# Lesson 15: More nets, more surface area

## Goals

- Draw and assemble a net for the prism or pyramid shown in a given drawing.
- Interpret (using words and other representations) two-dimensional representations of prisms and pyramids.
- Use a net without gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.

## Learning Targets

- I can calculate the surface area of prisms and pyramids.
- I can draw the nets of prisms and pyramids.

## Lesson Narrative

This lesson further develops students' ability to visualise the relationship between nets and polyhedra and their capacity to reason about surface area.

Previously, students started with nets and visualised the polyhedra that could be assembled from the nets. Here they go in the other direction—from polyhedra to nets. They practice mentally unfolding three-dimensional shapes, drawing two-dimensional nets, and using them to calculate surface area. Students also have a chance to compare and contrast surface area and volume as measures of two distinct attributes of a three-dimensional shape.

### Building On

- Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

### Addressing

- Find the volume of a right cuboid with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right cuboids with fractional edge lengths in the context of solving real-world and mathematical problems.
- Represent three-dimensional shapes using nets made up of rectangles and triangles, and use the nets to find the surface area of these shapes. Apply these techniques in the context of solving real-world and mathematical problems.

### Instructional Routines

- Compare and Connect
- Discussion Supports

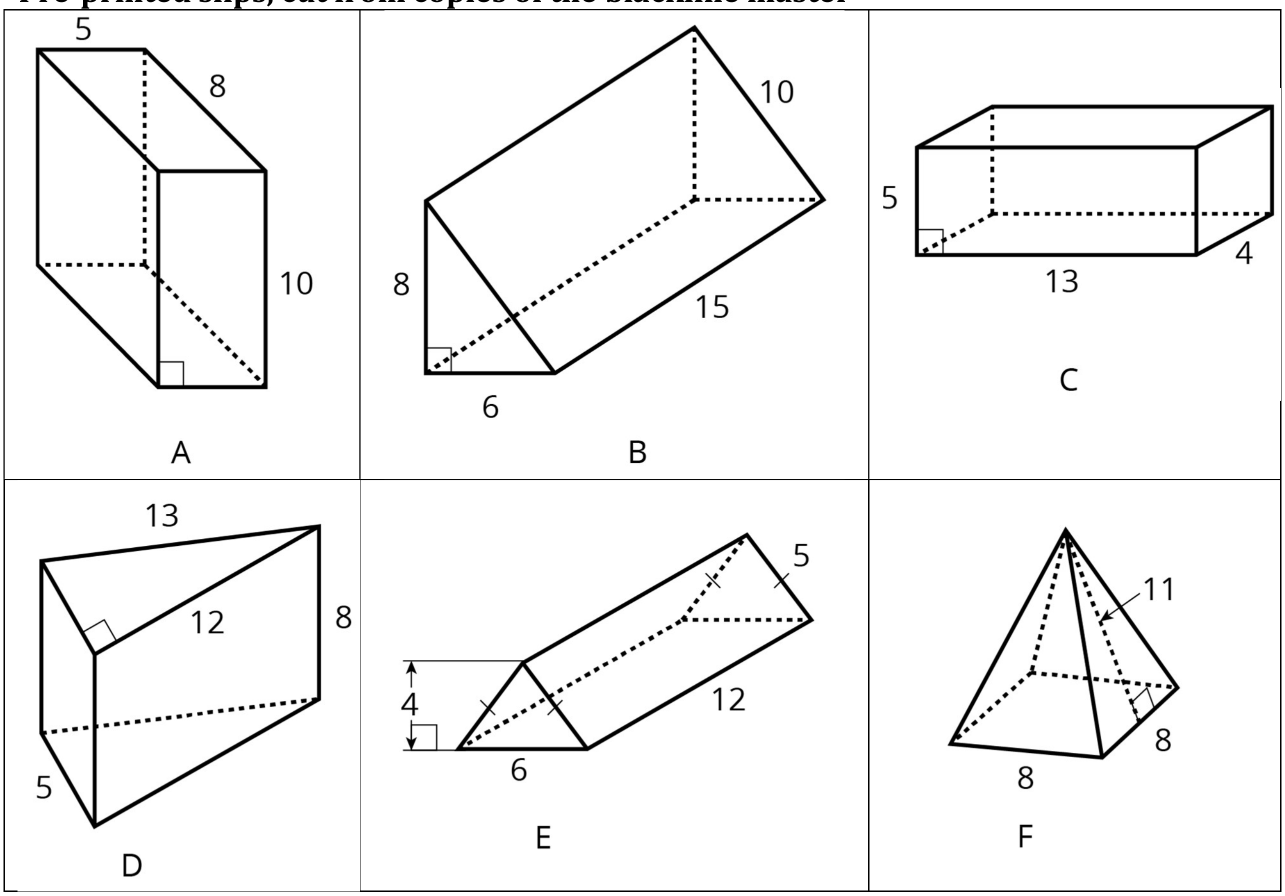
- Notice and Wonder
- Think Pair Share

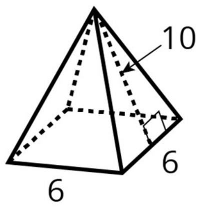
### Required Materials

**Demonstration nets with and without flaps**
**Geometry toolkits**
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles.

**Glue or glue sticks**

**Pre-printed slips, cut from copies of the blackline master**



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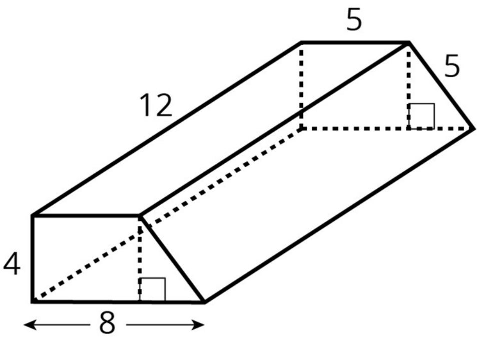


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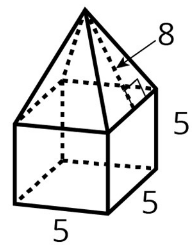


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**Sticky tape**

### Required Preparation

Copy and cut the blackline master for the Building Prisms and Pyramids activity. Make one copy for every 9 students, so that each student gets one drawing of a polyhedron. Consider assignments of polyhedra in advance.

