

EGYPTIAN FRACTIONS



The Ancient Egyptians started writing numbers about 5000 years ago. Their numbers were based on the decimal system; that is, they counted in tens, hundreds and thousands as we do today.

Papyrus

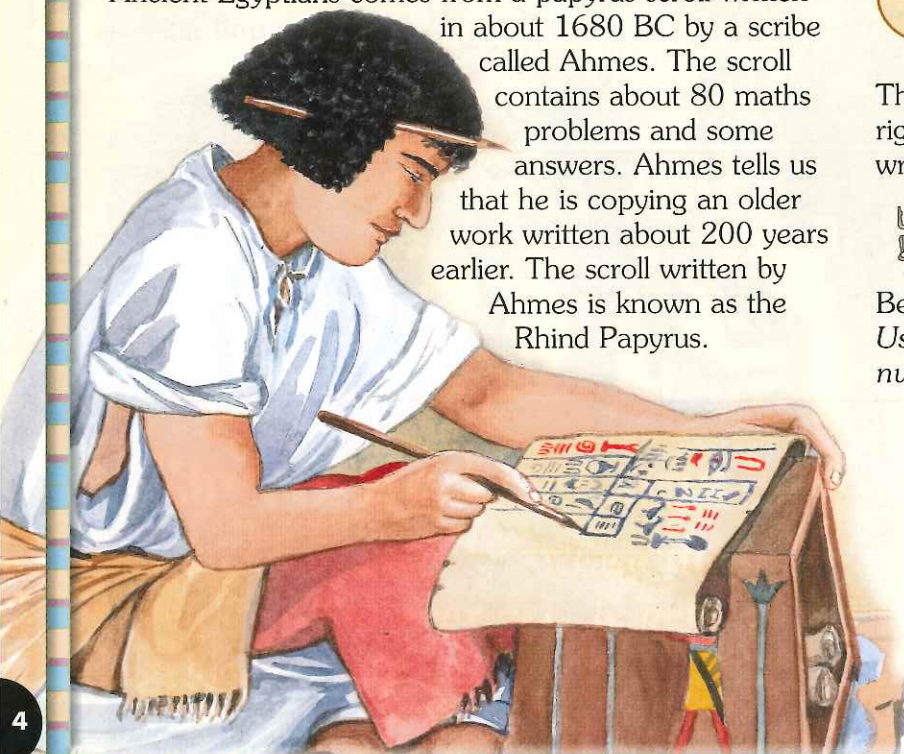
The Egyptians recorded their numbers by carving them onto stone or painting them on pottery or papyrus 'paper'.



Papyrus was made from thin layers of dried papyrus reeds that could be found growing along the banks of the Nile.

Ahmes the scribe

Much of what we know about the mathematics of Ancient Egyptians comes from a papyrus scroll written in about 1680 BC by a scribe called Ahmes. The scroll contains about 80 maths problems and some answers. Ahmes tells us that he is copying an older work written about 200 years earlier. The scroll written by Ahmes is known as the Rhind Papyrus.

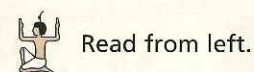


Egyptian numbers

Here are the Ancient Egyptians symbols, called hieroglyphs, for numbers up to a million.

Number	hieroglyph	description
1		a stick
10	∩	a basket handle
100	☉	a spiral of rope
1000	🌸	a lotus flower
10 000	☞	a bent finger
100 000	🐸	a tadpole
1 000 000	🧎	a kneeling man

The Egyptians wrote their hieroglyphs left to right, right to left or up and down. The direction of the writing is shown by the way they are facing.



Read from left.



