

Lesson 2: Changing temperatures

Goals

- Determine the final temperature given the starting temperature and the change in temperature, and explain (orally and using other representations) the solution method.
- Explain (orally) how to create a number line diagram that represents adding directed numbers.
- Write an addition equation to represent a situation involving a temperature increase or decrease.

Learning Targets

- I can use a number line to add positive and negative numbers.

Lesson Narrative

In this lesson, students represent addition of directed numbers on a number line. There are different ways to do this; in this unit, the convention is that each addend is represented by an arrow and the sum is represented as a point on the number line. Positive addends are represented by arrows that point to the right, and negative addends by arrows that point to the left. The first arrow starts at zero; the next arrow starts where the first arrow ends. The sum is represented by a point on the number line where the arrow for the last addend ends.

This lesson uses the context of temperature to help students make sense of the addition equations. Students see that an increase in temperature can be represented as adding a positive value and a decrease in temperature can be represented as adding a negative value. Students use quantitative contexts like temperature to aid in abstract reasoning about numeric expressions with directed numbers.

Addressing

- Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
- Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Building Towards

- Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
-

Instructional Routines

- Three Reads
- Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

Required Preparation

If desired, prepare to display a map showing the locations of:

- Houston, TX
- Orlando, FL
- Salt Lake City, UT
- Minneapolis, MN
- Fairbanks, AK

Student Learning Goals

Let's add directed numbers.

2.1 Which One Doesn't Belong: Arrows

Warm Up: 5 minutes

In this warm-up, students compare four number line diagrams with arrows. To give all students access to the activity, each diagram has one obvious reason it does not belong. Students will use diagrams like these later in the lesson to represent sums of directed numbers, but for this activity, the goal is to just get them used to analysing these types of diagrams carefully before they have to interpret them in terms of rational number arithmetic.

Instructional Routines

- Which One Doesn't Belong?

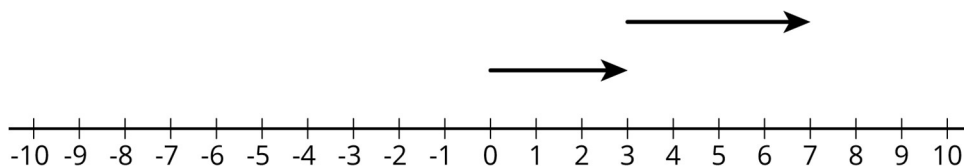
Launch

Arrange students in groups of 2–4. Display the image of the four figures for all to see. Ask students to indicate when they have noticed one figure that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason *each* figure doesn't belong.

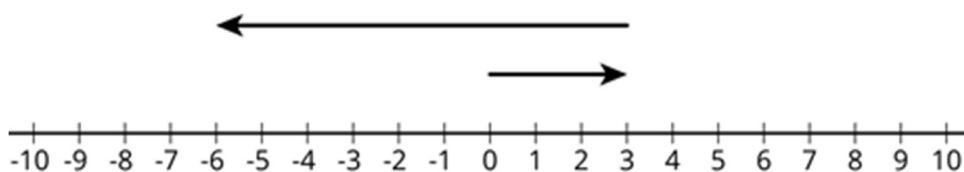
Student Task Statement

Which pair of arrows doesn't belong?

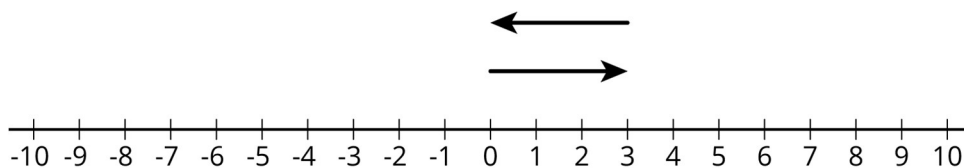
1.