### Student Learning Goals

Let's draw nets and find the surface area of polyhedra.

## 15.1 Notice and Wonder: Wrapping Paper

**Warm Up: 5 minutes**

This warm-up prompts students to think about a prism and its measurements in context and to consider potential questions that could be asked and answered.

Given their recent work, students are likely to notice and wonder about surface area, nets, and the missing height of the box. Students may also wonder about the volume of the box, given their geometric work in KS2.

When prompted about how to find the surface area or the volume of the box, students will likely ask about the missing measurement. This is an opportunity for them to practise making a reasonable estimate.

### Instructional Routines

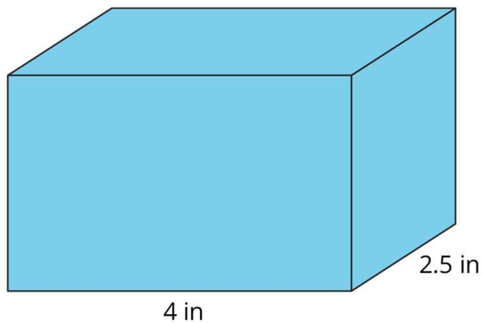
- Notice and Wonder

### Launch

Arrange students in groups of 2. Give students a minute of quiet time to observe the image. Ask students to be prepared to share at least one thing they notice and one thing they wonder about the picture. Ask them to give a signal when they have noticed or wondered about something.

### Student Task Statement

Kiran is wrapping this box of sports cards as a present for a friend.




What do you notice? What do you wonder?

### Student Response

Answers vary. Possible responses:

- Notice: the given side lengths of the box, the height looks like the same length as the width, the box being covered in wrapping paper, the box being a cuboid, the area of the top and bottom faces being 10 square inches each
- Wonder: the missing side length, how many cards are in the box, the volume of the box, the surface area of the box, how much wrapping paper it will take to cover the box, how would you count the overlapped paper on the edges that are wrapped

### Activity Synthesis

Invite students to share their observations and questions. Record the responses for all to see. If no students wonder about the surface area, the amount of wrapping paper needed, or the volume of the box, bring these questions up.

Tell students to choose either a question about surface area or one about volume and give them a minute to discuss with a partner *how* they would find the answer to the question. If students suggest that it cannot be done because of missing information, ask them to estimate the missing information.

Select a couple of students to share how they would find the surface area or the volume of the given box. After each response, poll the class on whether they agree or disagree.

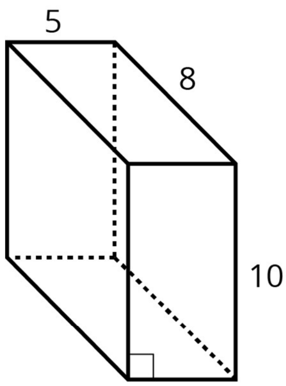
## 15.2 Building Prisms and Pyramids

**30 minutes**

Previously, students used a given net of a polyhedron to find its surface area. Here they use a given polyhedron to draw a net and then calculate its surface area.

Use the provided polyhedra to differentiate the work for students with varying degrees of visualisation skills. Cuboids (A and C), triangular prisms (B and D), and square-based pyramids (F and G) can be managed by most students. Triangular prism E requires a little more interpretive work (i.e., the measurements of some sides may not be immediately apparent to students). Trapezium-based prism H and polyhedron I (a composite of a cube and a square-based pyramid) require additional interpretation and reasoning.

As students work, remind them of the organisational strategies discussed in previous lessons, i.e., labelling polygons, showing measurements on the net, etc.

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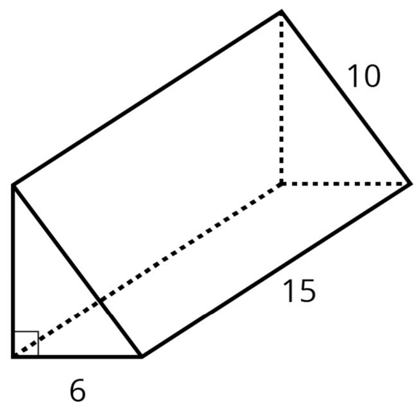


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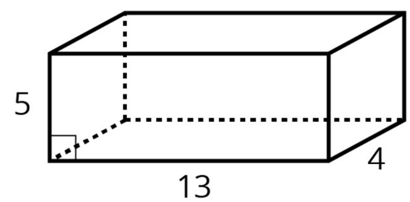


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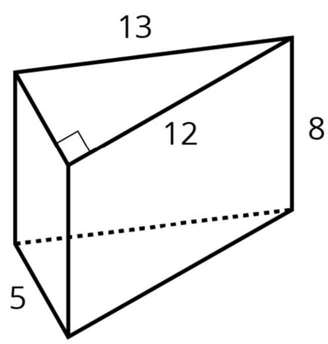


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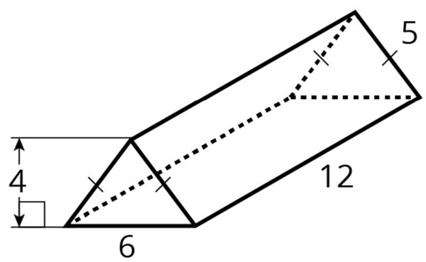


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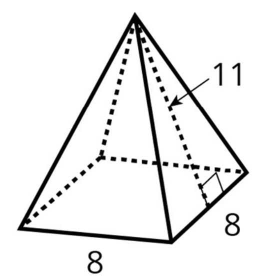


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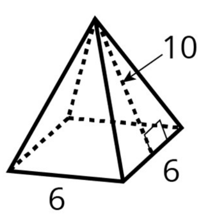