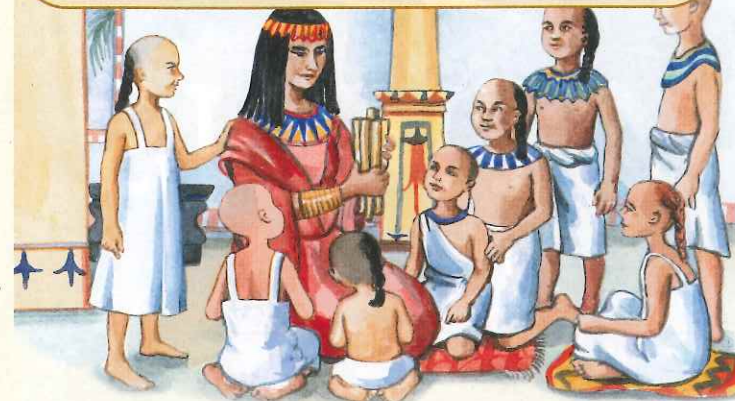
Read from right.

Below are some numbers written as hieroglyphs. Use the table above to work out what these numbers are:

(a)

(b)

Tahoser's fraction problem



Here's a problem like those written by Ahmes:

In a small Egyptian town, Tahoser made her way home from the market. The children had been good all afternoon so Tahoser decided they would have sugar cane. She had bought 5 lengths of cane to share between her 8 children. "How should I divide them?" she asked herself. "I can cut each cane into eight pieces and give each child an equal number of pieces or..."

Is there a better way of dividing up the canes that involves less cutting?

Writing Egyptian fractions

All fractions written by the Egyptians, apart from $\frac{2}{3}$, and $\frac{3}{4}$ were **unit fractions**; that is, they had 1 as the numerator. Fractions in ancient Egypt were shown by putting a hieroglyph of a mouth above a number.

$\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{10}$ $\frac{1}{1000}$

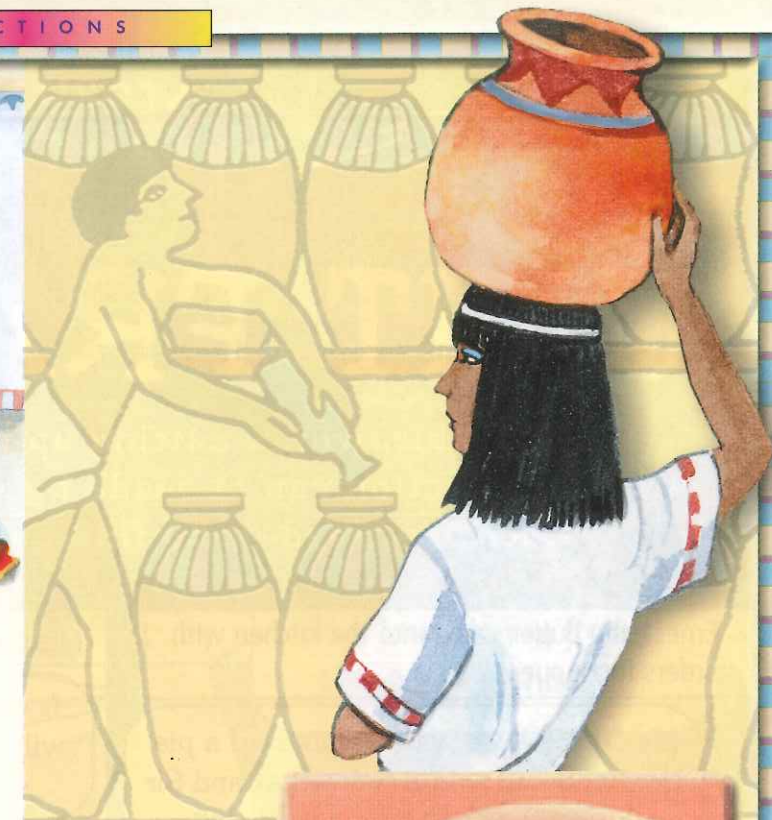
If the denominator was a big number then the mouth hieroglyph would be put at the front of the number.

$\frac{1}{359}$

(c) What unit fraction is this?

There were special hieroglyphs for the three most often used fractions:

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$



The hieroglyph of the Eye of Horus was a powerful symbol used as a protection from evil.



