

Lesson 10: Tax and tip

Goals

- Comprehend “sales tax” and “tip” as two contexts that involve adding a percentage of the initial amount.
- Explain (orally) how to calculate the total cost including a tax or tip, given the “subtotal” and the percentage.
- Explain (orally) how to determine the percentage of the subtotal that a tax or tip is.

Learning Targets

- I understand and can solve problems about sales tax and tip.

Lesson Narrative

In this lesson students are introduced to contexts involving sales tax and tips whilst on a holiday in the US. This provides a wider variety of tax rates than in the UK! They can use bar models and double number lines from their earlier work, but the lesson provides an opportunity to be more efficient by using an equation of the form $y = kx$. For example, if the tax rate is 6.2% they can calculate the tax, T , for any price, p , using the equation $T = 0.062p$. They do not necessarily write this equation out with variables, but rather repeatedly use it with specific values of p . By repeatedly calculating the tax for different prices and then generalising the process they are engaging in expressing regularity in repeated reasoning.

Questions about rounding naturally come up in this lesson. This lesson primarily involves dollar amounts, so it is sensible to round to the nearest cent (the nearest hundredth of a dollar).

Addressing

- Use proportional relationships to solve multistep ratio and percentage problems.
Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Building Towards

- Use proportional relationships to solve multistep ratio and percentage problems.
Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
 - Collect and Display
 - Clarify, Critique, Correct
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- Three Reads
- Notice and Wonder
- Think Pair Share

Required Materials

Four-function calculators

Required Preparation

It is recommended that students be provided access to four-function calculators so that they can focus on reasoning about how numbers are related to each other, representing those relationships, and deciding which operations are appropriate (rather than focusing on calculation.)

Student Learning Goals

Let's learn about sales tax and tips.

10.1 Notice and Wonder: The Price of Sunglasses

Warm Up: 5 minutes

The purpose of this warm-up is to introduce students to the meaning of sales tax.

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students to think of at least one thing they notice or wonder. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have at least one thing they noticed or wondered.

Student Task Statement

You are on vacation in the US and want to buy a pair of sunglasses for \$10 or less. You find a pair with a price tag of \$10. The cashier says the total cost will be \$10.45.



What do you notice? What do you wonder?

Student Response

Answers vary. Sample responses:

Students may notice:

- The price of the sunglasses is \$10
- The total cost is more than the price of the sunglasses.
- Sales tax was added to the cost of the sunglasses.

Students may wonder:

- If the cashier made a mistake.
- Why the total cost is more than the price listed on the sunglasses.
- What the tax was on the sunglasses.
- If something else was purchased to make the price more.

Activity Synthesis

Ask students to share what they noticed and wondered. Record and display the responses for all to see. Students are likely to notice that the total cost is more than the price listed on the sunglasses. Ask students to share why they think the amount shown on the cash register is more than the price of the glasses. Ask them if they have ever heard of sales tax before, and if some have, ask them to share their understanding.

Tell students that sales tax is a fee (an amount of money) paid to the government. The amount of tax is a percentage of the price of the item. Different states charge different sales tax percentages, and additionally some local governments like counties and cities also charge a sales tax.

To start to help make sense of how sales tax works, ask questions like:

- How much sales tax is being collected on the \$10 sunglasses? (\$0.45 or 45 cents)
- 45 cents is what percentage of \$10? (It's 4.5%)
- What is the sales tax rate for our local area? (Varies based on location.)

10.2 Shopping in Two Different Cities

20 minutes

In this activity, students work with tax rates. Because students reason repeatedly about the same percentage of different quantities, they have the opportunity to represent this

process as an equation of the form $y = kx$. Students should see connections to their previous work with percentage increase. As students work, monitor for different strategies, especially students who note that they can always multiply by the same factor and students who set up and use an equation.