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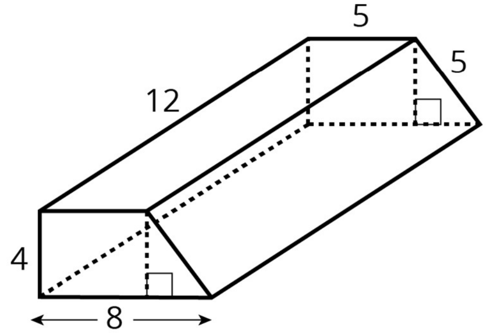


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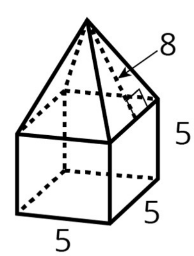



I

## Instructional Routines

- Compare and Connect

## Launch

Arrange students in groups of 2–3. Give each student in the group a different polyhedron from the blackline master and access to their geometry toolkits. Students need graph paper and a straightedge from their toolkits.

Explain to students that they will draw a net, find its surface area, and have their work reviewed by a peer. Give students 4–5 minutes of quiet time to draw their net on graph paper and then 2–3 minutes to share their net with their group and get feedback. When the group is sure that each net makes sense and all polygons of each polyhedron are accounted for, students can proceed and use the net to help calculate surface area.

If time permits, prompt students to cut and assemble their net into a polyhedron. Demonstrate how to add flaps to their net to accommodate gluing or taping. There should be as many flaps as there are edges in the polyhedron. (Remind students that this is different than the number of edges in the polygons of the net.)

*Representation: Internalise Comprehension.* Activate or supply background knowledge about calculating surface area. Some students may benefit from watching a physical demonstration of how to draw the net for a sample prism. Invite students to engage in the process by offering suggested directions as you demonstrate.
*Supports accessibility for: Visual-spatial processing; Organisation*

## Anticipated Misconceptions

Students may know what polygons make up the net of a polyhedron but arrange them incorrectly on the net (i.e., when cut and assembled the faces overlap instead of meeting at shared edges, or the faces are oriented incorrectly or are in the wrong places). Suggest that students label some faces of the polyhedron drawing and transfer the adjacencies they see to the net. If needed, demonstrate the reasoning (e.g., “Face 1 and face 5 both share the edge that is 7 units long, so I can draw them as two attached rectangles sharing a side that is 7-unit long.”)

It may not occur to students to draw each face of the polyhedron to scale. Remind them to use the grid squares on their graph paper as units of measurement.

If a net is inaccurate, this becomes more evident when it is being folded. This may help students see which parts need to be adjusted and decide the best locations for the flaps. Reassure students that a few drafts of a net may be necessary before all the details are worked out, and encourage them to persevere.

## Student Task Statement

Your teacher will give you a drawing of a polyhedron. You will draw its net and calculate its surface area.

1. What polyhedron do you have?

2. Study your polyhedron. Then, draw its net on graph paper. Use the side length of a grid square as the unit.

3. Label each polygon on the net with a name or number.

4. Find the surface area of your polyhedron. Show your thinking in an organised manner so that it can be followed by others.

### Student Response

1. Answers vary depending on polyhedron received. A and C are cuboids. B, D and E are triangular prisms. F and G are square-based pyramids. H is a trapezium-based prism. I is a composite of a cube and a square-based pyramid.

2. Net drawings vary. A and C should have 6 rectangles. B, D, and E should have 5 polygons: 2 right-angled triangles and 3 rectangles. F and G should have 5 polygons: 1 square and 4 triangles. H should have 6 polygons: 2 trapeziums and 4 rectangles. I should have 9 polygons: 5 squares and 4 triangles.

3. Answers vary.

4. Answers vary.

    - A: 340 square units. $2(5 \times 8) + 2(5 \times 10) + 2(8 \times 10) = 340$
    - B: 408 square units. $2(\frac{1}{2} \times 6 \times 8) + (6 \times 15) + (8 \times 15) + (10 \times 15) = 408$
    - C: 274 square units. $2(13 \times 4) + 2(13 \times 5) + 2(4 \times 5) = 274$
    - D: 300 square units. $2(\frac{1}{2} \times 5 \times 12) + (5 \times 8) + (12 \times 8) + (13 \times 8) = 300$
    - E: 216 square units. $2(\frac{1}{2} \times 6 \times 4) + (6 \times 12) + 2(5 \times 12) = 216$
    - F: 240 square units. $4(\frac{1}{2} \times 8 \times 11) + (8 \times 8) = 240$
    - G: 156 square units. $4(\frac{1}{2} \times 6 \times 10) + (6 \times 6) = 156$
    - H: 316 square units. The trapezium shaped base can be decomposed into a 5-by-4 rectangle and a right-angled triangle with a base of 3 units and a height of 4. $2(5 \times 4) + 2(\frac{1}{2} \times 3 \times 4) + (8 \times 12) + 2(5 \times 12) + (4 \times 12) = 316$
    - I: 205 square units. $5(5 \times 5) + 4(\frac{1}{2} \times 5 \times 8) = 205$

### Activity Synthesis

Ask students who finish their calculation to find another person in the class with the same polyhedron and discuss the following questions (displayed for all to see):

- Do your calculations match? Should they?
- Do your nets result in the same polyhedra? Should they?
- Do your models match the picture you were given? Why or why not?

If time is limited, consider having the answer key posted somewhere in the classroom so students could quickly check their surface area calculations.

Reconvene briefly for a whole-class discussion. Invite students to reflect on the process of drawing a net and finding surface area based on a picture of a polyhedron. Ask questions such as:

- How did you know that your net shows all the faces of your polyhedron?
- How did you know where to put each polygon or how to arrange all polygons so that, if folded, they can be assembled into the polyhedron in the drawing?
- How did the net help you find surface area?

