Lesson 6: Using equations to solve problems

Goals

- Generate an equation for a proportional relationship, given a description of the situation but no table.
- Interpret (orally) each part of an equation that represents a proportional relationship in an unfamiliar context.
- Use an equation to solve problems involving a proportional relationship, and explain (orally) the reasoning.

Learning Targets

- I can find missing information in a proportional relationship using the constant of proportionality.
- I can relate all parts of an equation like $y = kx$ to the situation it represents.

Lesson Narrative

In the previous two lessons students learned to represent proportional relationships with equations of the form $y = kx$. In this lesson they continue to write equations, and they begin to see situations where using the equation is a more efficient way of solving problems than other methods they have been using, such as tables and equivalent ratios.

The activities introduce new contexts and, for the first time, do not provide tables; students who still need tables should be given a chance to realise that and create tables for themselves. The activities are intended to motivate the usefulness of representing proportional relationships with equations, while at the same time providing some scaffolding for finding the equations.

Building On

- Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a\colon b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

Addressing

- Recognise and represent proportional relationships between quantities.

- Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Three Reads
- Discussion Supports
- Number Talk
- Think Pair Share

Required Materials

Four-function calculators

Student Learning Goals

Let's use equations to solve problems involving proportional relationships.

6.1 Number Talk: Quotients with Decimal Points

Warm Up: 5 minutes

The purpose of this Number Talk is to elicit the strategies and understandings that students have for determining how the size of a quotient changes when the divisor or dividend is multiplied by a power of 10. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to check the reasonableness of their answers. While four problems are given in this Number Talk it may not be possible to share answers for all of them.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Arrange students in groups of 2. Display the first question. Give students 2 minutes of quiet think time and ask them to give a signal when they have an answer, and reasoning, to support their answer. Follow with a whole-class discussion. Display the second question and give students 1 minute of quiet think time.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Without calculating, order the quotients of these expressions from least to greatest.

$42.6 \div 0.07$

$42.6 \div 70$

$42.6 \div 0.7$

$426 \div 70$

Place the decimal point in the appropriate location in the quotient: $42.6 \div 7 = 608571$

Use this answer to find the quotient of *one* of the previous expressions.

Student Response

- b, d, c, a
  - 6.08571
  - Answers vary. Possible responses in order a through d: 608.571; 0.608571; 60.8571; 6.08571

Activity Synthesis

Ask students to share where they placed the decimal point in the second question and their reasoning. After students share, ask the class if they agree or disagree. Ask selected students, who chose different problems to solve, to share quotients for the problems in the first question. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

Emphasise student reasoning based in place value that involves looking at the relationship between the dividend and divisor to determine the size of the quotient.

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

6.2 Concert Ticket Sales

15 minutes

This activity requires students to work with larger numbers, which is intended to encourage students to use an equation and notice the efficiencies of doing so. It also emphasises the interpretation of the constant of proportionality in the context. In this case, the constant represents the cost of a single ticket, and makes it easy to identify which singer would make more money for similar ticket sales in a concert series. Note that asking students to give the revenues for different ticket sales encourages looking for and expressing regularity in repeated reasoning. The last set of questions ask students to interpret the constant of proportionality as represented in an equation in terms of the context.

Monitor for students who solve the problems using the following strategies and invite them to share during the whole-class discussion.

- writing many calculations, without any organisation
- creating a table to organise their results
- writing an equation to encapsulate repeated reasoning

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports

Launch

Provide access to calculators. Consider using the names of actual performers to make the task more interesting to students.

Representation: Internalise Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

A performer expects to sell 5 000 tickets for an upcoming concert. They want to make a total of £311 000 in sales from these tickets.

1. Assuming that all tickets have the same price, what is the price for one ticket?
2. How much will they make if they sell 7 000 tickets?
3. How much will they make if they sell 10 000 tickets? 50 000? 120 000? A million? x tickets?

4. If they make £404 300, how many tickets have they sold?

5. How many tickets will they have to sell to make £5 000 000?

Student Response

1. £62.20

2. £435 400

3. £622 000; £3 110 000; £7 464 000; £62 200 000; 62.2x

4. 6 500

5. 80 386

number of tickets sold	earnings in pounds
5 000	311 000
1	62.20
7 000	435 400
10 000	622 000
50 000	3 110 000
120 000	7 464 000
1 000 000	62 200 000
6 500	404 300
80 386	5 000 009.20
x	62.20x

Activity Synthesis

Select student responses to be shared with the whole class in discussion. Sequence their explanations from less efficient and organised to more efficient and organised. Discuss how the solutions are the same and different, and the advantages and disadvantages of each method. An important part of this discussion is correspondences and connections between different approaches.

Speaking, Listening: Discussion Supports. Use this routine to support whole-class discussion. For each explanation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they heard to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of

others.
Design Principle(s): Support sense-making

6.3 Recycling

15 minutes

This activity is intended to further develop students' ability to write equations to represent proportional relationships. It involves work with decimals and asks for equations that represent proportional relationships of different pairs of quantities, which increases the challenge of the task.

Students may solve the first two problems in different ways. Monitor for different solution approaches such as: using computations, using tables, finding the constant of proportionality, and writing equations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Three Reads
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to calculators. Give 5 minutes quiet work time followed by sharing work with a partner.