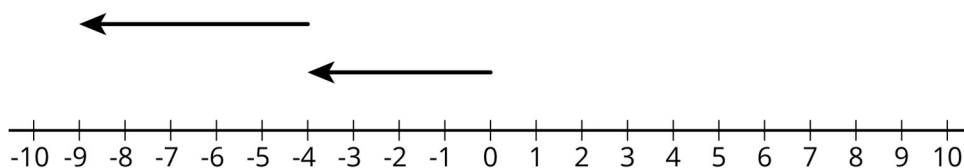
2.



3.



4.



Student Response

1. The only one where both arrows point right.
2. The only one where the arrows point in opposite directions and are different lengths.
3. The only one where the arrows point in opposite directions and are the same length.
4. The only one where both arrows point left.

Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular figure might not belong. Record and display the responses for all to see. After each response, poll the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which diagram does not belong, attend to students' explanations and ensure the reasons given make sense.

Ask the students what they think the arrows might represent. After collecting responses, say we are going to represent positive and negative numbers and their sums using arrows on a number line.

2.2 Warmer and Colder

15 minutes

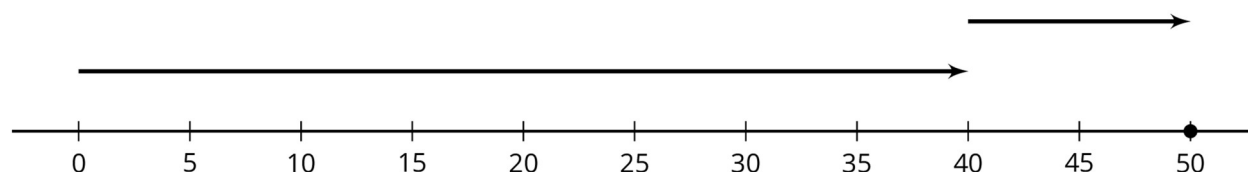
In this activity, the context of temperature is used to help students make sense of adding directed numbers. First students reason about temperature increases and decreases. They represent these increases and decreases on a number line, and then connect these temperature changes with adding positive numbers for increases and adding negative numbers for decreases. Students repeatedly add numbers to 40 and then to -20 to see that adding a positive number is the same as moving to the right on the number line and adding a negative number is the same as moving to the left on the number line.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Ask them, “If the temperature starts at 40 degrees and increases 10 degrees, what will the final temperature be?” Show them this number line:



Explain how the diagram represents the situation, including the start temperature, the change, and the final temperature. Point out that in the table, this situation is represented by an equation where the initial temperature and change in temperature are added together to find the final temperature.

Next, ask students to think about the change in the second row of the table. Give students 1 minute of quiet work time to draw the diagram that shows a decrease of 5 degrees and to think about how they can represent this with an *addition* equation. Have them discuss with a partner for 1 minute. Ask a few students to share what they think the addition equation should be. Be sure students agree on the correct addition equation before moving on. Tell students they will be answering similar questions,

- first by reasoning through the temperature change using whatever method makes sense,
- then drawing a diagram to show the temperature change, and

- finally, by writing an equation to represent the situation.

Give students 4 minutes of quiet work time followed by partner and then whole group discussion.

Representation: Internalise Comprehension. Use colour coding and annotations to highlight connections between the diagram and the situation. For example, annotate the diagram to show how the start temperature, the change, and the final temperature are represented. Encourage students to continue to annotate the number line diagrams for each situation in the task.