*Representing, Conversing: Compare and Connect.* Use this routine to help students consider their audience when preparing a visual display of their work. Ask students to prepare a visual display that shows their net drawings and surface area calculations. Students should consider how to display their calculations so that another student can interpret them. Some students may wish to add notes or details to their drawings to help communicate their thinking. When students find another person in the class with the same polyhedron, provide 2–3 minutes of quiet think time for students to read and interpret each other's drawings before they discuss the questions on display.
*Design Principle(s): Cultivate conversation; Maximise meta-awareness*

## 15.3 Comparing Boxes

**Optional: 15 minutes**

In this activity, students compare the surface areas and volumes of three cuboids given nets that are not on a grid. To do this, they need to be able to visualise the three-dimensional forms that the two-dimensional nets would take when folded.

In KS2, students had learned to distinguish area and volume as measuring different attributes. This activity clarifies and reinforces that distinction.

### Instructional Routines

- Discussion Supports
- Think Pair Share

### Launch

Keep students in the same groups of 2–3. Tell students that this activity involves working with both volume and surface area. To refresh students' understanding of volume from KS2, ask students:

- “When we find the volume of a cuboid, what are we measuring?”
- “How is volume different than surface area?”
- “How might we find the volume of a cuboid?”

Reiterate that volume measures the number of unit cubes that can be packed into a three-dimensional shape and that we can find the number of unit cubes in a cuboid by multiplying the side lengths of the cuboid.

Give students 1–2 minutes to read the task statement and questions. Ask them to think about how they might go about answering each question and to be prepared to share their ideas. Give students a minute to discuss their ideas with their group. Then, ask groups to collaborate: each member should perform the calculations for one cuboid (A, B, or C). Give students 5–7 minutes of quiet time to find the surface area and volume for their cuboid and then additional time to compare their results and answer the questions.

*Action and Expression: Internalise Executive Functions.* Provide students with a graphic organiser for recording measurements and calculations of surface areas and volumes of the boxes.
*Supports accessibility for: Language; Organisation*

### Anticipated Misconceptions

Students should have little trouble finding areas of rectangles but may have trouble keeping track of pairs of measurements to multiply and end up making calculation errors. Suggest that they label each polygon in the net and the corresponding written work and double-check their calculations to minimise such errors.

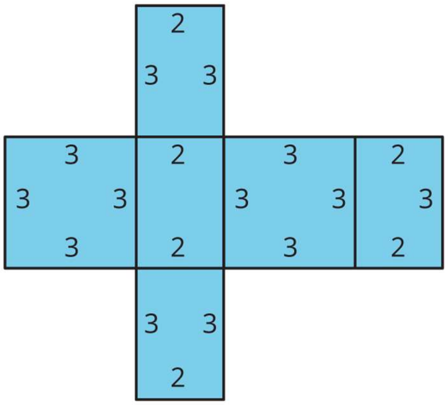
If students struggle to find the volume of their cuboid using information on a net, suggest that they sketch the cuboid that can be assembled from the net and label the edges of the cuboid.

Students may need a reminder that area is measured in square units and volume is measured in cubic units.

### Student Task Statement

Here are the nets of three cardboard boxes that are all cuboids. The boxes will be packed with 1-centimetre cubes. All lengths are in centimetres.

A

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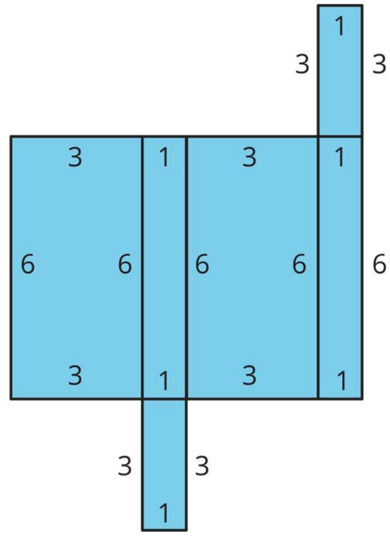


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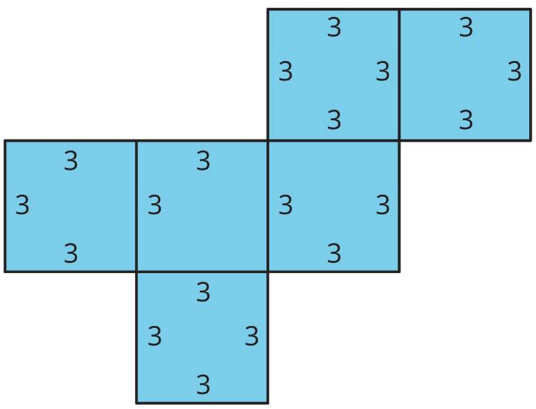



1. Compare the surface areas of the boxes. Which box will use the least cardboard? Show your reasoning.

2. Now compare the volumes of these boxes in cubic centimetres. Which box will hold the most 1-centimetre cubes? Show your reasoning.

### Student Response

1. The surface area of A is 42 square centimetres. A: $4(2 \times 3) + 2(3 \times 3) = 42$. The surface area of B and C is 54 square centimetres. B: $2(3 \times 6) + 2(3 \times 1) + 2(6 \times 1) = 54$. C: $6(3 \times 3) = 54$. Box A uses the least cardboard. Boxes B and C require the same amount of cardboard, both more cardboard than A.

