

Lessons 4-6 – Make Your Own Sundial!

Open the GeoGebra app and <https://www.geogebra.org/m/a4v7fzkt>

Day & Night Flight Map

1. Open *Terminator Line - Equirectangular Projection* in the *Map Projection* chapter and:
 - a. Animate or freeze the slider for “ t ” by clicking the play button at the bottom-left of the screen (*N.B. slower speeds are easier to watch!*);
 - b. experiment with the sliders for “ α ” and “ T ”.
 - i. What exactly do these variables represent?
 - ii. What does T affect – and why?
2. This type of map is called an *equirectangular projection* of the globe. It is closely related to the Mercator projection. Look online to answer:
 - a. How is a Mercator projection made?
 - b. What is the main difference between these two projections?
 - c. What other well-known projections are there of the globe and why is there more than one?
 - i. Have a look at the Gall-Peters Projection online. Why is the Mercator Projection sometime criticised as unfair in its representation of countries on Earth? (*Hint: consider latitudes and historical power relations.*)
 - ii. What is the benefit of using the Mercator Projection for the purposes of this GeoGebra graph?
 - d. What is a “terminator line”?
3. Go back to the map. Select the “Open in App” option in the menu to the top-right of the screen (click on the three vertical dots).
 - a. Carefully examine the x- and y-axes.
 - i. What do they measure?
 - ii. What other units would be appropriate to use?
 - iii. How do you convert between these units?
 - b. Approximately what coordinates would Greenwich, London have?
 - i. Use an appropriate slider to help you – which one does this?
 - ii. What do you think is meant by the “Greenwich Meridian”?
 - iii. Look up the GPS coordinates of Greenwich, London. How do these relate to λ and α ?
 - iv. When $t = 0$, what time is it at $(15^\circ, 51^\circ)$? (*N.B. pay attention to the units!*)
 - c. When $t=1$, what time is it:
 - i. at $(-15^\circ, 0^\circ)$?
 - ii. In Greenwich Mean Time (GMT)?
 - d. Use an appropriate slider to determine approximately:
 - i. what time is sunset on Midsummer Day?

- ii. In which month(s) do people in Cuba have the smallest shadows?
- e. How can you determine the length of the day for any given latitude using one of the GeoGebra tools (to the top-left of the screen)? (*Hint: consider the relationship between x , t and ω from the last lesson.*)
- f. Find the length of the day at $+51^\circ$ latitude on Midsummer Day (21st June).
- g. Open the graph you made last lesson. How does it compare?
- h. How long is the day on 21st March?
- i. Imagine if an asteroid collided with the Earth and the polar axis changed direction. Find and an appropriate slider in the Algebra pane on the left of the screen and show it on-screen to see what happens if the Obliquity of the Ecliptic (tilt of the Earth) were more than 23.5° .
 - i. The equator of the planet Uranus is nearly at right angles to its orbit. What effect would this have on Earth for:
 1. the length of the day?
 2. the seasons?

Day Length Formula – a New Angle?

4. How would you describe the vector that points to the zenith (the point in the sky directly above you)? (*Hint: what significant points does the infinite straight line it occupies pass through?*)
5. Imagine a vector pointing towards the Sun (from your eye).
 - a. Can you find a general expression for the **angle** between this vector and the one that points to the zenith? (*Hint: draw a diagram!*)
6. Now look at the (interactive) diagram *Angle between the Sun and the Zenith*.
 - a. How does it compare with your diagram? What would you draw differently as a result?
 - b. Remembering the last lesson, can you explain why the angles shown are labelled " α " and " θ "?
7. In the chapter entitled *Frames of Reference*, open *Transforming Between 3D Cartesian Coordinate Systems on the Surface of the Earth* (made using the GeoGebra Geometry Calculator).
 - a. How does your diagram compare with this? Which is more helpful – why?
 - b. Can you prove that the angles shown at point A are correctly labelled " $\alpha - \theta$ " and " θ "?
 - c. Click on A and drag it around. What do you notice?
 - d. Recreate this diagram using pencil and paper. Label all the other angles you can see using the appropriate variables.
8. Have a go at finding the expression for ϕ' labelled in *Angle between the Sun and the Zenith*. (*Hint: consider the rules you have learnt concerning **both** right-angled **and** non-right-angled triangles.*)

9. What value does ϕ' have at sunset? Why?
10. Answer the questions on the IWB.
11. What is the relationship between ϕ' and your own shadow?

Make Your Own Sundial

12. Imagine that in questions 4 and 5 that it is High Noon and you are standing up straight, outside on the football pitch, on a clear, sunny day.
 - a. What will the bearing of the Sun be? Is this the same everywhere?
 - b. What will the bearing of your shadow be?
 - c. How long will your shadow be?
13. Roughly how would these answers to question 12 vary at other times of:
 - a. day?
 - b. year?
14. Look at the questions on the IWB and follow the instructions...
15. Look at the “hour lines” on the teacher’s sundial in GeoGebra on the IWB. What do you think will happen for negative values of α . Why?
16. The teacher will print one of each type of sundial made by your learning group so that you can construct them and go outside to test them!
 - a. Before doing so, what equipment do you need to position your sundial accurately?
 - b. How would you have designed a “poke” sundial for use in Shakespeare’s time? What would people have had to do with them to tell the time when asked?
 - c. Would it have been worth doing those things? (*Hint: consider how else people told the time and how accurate they needed it to be back then...*)