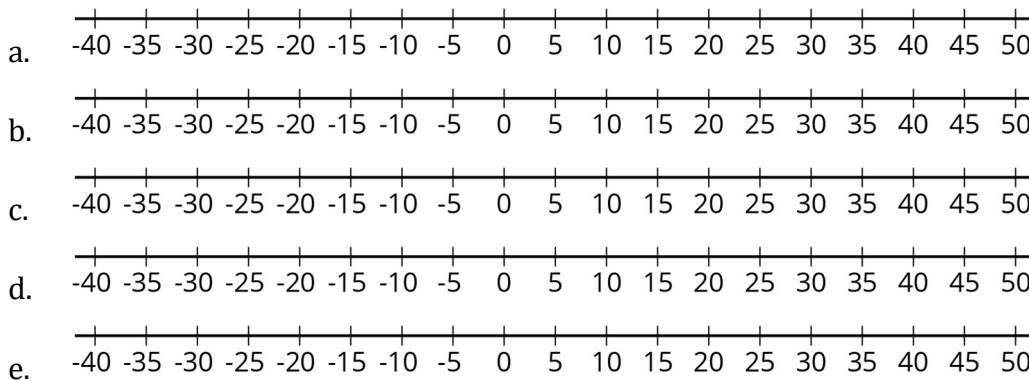
Supports accessibility for: Visual-spatial processing Listening, Representing: Discussion Supports. Demonstrate thinking aloud to describe possible approaches to represent the temperature change on a number line. Talk through your reasoning while you are representing and connecting the change on the number line and in the equation. This helps students hear the language used to explain mathematical reasoning and to see how that mathematical language connects to a visual representation.

Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

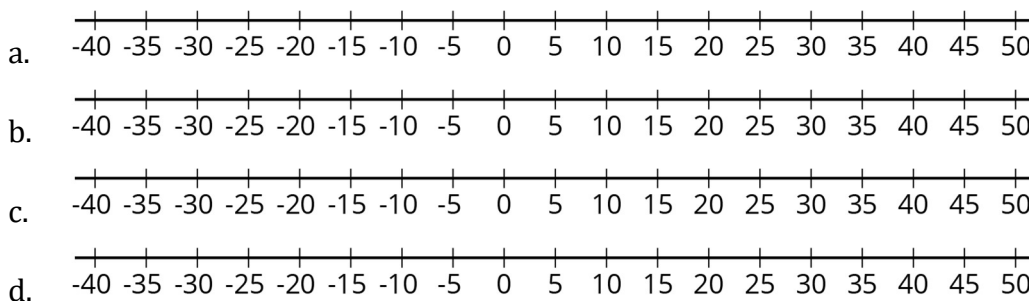
1. Complete the table and draw a number line diagram for each situation.

	start (°C)	change (°C)	final (°C)	addition equation
a	+40	10 degrees warmer	+50	$40 + 10 = 50$
b	+40	5 degrees colder		
c	+40	30 degrees colder		
d	+40	40 degrees colder		
e	+40	50 degrees colder		



2. Complete the table and draw a number line diagram for each situation.

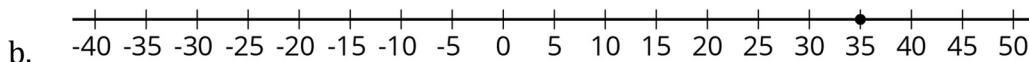
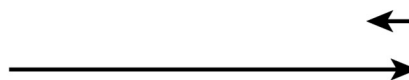
	start (°C)	change (°C)	final (°C)	addition equation
a	-20	30 degrees warmer		
b	-20	35 degrees warmer		
c	-20	15 degrees warmer		
d	-20	15 degrees colder		



Student Response

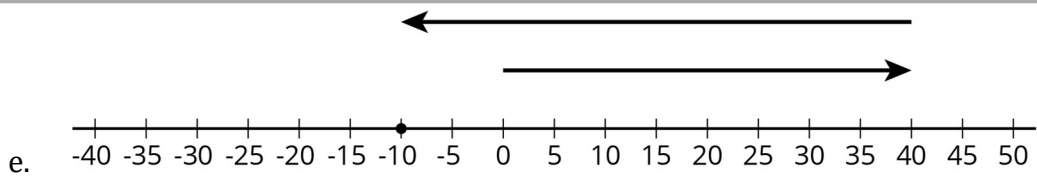
	start (°C)	change (°C)	final (°C)	addition equation
a	+40	10 degrees warmer	+50	$40 + 10 = 50$
b	+40	5 degrees colder	+35	$40 + -5 = 35$
c	+40	30 degrees colder	+10	$40 + -30 = 10$
d	+40	40 degrees colder	0	$40 + -40 = 0$
e	+40	50 degrees colder	-10	$40 + -50 = -10$

a. [shown in the launch]