In this activity, the tax rates are whole percentages so that students do not have to deal with rounding. The next activity deals with a tax rate that is a fractional percentage.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Three Reads

Launch

Provide access to calculators. Arrange students in groups of 2. Make sure students understand the situation by asking questions like, "How much would you have to pay for the paper towels in City 1? And for the lamp in City 2?" (\$8.48 and \$27)

Tell students, "In some places in the US, there are different sales tax rates for different types of items (clothing, food, medicine, cars, etc.), but the cities in this question have a single sales tax rate for all items."

Give students 3–5 minutes of quiet work time. Afterwards, give students the option to work with a partner or to continue to work alone. Follow with a whole-class discussion.

Reading, Writing: Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., There are two cities. Each city has a different tax rate.). If needed, discuss the meaning of unfamiliar terms at this time. Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., price, sales tax, and total cost). In the third read, ask students to brainstorm possible mathematical solution strategies to complete the table. This will help students connect the language in the word problem, the table, and the reasoning needed to solve the problem while keeping the intended level of cognitive demand in the task.

Design Principle(s): Support sense-making

Student Task Statement

Different cities have different sales tax rates. Here are the sales tax charges on the same items in two different cities. Complete the tables.

City 1

item	price (dollars)	sales tax (dollars)	total cost (dollars)
paper towels	8.00	0.48	8.48
lamp	25.00	1.50	

pack of gum	1.00		
laundry soap	12.00		
	x		

City 2

item	price (dollars)	sales tax (dollars)	total cost (dollars)
paper towels	8.00	0.64	8.64
lamp	25.00	2.00	
pack of gum	1.00		
laundry soap	12.00		
	x		

Student Response

City 1

item	price (dollars)	sales tax (dollars)	total cost (dollars)
paper towels	8.00	0.48	8.48
lamp	25.00	1.50	26.50
pack of gum	1.00	0.06	1.06
laundry soap	12.00	0.72	12.72
	x	$0.06x$	$1.06x$

City 2

item	price (dollars)	sales tax (dollars)	total cost (dollars)
paper towels	8.00	0.64	8.64
lamp	25.00	2.00	27.00
pack of gum	1.00	0.08	1.08
laundry soap	12.00	0.96	12.96

	x	$0.08x$	$1.08x$
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Activity Synthesis

Select previously identified students to share the sales tax they calculated for laundry soap in each city. Have students share how they calculated the sales tax.

Have some students share the equations for the last row of each tables. Make sure students see the connection between this row and their previous work on percentage increase. Point out that sometimes we want to know just the amount of the tax, $0.06x$, and sometimes we want to know the total cost, which is the price plus the cost, $x + 0.06x = 1.06x$.

Tell students that, when there is a certain tax that gets applied to a class of goods, it is called a *tax rate*. Tax rates are usually described in terms of percentages.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “First, I _____ because . . .”, “I noticed ____ so I...”, “Why did you . . .?”, “I agree/disagree because....”
Supports accessibility for: Language; Social-emotional skills

10.3 Shopping in a Third City

Optional: 5 minutes

The purpose of this activity is for students to encounter a situation in which rounding error makes it look like the relationship between the price of an item and the sales tax is not quite proportional. Students should realise this is due to having a fractional percentage for the tax rate and the custom of rounding dollar amounts to the nearest cent.

Instructional Routines

- Clarify, Critique, Correct
- Think Pair Share

Launch

Provide access to four-function calculators. Keep students in the same groups of 2. Allow students 2 minutes quiet work time followed by partner and whole-class discussions.

Again, tell students that the tax rate for items in City 3 is the same for all types of items.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students calculate the tax rate for each item before estimating the tax rate of City 3.

Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

Some students may say that the relationship is not proportional. Remind them of the activity in a previous unit where they measured the length of the diagonal and the perimeter of several squares and determined that there was really a proportional relationship, even though measurement error made it look like there was not an exact constant of proportionality.

