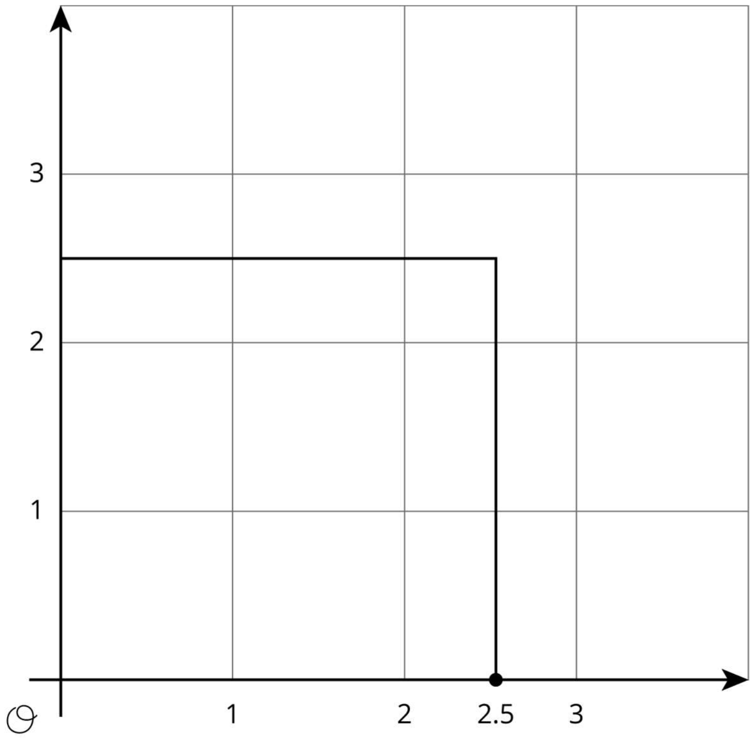
Launch

Give students access to four-function calculators. Display the diagram for all to see. Ask students what is the same and what is different about this diagram and diagrams they have seen in earlier activities. Arrange students in groups of 2. Give students 2–3 minutes of quiet work time followed by partner and whole-class discussions.

Action and Expression: Internalise Executive Functions. Provide students with a graphic organiser for data collection and organising information.
Supports accessibility for: Language; Organisation Writing, Speaking, Listening: Stronger and Clearer Each Time. After students have had time to identify a point on the number line that is closer to $\sqrt{3}$, ask them to write a brief explanation of their reasoning. Give students time to meet with 2–3 partners, to share and get feedback on their writing. Display prompts that students can ask each other that will help students strengthen their ideas and clarify their language. For example, "How do you know that $\sqrt{3}$ is between 1.5 and 2?" and "How did you determine the area of the square that you drew?" Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine both their ideas and their verbal and written output.
Design Principles(s): Optimise output (for explanation); Maximise meta-awareness

Student Task Statement

Diego said that he thinks that $\sqrt{3} \approx 2.5$.



1. Use the square to explain why 2.5 is not a very good approximation for $\sqrt{3}$. Find a point on the number line that is closer to $\sqrt{3}$. Draw a new square on the axes and use it to explain how you know the point you plotted is a good approximation for $\sqrt{3}$.

2. Use the fact that $\sqrt{3}$ is a solution to the equation $x^2 = 3$ to find a decimal approximation of $\sqrt{3}$ whose square is between 2.9 and 3.1.

Student Response

Answers vary. Sample response:

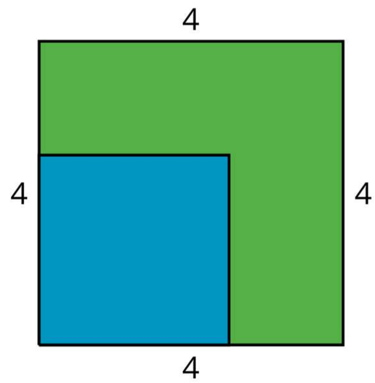
1. Any number between 1.5 and 2 will give a better approximation, and the corresponding square will have an area between 2.25 and 4. Because you can see the area of the square, you can know that the approximation is better.

2. 1.73

Are You Ready for More?

A farmer has a grassy patch of land enclosed by a fence in the shape of a square with a side length of 4 metres. To make it a suitable home for some animals, the farmer would like to carve out a smaller square to be filled with water, as in the figure.

What should the side length of the smaller square be so that half of the area is grass and half is water?