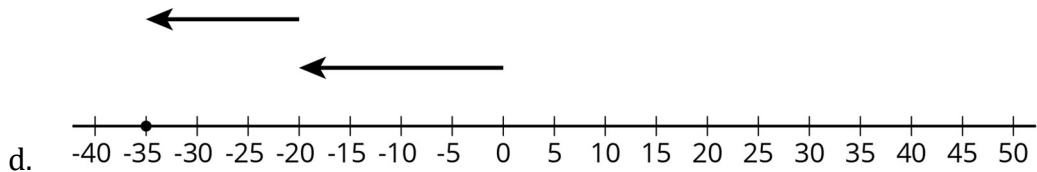
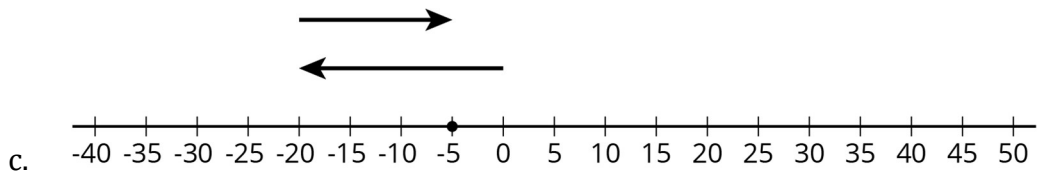
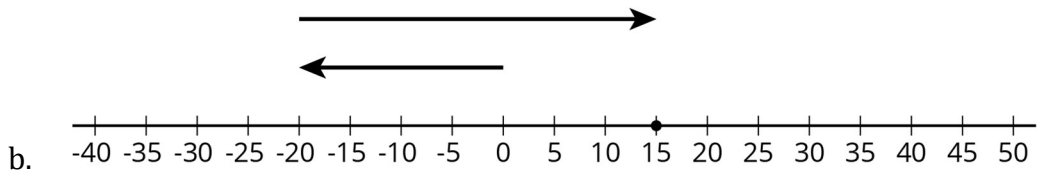
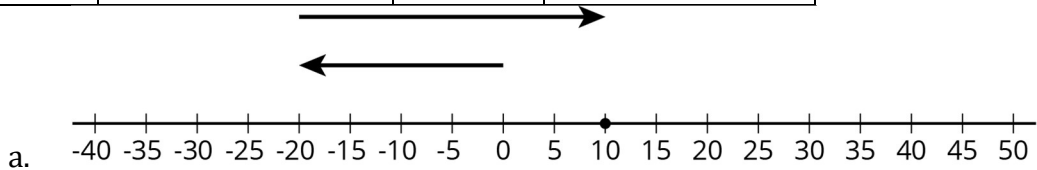


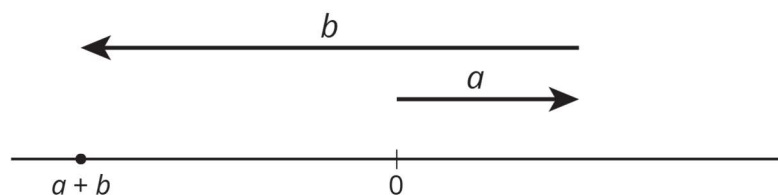
c. A number line with an arrow pointing from 0 to 40, another arrow pointing from 40 to 10, and a dot at 10.

d. A number line with an arrow pointing from 0 to 40, another arrow pointing from 40 to 0, and a dot at 0.



	start (°C)	change (°C)	final (°C)	addition equation
a	-20	30 degrees warmer	+10	$-20 + 30 = 10$
b	-20	35 degrees warmer	+15	$-20 + 35 = 15$
c	-20	15 degrees warmer	-5	$-20 + 15 = -5$
d	-20	15 degrees colder	-35	$-20 + -15 = -35$



Are You Ready for More?