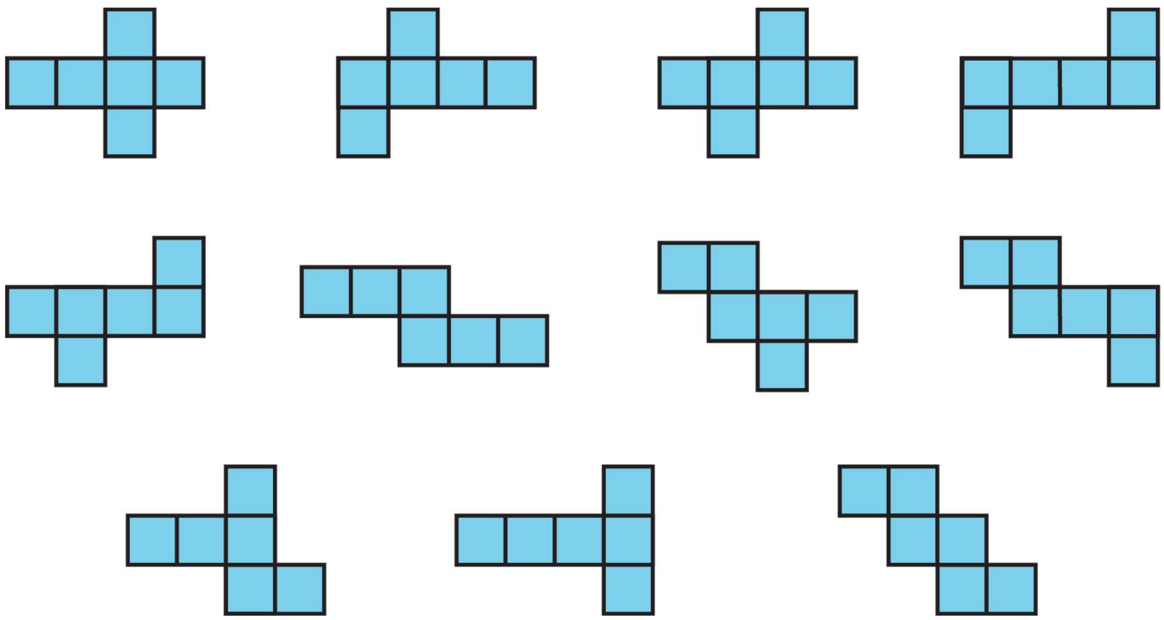
2. The volume of A and B is 18 cubic centimetres. A: $3 \times 2 \times 3 = 18$. B: $6 \times 1 \times 3 = 18$. The volume of C is 27 cubic centimetre. C: $3 \times 3 \times 3 = 27$. Box C fits the most 1-centimetre cubes. A and B fit the same number of cubes, but fewer than C.

### Are You Ready for More?

Shape C is a net of a cube. Draw a different net of a cube. Draw another one. And then another one. How many different nets can be drawn and assembled into a cube?

### Student Response

There are 11 different nets for a cube. Any other net would be congruent to one of these.



### Activity Synthesis

Select a few students to share the surface area and volume of each cuboid. After each person shares, poll those who worked on the same cuboid for agreement or disagreement. Record the results on the board.

Invite students to share a few quick observations about the relationship between the surface areas and volumes for these three cuboids, or between the amounts of material needed to build the boxes and the number of cubes that they can contain. Discuss questions such as:

- "If these cuboids are boxes, which cuboid—B or C—would take more material to build? Which can fit more unit cubes?" (B and C would likely take the same amount of material to build since their surface areas are the same. C has a greater volume than B, so it can fit more unit cubes.)
- "Which cuboid—A or B—would take more material to build? Which can fit more unit cubes?" (A and B can fit the same number of unit cubes but, B would require more material to build.)
- "If two cuboids have the same surface area, would they also have the same volume? How do you know?" (No, cuboids A, B, and C are examples of how two shapes with the same volume may not have the same surface area, and vice versa.)

Students will gain more insights into these ideas as they explore squares, cubes, and exponents in upcoming lessons. If students could benefit from additional work on distinguishing area and volumes as different measures, do the optional lesson Distinguishing Between Surface Area and Volume.

*Representing, Speaking: Discussion Supports.* Use this routine to support the use of mathematical language when comparing surface area and volume. Give students 3–4 minutes to write a response to the following prompt: “If two cuboids have the same surface area, their volume will *always/sometimes/never* be the same because . . .” Invite students to discuss their responses with a partner before selecting 1–2 students to share with the whole class. Listen for and call students' attention to how the use of examples and counterexamples can help justify their reasoning.
*Design Principle(s): Optimise output (for justification); Support sense-making*

### Lesson Synthesis

To highlight some key points from the lesson, display a picture of a prism or a pyramid and a drawing of its net. Discuss these questions:

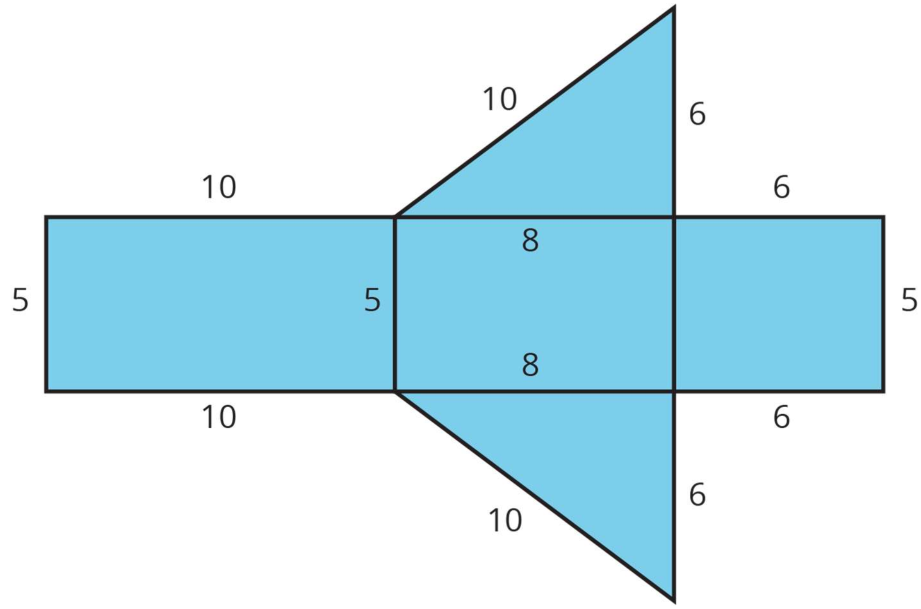
- “Can you find the surface area of a simple prism or pyramid from a picture, if all the necessary measurements are given?”
- “Can you find the surface area from a net, if all the measurements are given?”
- “Which might be more helpful for calculating surface area—a picture of a polyhedron or a net?” (If the polyhedron is simple—e.g., a cube, a square-based pyramid, etc.—and does not involve hidden faces with different measurements or require a lot of visualising, either a picture or a net can work. Otherwise, a net may be more helpful because we can see all of the faces at once and can find the area of each polygon more easily. A net may also help us keep track of our calculations and notice missing or extra areas.)