Student Response

The area enclosed by the fence is 16 square metres, so we want the area of both the grassy region and the water region to be 8 square metres. For the blue square in the figure to have an area of 8 square metres, the side length needs to be $\sqrt{8}$ metres, or about 2.8 metres.

Activity Synthesis

Invite one or two students to share their squares. Then tell students, "The square of a point on the number line can be visualised as the area of a literal square. This can help us estimate the value of a square root. Simply squaring the number can as well. Let's check the squares of some numbers that are potential approximations of $\sqrt{3}$."

Then ask students to suggest decimal approximations, and check together as a class by finding their squares. Students should be using each guess to make a better guess next. For example, if they try 1.5, then the square is 2.25, which is too low. This suggests trying bigger. Because we know that 2 is too big (because $2^2 = 4$, it should be somewhere in between 1.5 and 2.) For example, students might suggest this order:

$1^2 = 1$ and $2^2 = 4$

$1.5^2 = 2.25$

$1.8^2 = 3.24$

$1.7^2 = 2.89$

$1.72^2 = 2.9584$

$1.73^2 = 2.9929$

So 1.73 is a pretty good approximation of $\sqrt{3}$.

Lesson Synthesis

The goal of this discussion is to check that students know how to approximate square roots. In the previous lesson, students learned that some square roots, $\sqrt{2}$ in particular, are not rational. But they are still numbers, and we can reason about their approximate value using more familiar numbers.

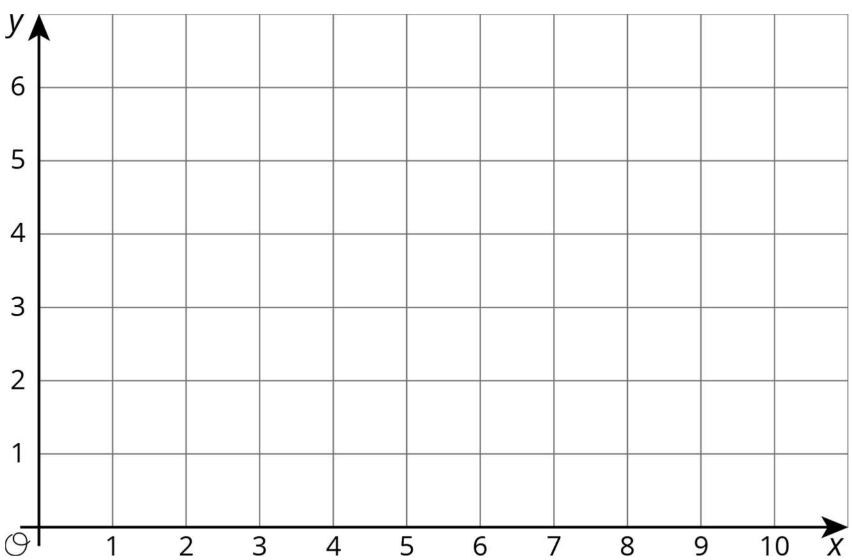
- "How can you approximate the value of $\sqrt{130}$?" ($\sqrt{130}$ is somewhere between 11 and 12 because $11^2 = 121$ and $12^2 = 144$.)
- "So we know $\sqrt{130}$ is somewhere between 11 and 12. Can we get more accurate than that? How?" (We could try squaring numbers from 11 to 12 like 11.1, 11.2, etc. . . to find the one closest to 130.)

4.4 Approximating $\sqrt{18}$

Cool Down: 5 minutes

Student Task Statement

Plot $\sqrt{18}$ on the x-axis. Consider using the grid to help.

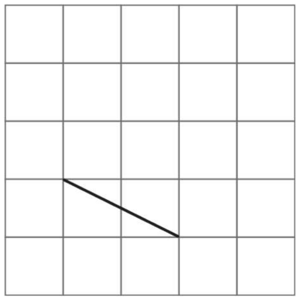


Student Response

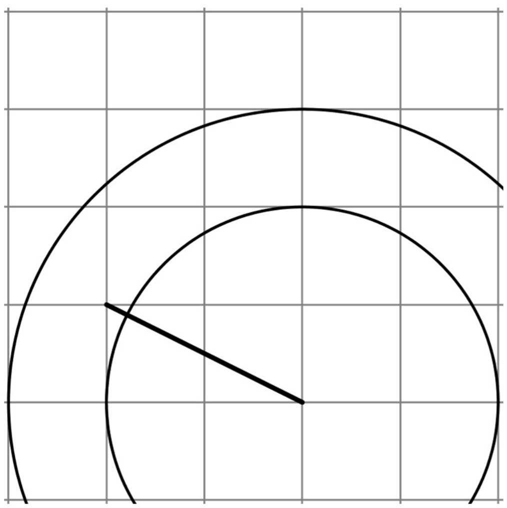
About 4.2.