For the numbers a and b represented in the figure, which expression is equal to $|a + b|$?

$|a| + |b|$

$|a| - |b|$

$|b| - |a|$

Student Response

$|a| - |b|$

Activity Synthesis

Ask students,

- “How can we represent an increase in temperature on a number line?” (An arrow pointing to the right.)
 - “How can we represent a decrease in temperature on a number line?” (An arrow pointing to the left.)
 - “How are positive numbers represented on a number line?” (Arrows pointing to the right.)
 - “How are negative numbers represented on a number line?” (Arrows pointing to the left.)
 - “How can we represent a sum of two numbers?” (Put the arrows so the tail of the second is at the tip of the first.)
 - “How can we determine the sum from the diagram?” (It is at the tip of the second arrow.)
 - “What happens when we add a positive number to another number?” (We move to the right on the number line.)
 - “What happens when we add a negative number to another number?” (We move to the left on the number line.)
-

2.3 Winter Temperatures

10 minutes (there is a digital version of this activity)

In this activity, students use what they learned in the previous activity to find temperature differences and connect them to addition equations. Students who use number line diagrams are using tools strategically. Students may draw number line diagrams in a variety of ways; what matters is that they can explain how their diagrams represent the situation. Students may think of these questions in terms of subtraction; that is completely correct, but the discussion should focus on how to think of these situations in terms of addition. Students will have an opportunity to connect addition and subtraction in a future lesson.

Instructional Routines

- Three Reads

Launch

Before students start working, it may be helpful to display a map of the United States and point out the locations of the cities in the problem. Explain that in the northern hemisphere, it tends to be colder the farther north you are.

If students are using the digital version, ask them to think about and respond to the questions before testing out their conclusions with the embedded applet. If students jump right to using the applet, they might skip some deep thinking that results from figuring out how to represent the initial temperature, change in temperature, and final temperature, as well as miss out on practice representing these operations with a number line.

Reading: Three Reads. Use this routine to support reading comprehension of this problem without solving it for students. Use the first read to orient students to the situation by asking students to describe it without using numbers (e.g., this problem is about temperatures in various cities; each city is warmer or colder than another). For the second read, identify the important quantities by asking students what can be counted or measured for each city (e.g., Houston is the only city with the actual temperature stated; the other temperatures given are relative values, such as “10 degrees warmer.”). For the third read, brainstorm strategies that can be used to find the temperatures in the other cities. This helps students to comprehend the problem, identify mathematical relationships in the text, and develop some problem solving approaches.

Design Principle(s): Support sense-making

Anticipated Misconceptions

If students struggle to find the temperature in Minneapolis, thinking that $8 - 20$ doesn't have an answer, suggest they represent the decrease in temperature as $8 + (-20)$ and use a number line to reason about the resulting temperature.

Student Task Statement

One winter day, the temperature in Houston is 8° Celsius. Find the temperatures in these other cities. Explain or show your reasoning.

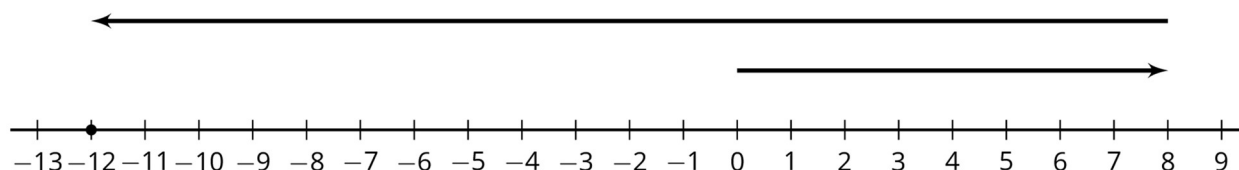
1. In Orlando, it is 10° warmer than it is in Houston.
2. In Salt Lake City, it is 8° colder than it is in Houston.
3. In Minneapolis, it is 20° colder than it is in Houston.
4. In Fairbanks, it is 10° colder than it is in *Minneapolis*.
5. Write an addition equation that represents the relationship between the temperature in Houston and the temperature in Fairbanks.