## 15.4 Surface Area of a Triangular Prism

**Cool Down: 5 minutes**

### Student Task Statement

1. In this net, the two triangles are right-angled triangles. All quadrilaterals are rectangles. What is its surface area in square units? Show your reasoning.




2. If the net is assembled, which of the following polyhedra would it make?

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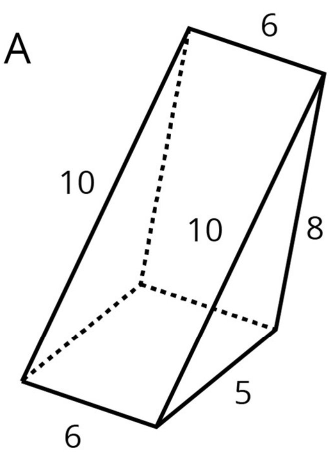


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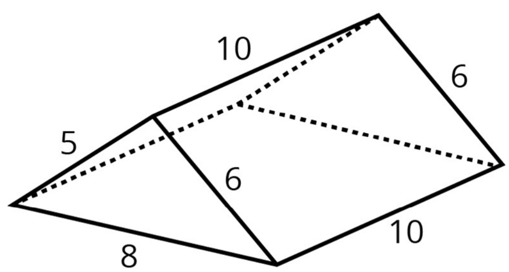


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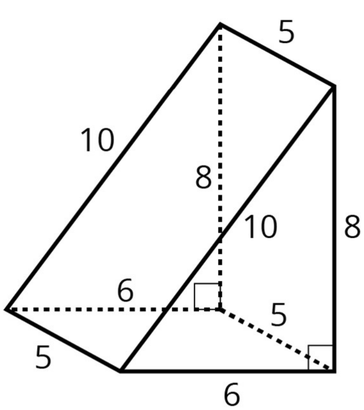


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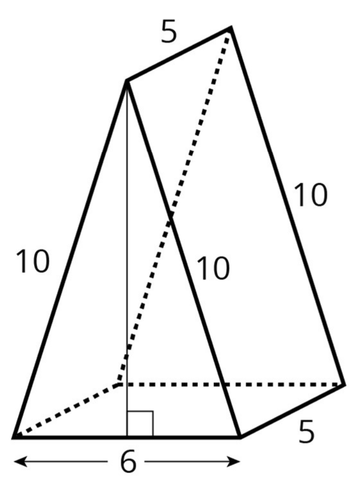


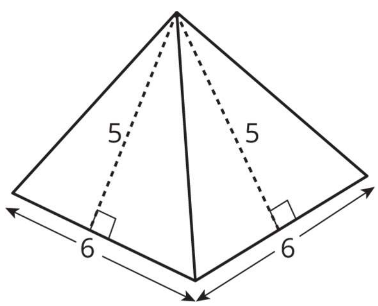
## Student Response

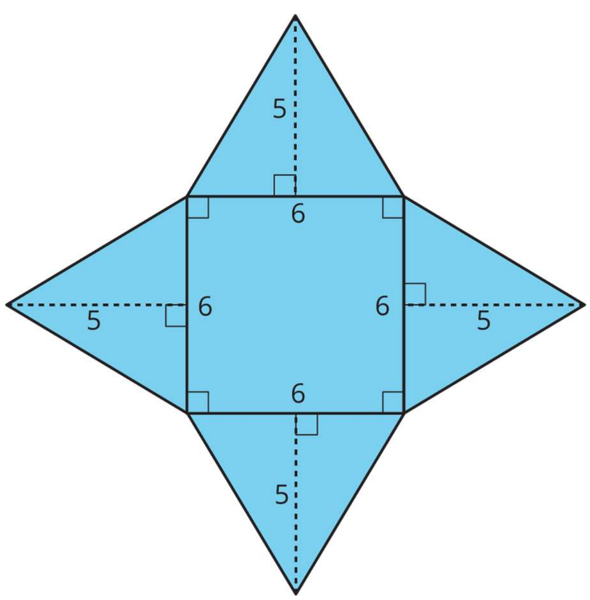
1. The surface area is 168 square units. Explanations vary. Sample response: There are two triangular faces with area of 24 square units each. $\frac{1}{2} \times 6 \times 8 = 24$. There is a rectangular face with area of 50 square units. $10 \times 5 = 50$. There is one rectangular face with area of 40 square units. $5 \times 8 = 40$. There is one rectangular face with area $5 \times 6 = 30$ square units. $2 \times 24 + 50 + 40 + 30 = 168$

2. Prism C

## Student Lesson Summary

The surface area  of a polyhedron is the sum of the areas of all of the faces. Because a net shows us all faces of a polyhedron at once, it can help us find the surface area. We can find the areas of all polygons in the net and add them.