Student Lesson Summary

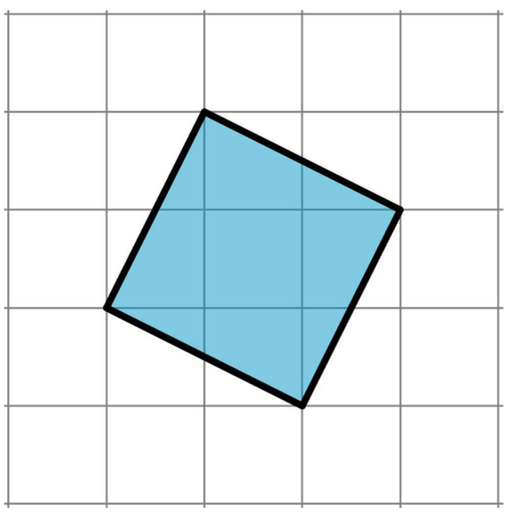
Here is a line segment on a grid. What is the length of this line segment?



By drawing some circles, we can tell that it’s longer than 2 units, but shorter than 3 units.



To find an exact value for the length of the segment, we can build a square on it, using the segment as one of the sides of the square.



The area of this square is 5 square units. (Can you see why?) That means the exact value of the length of its side is $\sqrt{5}$ units.

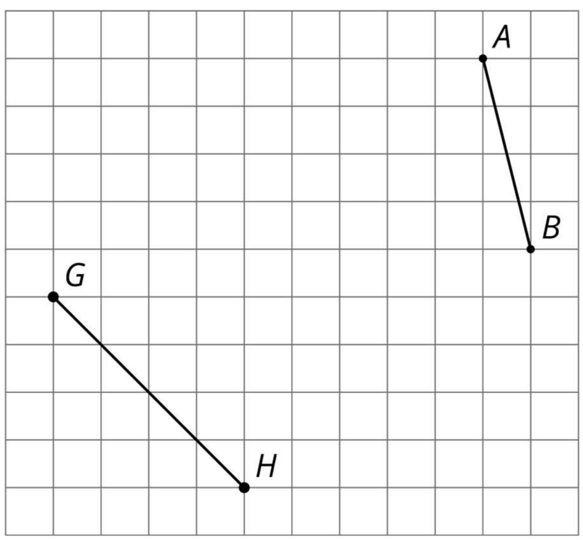
Notice that 5 is greater than 4, but less than 9. That means that $\sqrt{5}$ is greater than 2, but less than 3. This makes sense because we already saw that the length of the segment is in between 2 and 3.

With some arithmetic, we can get an even more precise idea of where $\sqrt{5}$ is on the number line. The image with the circles shows that $\sqrt{5}$ is closer to 2 than 3, so let's find the value of 2.1^2 and 2.2^2 and see how close they are to 5. It turns out that $2.1^2 = 4.41$ and $2.2^2 = 4.84$, so we need to try a larger number. If we increase our search by a tenth, we find that $2.3^2 = 5.29$. This means that $\sqrt{5}$ is greater than 2.2, but less than 2.3. If we wanted to keep going, we could try 2.25^2 and eventually narrow the value of $\sqrt{5}$ to the hundredths place. Calculators do this same process to many decimal places, giving an approximation like $\sqrt{5} \approx 2.2360679775$. Even though this is a lot of decimal places, it is still not exact because $\sqrt{5}$ is irrational.

Lesson 4 Practice Problems

1. **Problem 1 Statement**

   a. Find the exact length of each line segment.



   b. Estimate the length of each line segment to the nearest tenth of a unit. Explain your reasoning.