Liquid fractions

For measuring grain and for measuring the volume of liquids such as milk or medicine, the Egyptians used a measure called the heqat. The heqat was equal to about 4.75 litres. For fractions of the heqat the Egyptians had a completely different set of hieroglyphs. They were based on the symbol of the Eye of Horus. The eye was split into six parts and each represented a different fraction.

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$
 $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$

When measuring volumes, the six parts of the Eye of Horus were used for fractions of a heqat.

What sum do you get when you add all the fractions of the eye together?



STORIES FROM

UNIT FRACTIONS

Mrs Crumble, whilst cutting up pies for supper, discovers that ANY unit fraction can be made by summing two different sized unit fractions. Read on to find out about her special recipe...

Ernest, the butler, came into the kitchen with orders for supper.

Ms Cumference wants a sixth of a pie, Her Ladyship wants twice that and Sir Cumference will have what's left.



Right, that's two sixths for her ladyship, which is a third...

Mrs Crumble started to cut a pie into three fractions.

It works out that Sir Cumference will get exactly half! That's three distinct unit fractions.

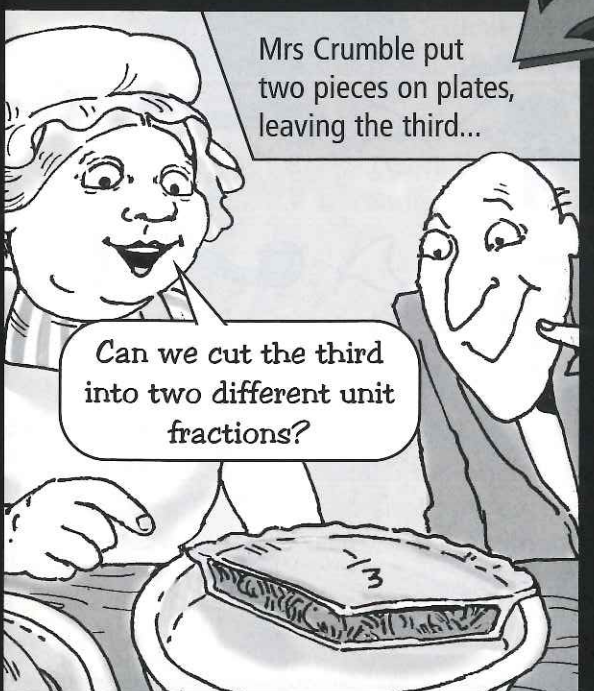


... and, look, $\frac{1}{2}$ is equal to $\frac{1}{3} + \frac{1}{6}$

That's right, m'deary.

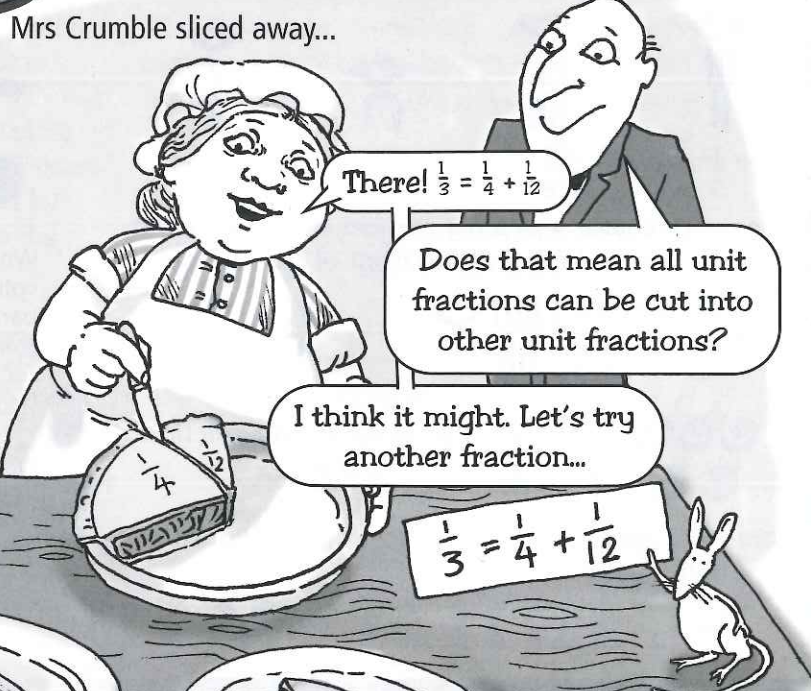
$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$$

Mrs Crumble sliced away...