Some students may say that the tax rate is exactly 7%. Prompt them to calculate what the sales tax would have been for the paper towels and the lamp if the tax rate were exactly 7%.

Some students may use 7.25% as the tax rate since that is what comes from the first item (paper towels) without checking this number against the tax on the other items provided. Prompt students to use the additional information they have to check their answer before proceeding to solve the row with laundry soap.

Student Task Statement

Here is the sales tax on the same items in City 3.

item	price (dollars)	sales tax (dollars)
paper towels	8.00	0.58
lamp	25.00	1.83
pack of gum	1.00	0.07
laundry soap	12.00	

1. What is the tax rate in this city?
2. For the sales tax on the laundry soap, Kiran says it should be \$0.84. Lin says it should be \$0.87. Do you agree with either of them? Explain your reasoning.

Student Response

1. The tax rate is about 7.3%, because this rate can give all the dollar amounts in the table when rounded to the nearest cent. (Note: 7.31% would also work.)
 - For the paper towels, $0.073 \times 8.00 = 0.584$, which rounds to \$0.58.
 - For the lamp, $0.073 \times 25.00 = 1.825$, which rounds to \$1.83.
 - For the pack of gum, $0.073 \times 1.00 = 0.073$, which rounds to \$0.07.
2. No, I don't agree with either of them. Since the sales tax on the lamp was \$1.83, both Kiran and Lin's answers are too small.
 - To get Kiran's answer of \$0.84, the tax rate would be just 7%, but then the tax on the lamp would have been only \$1.75.

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- To get Lin's answer of \$0.87, the tax rate could be 7.25%, but then the tax on the lamp would still have been only \$1.81.
 - The sales tax on the laundry soap should be \$0.88, because $0.073 \times 12.00 = 0.876$, which rounds to \$0.88.

Activity Synthesis

Remind students about when measurement error made it look like the relationship between the length of the diagonal and the perimeter of a square was not quite a proportional relationship.

Consider asking these discussion questions:

- "How did you determine the tax rates for the items in City 1 and City 2 from the previous activity?" (Divide the sales tax by the price.)
- "How did determining the tax rate for City 3 differ from the work you did for the other cities?" (Since the tax rates were not the same for each item, I had to determine what tax rate might give each value listed.)

Tell students that, when multiple pairs of values are known (as in this activity or the previous one), they should work to find an exact tax rate as they did here. If only 1 pair of values is known (as in the following activities), they may use the exact tax rate found from that pair of values. For example, if this activity had only given the price and sales tax for paper towels, we may assume that the tax rate is 7.25%.

Writing: Clarify, Critique, Correct. Present an incorrect response to the second question that reflects a possible misunderstanding from the class. For example, "Kiran is right because the tax rate is 7%." Ask students to identify the error (e.g., ask, "Do you agree with the author's reasoning? Why or why not?"), critique the reasoning, and write a correct explanation. This can help students to further understand the ways rounding errors can influence the appearance of a tax rate.

Design Principle(s): Maximise meta-awareness

10.4 Dining at a Restaurant

10 minutes

In this activity students use previously learned strategies to solve problems involving fractional percentages. Monitor for students using various strategies (double number line, table, unit rate, equation) and identify students using an equation of the form $y = kx$, especially to solve the last problem.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display

Launch

Keep students in groups of 2. Tell students that in some restaurants, people pay the server a *tip* in addition to paying for the meal. In the US, tips usually range between 10% and 20% of the cost of the meal. Provide access to calculators. Give students 3–5 minutes of quiet work time, followed by partner and whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisation and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organisation; Attention

Anticipated Misconceptions

Students may attempt to write an equation, but place numbers in the wrong place. Ask them what each piece of their equation means in this situation. In particular, monitor for students who struggle with understanding the second part the first question. Help these students understand by rephrasing the question as, "The total is what percentage of the subtotal?" and helping them to see that the answer should be greater than 100% since the total is greater than the subtotal.