A square-based pyramid has a square and four triangles for its faces. Its surface area is the sum of the areas of the square base and the four triangular faces:

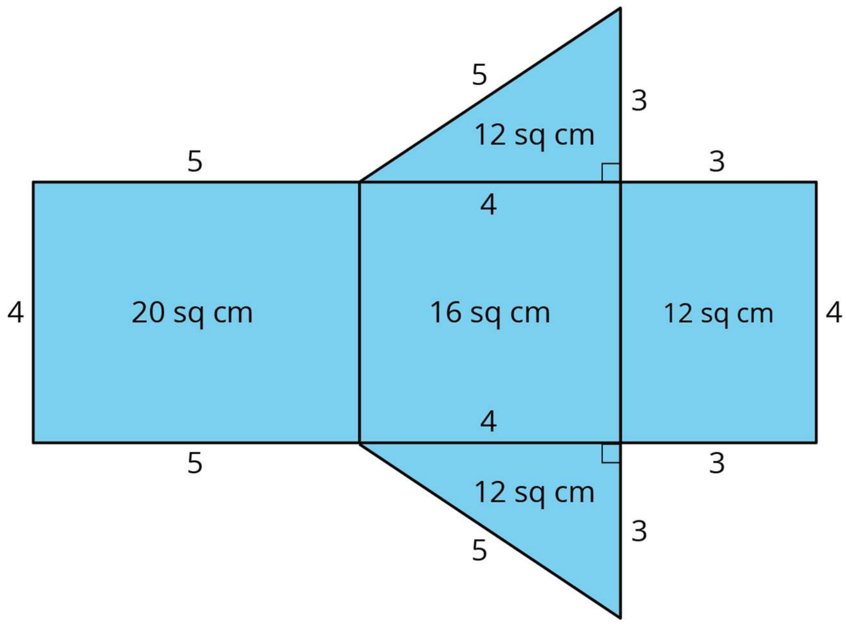
$$(6 \times 6) + 4 \times \left(\frac{1}{2} \times 5 \times 6\right) = 96$$

The surface area of this square-based pyramid is 96 square units.

## Lesson 15 Practice Problems

1. **Problem 1 Statement**

   Jada drew a net for a polyhedron and calculated its surface area.




   a. What polyhedron can be assembled from this net?

   b. Jada made some mistakes in her area calculation. What were the mistakes?

   c. Find the surface area of the polyhedron. Show your reasoning.

   **Solution**

   a. Triangular prism

   b. She calculated the areas of the two triangular faces incorrectly. The right-angled triangles have a base of 4 cm and a height of 3 cm, so the area of each should be $\frac{1}{2} \times 4 \times 3$ or 6 $cm^2$. Jada wrote “12 sq cm” for the area of each triangle.

   c. 60 $cm^2$. The triangular faces should be 6 sq cm each, so the surface area is $20 + 16 + 12 + 6 + 6$, or 60.

2. **Problem 2 Statement**

   A cereal box is 8 inches by 2 inches by 12 inches. What is its surface area? Show your reasoning. If you get stuck, consider drawing a sketch of the box or its net and labelling the edges with their measurements.

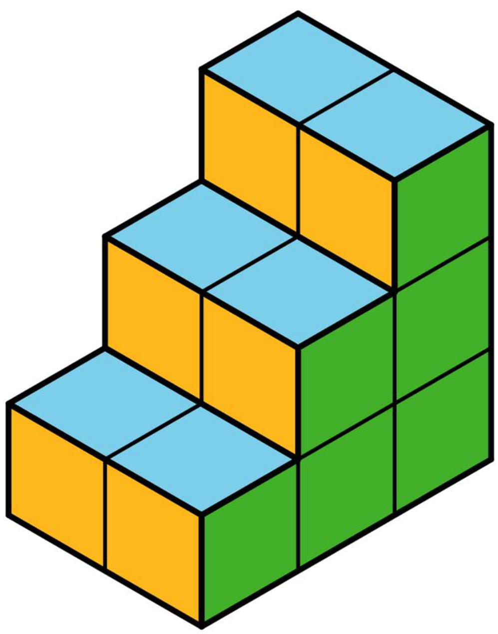
   **Solution**

272 square inches. Sample reasoning:

- The top and bottom faces are 2 inches by 8 inches each, so their combined area is $2(2 \times 8)$ or 32 square inches.
- The front and back faces are 8 inches by 12 inches each, so their combined area is $2(8 \times 12)$ or 192 square inches.
- The side faces are 2 inches by 12 inches each, so their combined area is $2(2 \times 12)$ or 48 square inches.
- The surface area is $32 + 192 + 48$ or 272 square inches.

3. **Problem 3 Statement**

Twelve cubes are stacked to make this shape.



a. What is its surface area?

b. How would the surface area change if the top two cubes are removed?

**Solution**

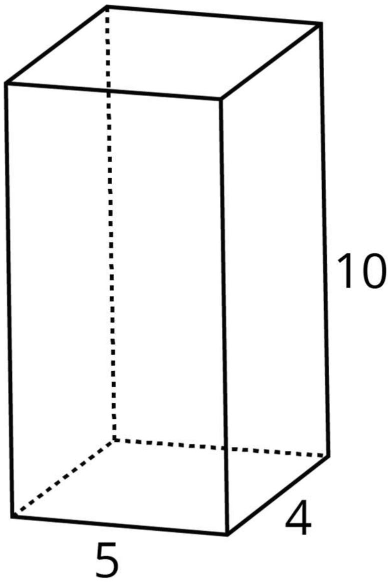
a. 36 square units

b. The surface area would decrease by 6 square units.

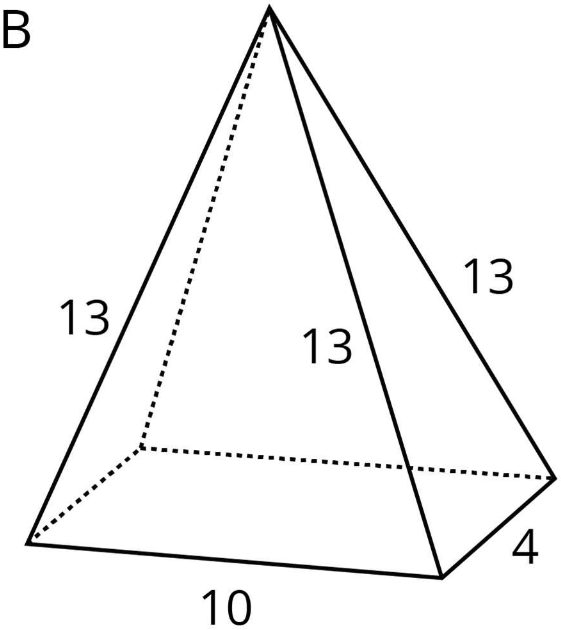
4. **Problem 4 Statement**

Here are two polyhedra and their nets. Label all edges in the net with the correct lengths.