Student Response

1. Orlando is 18°C
2. Salt Lake City is 0°C
3. Minneapolis is -12°C
4. Fairbanks is -22°C
5. $8 + (-20) + (-10) = -22$ or equivalent

Activity Synthesis

Ask students who used number lines to share their reasoning. If a student correctly describes the situation in terms of subtraction, acknowledge that perspective and then ask them if they can also think of it in terms of addition. If no students used a number line, demonstrate how to represent these situations a number line diagram. For example, we can draw this diagram for Minneapolis:



The initial temperature of 8 is represented by an arrow starting at 0 and going 8 units to the right. The decrease in temperature is represented by an arrow starting at 8 and going 20 units to the left. The final temperature is represented by a point at -12, where the second arrow ends. Remind them that we can represent this with an addition equation: $8 + (-20) = -12$

Lesson Synthesis

Main learning points:

- We can represent a decrease as adding a negative number.
- We can represent addition on a number line with two arrows, the second arrow starting where the first arrow ends.
- We can represent a negative addend on a number line as an arrow pointing to the left.

Discussion questions:

- “How can you represent an increase or decrease in temperature using an addition equation?”
- “How can you represent an addition equation on a number line?”

2.4 Stories about Temperature

Cool Down: 10 minutes

Anticipated Misconceptions

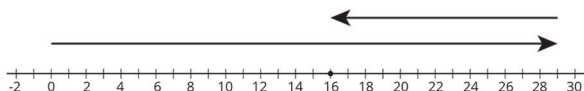
For the second question, some students might not pay attention to the word *to*, and interpret the question as the temperature dropped *by* 16° . Point out the word *to* in the question and tell them that their number line diagram should show the final temperature is 16°C .

Student Task Statement

1. Write a story about temperatures that this expression could represent: $27 + (-11)$
2. Draw a number line diagram and write an expression to represent this situation: “On Tuesday at lunchtime, it was 29°C . By sunset, the temperature had dropped to 16°C .”

Student Response

1. Answers vary. Sample response: It was 27 degrees at lunch time and by the evening the temperature has dropped 11 degrees.
2. Answers vary. Sample expression: $29 + (-13)$.



Student Lesson Summary

If it is 42° outside and the temperature increases by 7° , then we can add the initial temperature and the change in temperature to find the final temperature.

$$42 + 7 = 49$$

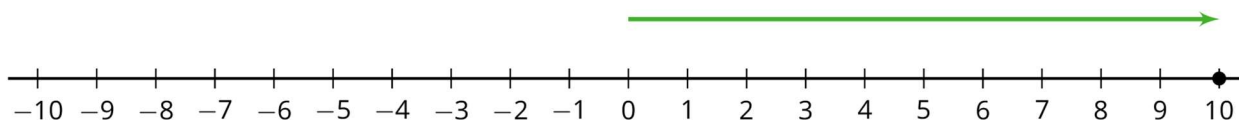
If the temperature decreases by 7° , we can either subtract $42 - 7$ to find the final temperature, or we can think of the change as -7° . Again, we can add to find the final temperature.

$$42 + (-7) = 35$$

In general, we can represent a change in temperature with a positive number if it increases and a negative number if it decreases. Then we can find the final temperature by adding the initial temperature and the change. If it is 3° and the temperature decreases by 7° , then we can add to find the final temperature.