Students might need a way to keep track of all the information. Suggest using a table that keeps track of original price and percentage.

Student Task Statement

1. Jada has a meal in a restaurant. She adds up the prices listed on the menu for everything they ordered and gets a subtotal of \$42.00.

Date: Sep. 12th	
Time: 6:55 PM	
Server: # 27	
Bread Stix	9.50
Chicken Parm	15.50
Chef Salad	12.00
Lemon Soda	2.00
Tea	3.00
Subtotal	42.00
Sales Tax	3.99
Total	45.99

- a. When the bill comes, it says they also need to pay \$3.99 in sales tax. What percentage of the subtotal is the sales tax?
 - b. After tax, the total is \$45.99. What percentage of the subtotal is the total?
 - c. They actually pay \$52.99. The additional \$7 is a tip for the server. What percentage of the subtotal is the tip?
2. The tax rate at this restaurant is 9.5%.

Date: Sep 12th	
Time: 6:04 PM	
Server: # 27	
Bread Stix	9.50
Ravioli Bites	10.50
Cheesecake	4.95
Subtotal	24.95
Sales Tax	_____
Total	_____

Date: Sep 12th	
Time: 7:12 PM	
Server: # 27	
Garden Salad	_____
Broccoli Bites	_____
Subtotal	_____
Sales Tax	1.61
Total	_____

This person's subtotal is \$24.95. How much will their sales tax be?

This person's sales tax is \$1.61. How much was their subtotal?

Student Response

1.
 - a. 9.5% because $3.99 \div 42 = 0.095$.
 - b. 109.5% because $45.99 \div 42 = 1.095$.
 - c. About 16.67% because $7 \div 42 \approx 16.67$.
2.
 - a. \$2.37 because $24.95 \times 0.095 = 2.37025$.
 - b. Answers vary from \$16.90 to \$16.99. Possible response: \$16.95 because $1.61 \div 0.095 \approx 16.95$.

Are You Ready for More?

Elena's cousins went to a restaurant. The part of the entire cost of the meal that was tax and tip together was 25% of the cost of the food alone. What could the tax rate and tip rate be?

Student Response

Answers vary. Sample response: 5% tax rate and 20% tip.

Activity Synthesis

Select students to share who used different strategies to solve the problems. Sequence them to show solutions that use diagrams first, then an equation like $y = kx$.

If no student uses the equation strategy ask students:

- "How we might use an equation to solve the problem?"
- "What are the two quantities being used in these problems?" (Sales tax and subtotal or tip and subtotal.)

Demonstrate to students how to use the equation to solve a problem.

Help students connect the different strategies.

Representing, Conversing: Collect and Display. While pairs are working, circulate and listen to student talk about their approaches for calculating sales tax, tips, subtotals, and total cost. Write down common or important phrases you hear students say to describe the relationships between the quantities onto a visual display. This will help students read and use mathematical language during their paired and whole group discussions.

Design Principle(s): Support sense-making

Lesson Synthesis

Students should understand that we reason about fractional percentages like 0.8% and 110.5% using the same strategies we did with percentages that were whole numbers, like 37%. Ask students:

- "Where did we see and use fractional percentages in this lesson?" (Sales tax)
- "What are strategies we can use to calculate fractional percentages (including sales tax)?" (Double number lines, tables, an equation)

10.5 A Restaurant in a Different City

Cool Down: 5 minutes

This cool-down assesses whether students understand how to calculate a tax rate based on a price before tax was added and the amount of tax added. Additionally, they must use proportional reasoning to determine the tax on another item with the same tax rate.

Student Task Statement

At a dinner, the meal cost \$22 and a sales tax of \$1.87 was added to the bill.

1. How much would the sales tax be on a \$66 meal?
2. What is the tax rate for meals in this city?

Student Response

1. \$5.61, because $22 \times 3 = 66$ and $1.87 \times 3 = 5.61$.
2. 8.5%, because $1.87 \div 22 = 0.085$.