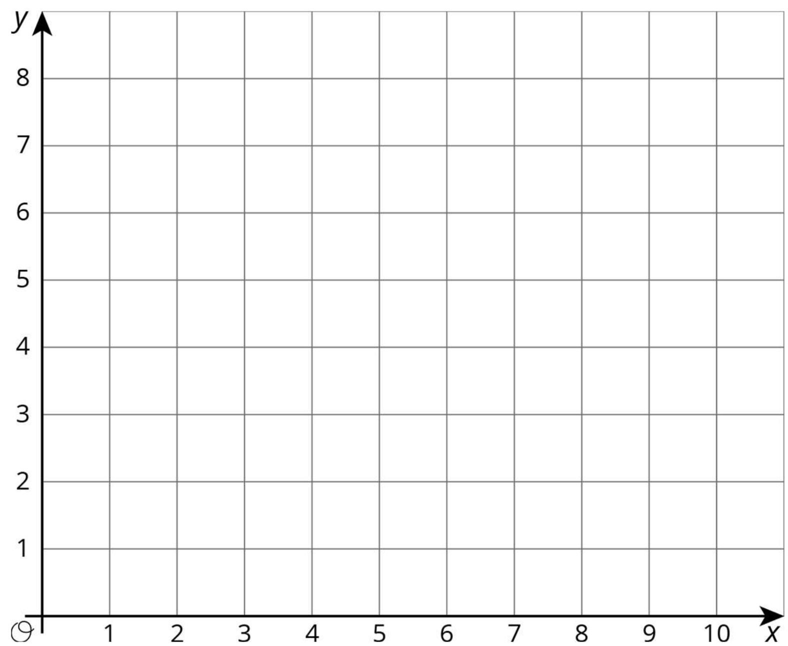
Solution

   a. $AB = \sqrt{17}, GH = \sqrt{32}$

   b. $AB \approx 4.1$, because $4.1^2 = 16.81$ and $4.2^2 = 17.64$. $GH \approx 5.7$, because $5.6^2 = 31.36$ and $5.7^2 = 32.49$.

2. **Problem 2 Statement**

Plot each number on the x-axis: $\sqrt{16}$, $\sqrt{35}$, $\sqrt{66}$. Consider using the grid to help.



Solution

$\sqrt{16}$ at 4, $\sqrt{35}$ a little less than 6, $\sqrt{66}$ a little more than 8

3. **Problem 3 Statement**

Use the fact that $\sqrt{7}$ is a solution to the equation $x^2 = 7$ to find a decimal approximation of $\sqrt{7}$ whose square is between 6.9 and 7.1.

Solution

Answers vary. Sample responses: 2.63, 2.64, 2.65, 2.66

4. **Problem 4 Statement**

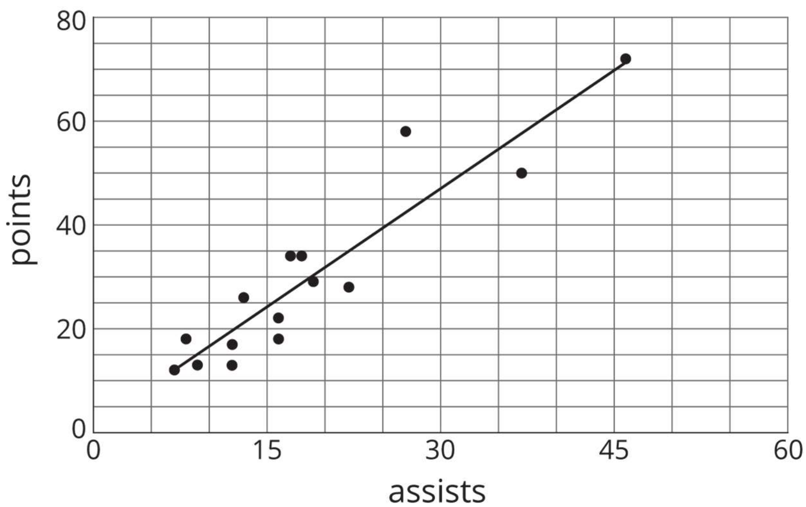
Graphite is made up of layers of graphene. Each layer of graphene is about 200 picometres, or 200×10^{-12} metres, thick. How many layers of graphene are there in a 1.6-mm-thick piece of graphite? Express your answer in standard form.

Solution

About 8×10^6. The thickness of the graphite is 1.6×10^{-3} metres. The number of layers of graphene is given by $\frac{1.6\times10^{-3}}{200\times10^{-12}} = 0.008 \times 10^9$. This number, in standard form, is 8×10^6, or about 8 million.

5. **Problem 5 Statement**

Here is a scatter plot that shows the number of assists and points for a group of hockey players. The model, represented by $y = 1.5x + 1.2$, is graphed with the scatter plot.



a. What does the gradient mean in this situation?

b. Based on the model, how many points will a player have if he has 30 assists?

Solution

a. For every assist, a player’s points have gone up by 1.5.

b. Approximately 46.2 points

6. **Problem 6 Statement**

The points (12,23) and (14,45) lie on a line. What is the gradient of the line?

Solution

$\frac{22}{2}$ (or 11)