Mrs Crumble put two pieces on plates, leaving the third...

Can we cut the third into two different unit fractions?



There! $\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$

Does that mean all unit fractions can be cut into other unit fractions?

I think it might. Let's try another fraction...

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$$

Mrs Crumble took a slice of cheese. It was one sixth of the whole piece.

Start with $\frac{1}{6}$. The next lowest unit fraction is $\frac{1}{7}$.



How do you know the other fraction?

$\frac{1}{6}$ is the same as $\frac{7}{42}$ and $\frac{1}{7}$ is the same as $\frac{6}{42}$...

So, if I cut $\frac{1}{7}$ away from $\frac{1}{6}$, I shall be left with $\frac{1}{42}$...

$$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$$

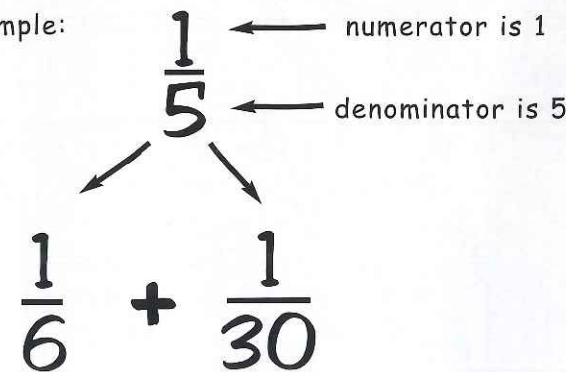
Ernest was puzzled WHY Mrs Crumble calculated with 42.

Mrs Crumble showed Ernest a recipe.

How did you know to change the fractions to 42ths?

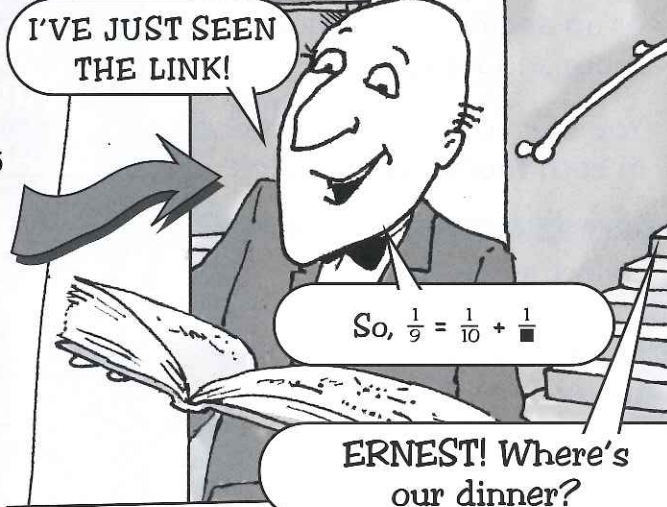
How to cut up a unit fraction into two distinct unit fractions

for example:



Write the next smallest unit fraction (add 1 to the denominator)

Multiply the other two denominators ($5 \times 6 = 30$) to get a common multiple



Ernest puzzled for a while, then...

I'VE JUST SEEN THE LINK!

So, $\frac{1}{9} = \frac{1}{10} + \frac{1}{90}$

ERNEST! Where's our dinner?

Can you see why Mrs Crumble's recipe works? Try these:

- (a) $\frac{1}{7} = \frac{1}{8} + \frac{1}{56}$
- (b) $\frac{1}{10} = \frac{1}{11} + \frac{1}{110}$
- (c) $\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$
- (d) $\frac{1}{20} = \frac{1}{21} + \frac{1}{420}$