$$3 + (-7) = -4$$

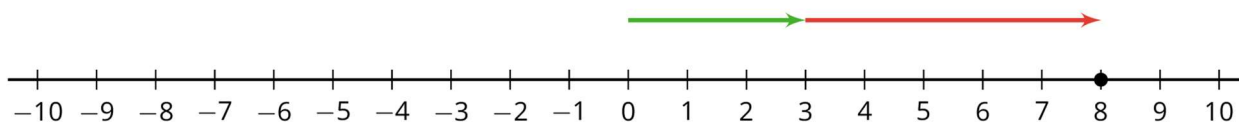
We can represent directed numbers with arrows on a number line. We can represent positive numbers with arrows that start at 0 and point to the right. For example, this arrow represents $+10$ because it is 10 units long and it points to the right.



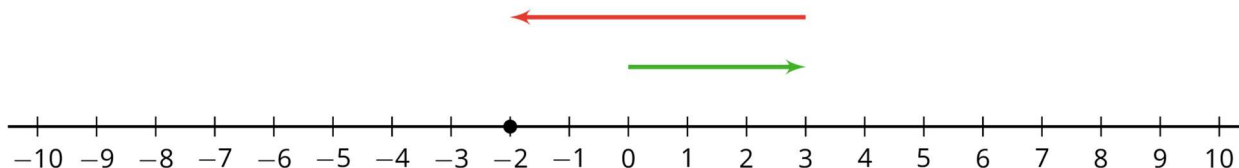
We can represent negative numbers with arrows that start at 0 and point to the left. For example, this arrow represents -4 because it is 4 units long and it points to the left.



To represent addition, we put the arrows “tip to tail.” So this diagram represents $3 + 5$:



And this represents $3 + (-5)$:



Lesson 2 Practice Problems

1. Problem 1 Statement

- The temperature is -2°C . If the temperature rises by 15°C , what is the new temperature?
- At midnight the temperature is -6°C . At midday the temperature is 9°C . By how much did the temperature rise?

Solution

- 13°C
- 15°C

2. Problem 2 Statement

Draw a diagram to represent each of these situations. Then write an addition expression that represents the final temperature.

- The temperature was 80°F and then fell 20°F .
- The temperature was -13°F and then rose 9°F .
- The temperature was -5°F and then fell 8°F .

Solution

Answers vary. Sample responses:

- A number line diagram with an arrow that starts at 80, is 20 units long, and points to the left. $80 + (-20)$
- A number line diagram with an arrow that starts at -13, is 9 units long, and points to the right. $-13 + 9$
- A number line diagram with an arrow that starts at -5, is 8 units long, and points to the left. $-5 + (-8)$

3. Problem 3 Statement

Complete each statement with a number that makes the statement true.

- ____ $< 7^{\circ}\text{C}$
- ____ $< -3^{\circ}\text{C}$
- $-0.8^{\circ}\text{C} < \text{____} < -0.1^{\circ}\text{C}$
- ____ $> -2^{\circ}\text{C}$

Solution

Answers vary. Sample responses:

- a. 5°C
- b. -8°C
- c. -0.7°C
- d. 4°C

4. Problem 4 Statement

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

- a. The number of wheels on a group of buses.

number of buses	number of wheels	wheels per bus
5	30	
8	48	
10	60	
15	90	

- b. The number of wheels on a train.

number of train coaches	number of wheels	wheels per train coach
20	184	
30	264	
40	344	
50	424	

Solution

- a. It could be proportional. The constant of proportionality would be 6 wheels per bus.
- b. It is not proportional. (Every train coach does not necessarily have the same number of wheels.)

5. Problem 5 Statement

Noah was assigned to make 64 cookies for the bake sale. He made 125% of that number. 90% of the cookies he made were sold. How many of Noah's cookies were left after the bake sale?

Solution

8 cookies. He made 80 cookies, because $(1.25) \times 64 = 80$.

72 were sold, because $(0.9) \times 80 = 72$.

Therefore 8 were left because $80 - 72 = 8$.



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