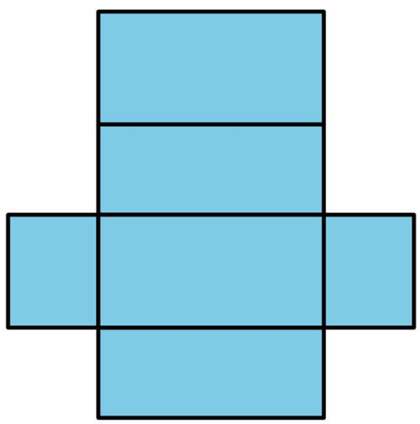
A



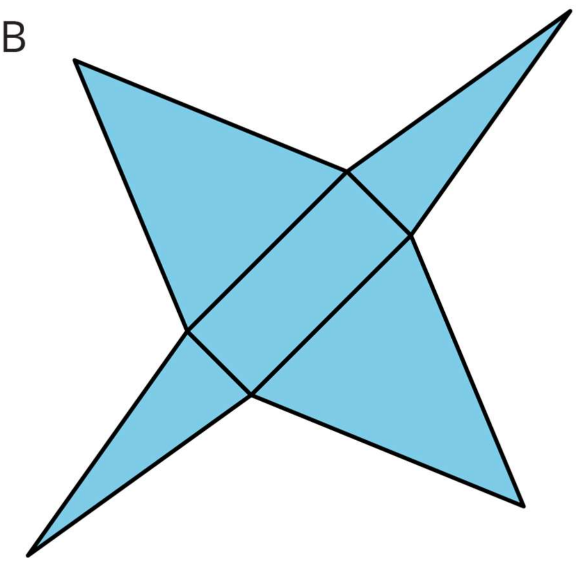

B



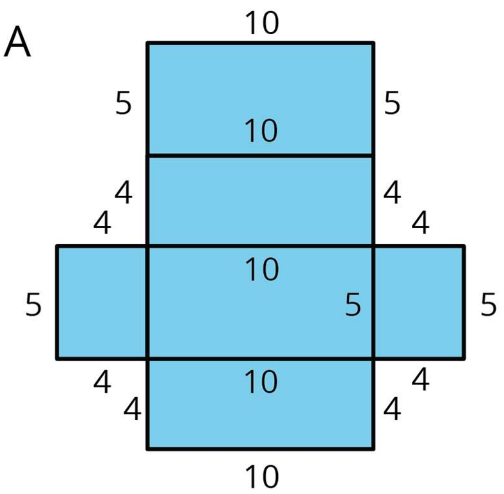

A

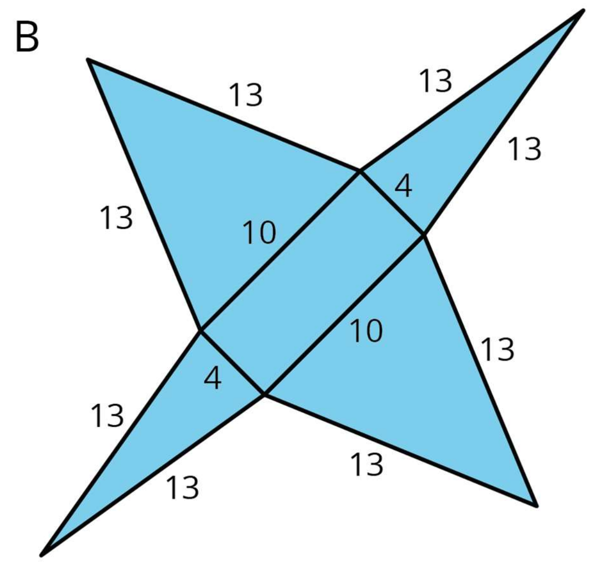


B



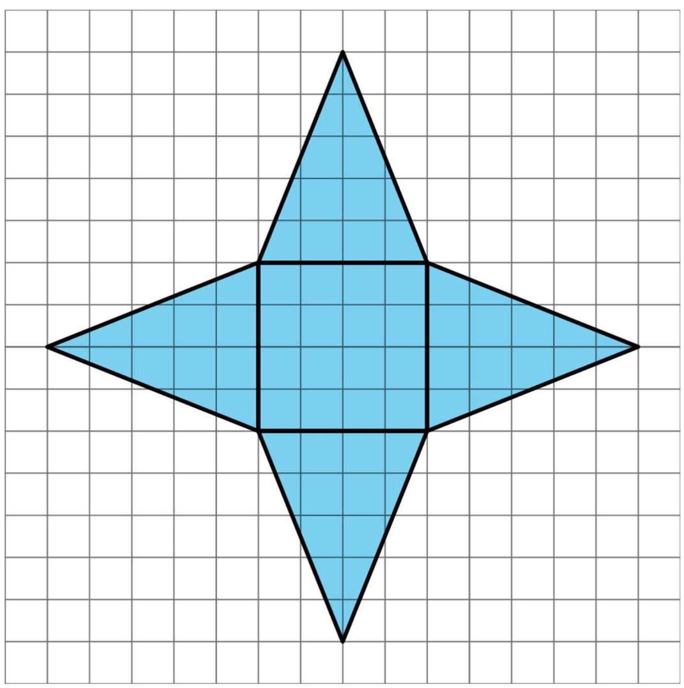
**Solution**




5. **Problem 5 Statement**

   a. What three-dimensional shape can be assembled from the net?



   b. What is the surface area of the shape? (One grid square is 1 square unit.)

**Solution**

   a. Square-based pyramid

   b. 56 square units. The area of the base is 16 square units. Each triangular face has a base of 4 units and a height of 5 units. This means each triangular face has an area of 10 square units. The total surface area is 56 square units, because $16 + 10 + 10 + 10 + 10 = 56$.