Student Lesson Summary

Many places have *sales tax*. A sales tax is an amount of money that a government agency collects on the sale of certain items. In the UK we call this VAT (Value Added Tax) and it is 20% for most goods and services. In the US, sales tax varies by product and state. For example, a state might charge a tax on all cars purchased in the state. Often the tax rate is given as a percentage of the cost. For example, a state's tax rate on car sales might be 2%, which means that for every car sold in that state, the buyer has to pay a tax that is 2% of the sales price of the car.

Fractional percentages often arise when a state or city charges a sales tax on a purchase. For example, the sales tax in Arizona is 7.5%. This means that when someone buys something, they have to add 0.075 times the amount on the price tag to determine the total cost of the item.

For example, if the price tag on a T-shirt in Arizona says \$11.50, then the sales tax is $(0.075) \times 11.5 = 0.8625$, which rounds to 86 cents. The customer pays $11.50 + 0.86$, or \$12.36 for the shirt.

The total cost to the customer is the item price plus the sales tax. We can think of this as a percentage increase. For example, in Arizona, the total cost to a customer is 107.5% of the price listed on the tag.

A *tip* is an amount of money that a person gives someone who provides a service. In the US, it is customary in many restaurants to give a tip to the server that is between 10% and 20% of the cost of the meal (in the UK, it is around 10%). If a person plans to leave a 15% tip on a meal, then the total cost will be 115% of the cost of the meal.

Lesson 10 Practice Problems

Problem 1 Statement

In a city in Ohio, the sales tax rate is 7.25%. Complete the table to show the sales tax and the total price including tax for each item.

item	price before tax (\$)	sales tax (\$)	price including tax (\$)
pillow	8.00		
blanket	22.00		
trash can	14.50		

Solution

item	price before tax (\$)	sales tax (\$)	price including tax (\$)
pillow	8.00	0.58	8.58
blanket	22.00	1.60	23.60
trash can	14.50	1.05	15.55

For the blanket and the trash can, the tax is rounded to the nearest cent: it is rounded up for the blanket and rounded down for the trash can.

Problem 2 Statement

The sales tax rate in New Mexico is 5.125%. Select **all** the equations that represent the sales tax, t , you would pay in New Mexico for an item of cost c ?

- a. $t = 5.125c$
- b. $t = 0.5125c$
- c. $t = 0.05125c$
- d. $t = c \div 0.05125$
- e. $t = \frac{5.125}{100}c$

Solution ["C", "E"]

Problem 3 Statement

Here are the prices of some items and the amount of sales tax charged on each in Nevada.

cost of item (\$)	sales tax (\$)
10	0.46
50	2.30
5	0.23

- What is the sales tax rate in Nevada?
- Write an expression for the amount of sales tax charged, in dollars, on an item that costs c dollars.

Solution

- 4.6%
- $0.046c$ or equivalent

Problem 4 Statement

Find each amount:

- 3.8% of 25
- 0.2% of 50
- 180.5% of 99

Solution

- 0.95
- 0.1
- 178.695

Problem 5 Statement

On Monday, the temperature was 60 degrees Fahrenheit. On Tuesday, the temperature was 18% more. How much did the temperature increase from Monday to Tuesday?

Solution

10.8 degrees Fahrenheit.

Problem 6 Statement

Complete the table. Explain or show your reasoning.

object	radius	circumference
ceiling fan	2.8 ft	

water bottle cap	13 mm	
bowl		56.5 cm
drum		75.4 in

Solution

object	radius	circumference
ceiling fan	2.8 ft	17.6 ft
water bottle cap	13 mm	82 mm
bowl	9 cm	56.5 cm
drum	12 in	75.4 in

The constant of proportionality is $2 \times \pi$. The given radii is multiplied by 6.28 to find the missing circumferences, and the given circumferences is divided by 6.28 to find the missing radii.



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