Representation: Internalise Comprehension. Represent the same information through different modalities by using tables. If students are unsure where to begin, suggest that they draw a table to help organise the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing Reading: Three Reads. Use this routine to support reading comprehension, without solving, for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., The situation involves weight of cans and the amount of money made from recycling). In the second read, ask students to look for quantities without focusing on specific values. Listen for, and amplify, the quantities that vary in relation to each other in this situation: number of aluminium cans; total weight of aluminium cans, in kilograms; money earned, in pounds. In the third read, ask students to brainstorm possible strategies to calculate the weight of aluminium in one can and the amount of money earned from one can. This helps students connect the language in the word problem and the reasoning needed to solve the problem keeping the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making

Anticipated Misconceptions

If students have trouble getting started, encourage them to create representations of the relationships, like a diagram or a table. If they are still stuck, suggest that they first find the weight and pound value of 1 can.

Student Task Statement

Aluminium cans can be recycled instead of being thrown in the garbage. The weight of 10 aluminium cans is 0.16 kilograms. The aluminium in 10 cans that are recycled has a value of £0.14.

1. If a family threw away 2.4 kg of aluminium in a month, how many cans did they throw away? Explain or show your reasoning.
2. What would be the recycled value of those same cans? Explain or show your reasoning.
3. Write an equation to represent the number of cans c given their weight w.
4. Write an equation to represent the recycled value r of c cans.
5. Write an equation to represent the recycled value r of w kilograms of aluminium.

Student Response

1. 150, because 2.4 is $(0.16) \times 15$, and $10 \times 15 = 150$.
2. £2.10, because $(0.14) \times 15 = 2.1$.
3. $c = 62.5w$
4. $r = 0.014c$
5. $r = 0.875w$

Here is one way to organise the given information and solutions in a table:

number of cans (c)	weight in kilograms (w)	recycled value in pounds (r)
10	0.16	0.14
150	2.4	2.10
1	0.016	0.014
62.5	1	
$62.5w$	w	
c		$0.014c$

	1	0.875
	w	$0.875w$

Are You Ready for More?

The EPA estimated that in 2013, the average amount of garbage produced in the United States was 4.4 pounds per person per day. At that rate, how long would it take your family to produce a ton of garbage? (A ton is 2 000 pounds.)

Student Response

Answers vary. Sample responses: A family of two would take about 32 and a half weeks. A family of three would take about 21 and a half weeks. A family of four would take about 15 and a half weeks.

Activity Synthesis

Select students to share their methods: using computations, using tables, finding the constant of proportionality, writing equations. If students did not use equations to solve the first two problems, ask them how they can use the equations they found later in the activity to answer the first two questions.

If time permits, highlight connections between the equations generated, illustrated by the sequence of equations below. $r = 0.014c \quad r = 0.014(62.5w) \quad r = 0.875w$

Lesson Synthesis

The activities in this lesson removed some scaffolds used in previous lessons (e.g., presenting a table) and included features (e.g., large numbers) intended to motivate use of equations. Remind students that throughout this lesson, they considered problem situations and created organised ways to get answers. Whether the numbers in the problem are whole numbers, large numbers, or decimals, if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. The situations provided demonstrate the efficiency of equations for certain types of problems. Finding how many tickets should be sold in order to earn £5 million and finding the relationship between number of cans and weight and recycled value are more elegantly and efficiently handled by equations than by calculations or tables.

- What were some helpful ways we organised information?
- What were some equations we found in this lesson?
- In each equation, what did the letters represent? What did the number mean? $y = 62.2x$, $c = 62.5w$, $r = 0.014c$, $r = 0.875w$

6.4 Granola

Cool Down: 5 minutes

Student Task Statement

Based on her recipe, Elena knows that 5 servings of granola have 1 750 calories.

1. If she eats 2 servings of granola, how many calories does she eat?
2. If she wants to eat 175 calories of granola, how many servings should she eat?
3. Write an equation to represent the relationship between the number of calories and the number of servings of granola.

Student Response

1. 700 calories. $1\,750 \div 5 = 350$, and $350 \times 2 = 700$.
2. $\frac{1}{2}$, because $175 = 350 \times \frac{1}{2}$.
3. If c represents the number of calories in s servings, then the equation could be either $c = 350s$ or $s = \frac{1}{350}c$.

Student Lesson Summary

Remember that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the easiest way to solve a problem.

For example, we know that Denali, the highest mountain peak in North America, is 20 310 feet above sea level. How many miles is that? There are 5 280 feet in 1 mile. This relationship can be represented by the equation

$f = 5\,280m$

where f represents a distance measured in feet and m represents the same distance measured miles. Since we know Denali is 20 310 feet above sea level, we can write

$20\,310 = 5\,280m$

So $m = \frac{20\,310}{5\,280}$, which is approximately 3.85 miles.

Lesson 6 Practice Problems

1. **Problem 1 Statement**

   A car is travelling down a highway at a constant speed, described by the equation $d = 65t$, where d represents the distance, in miles, that the car travels at this speed in t hours.

   a. What does the 65 tell us in this situation?

   b. How many miles does the car travel in 1.5 hours?

   c. How long does it take the car to travel 26 miles at this speed?

   Solution

   a. The car travels 65 miles in 1 hour. Or, the car is travelling 65 miles per hour. Or, 65 miles per hour is the constant of proportionality.

   b. The car travels 97.5 miles in 1.5 hours.

   c. It takes the car $\frac{2}{5}$ of an hour, or 0.4 hours, or 24 minutes to travel 26 miles.

2. **Problem 2 Statement**

   Elena has some bottles of water that each holds 17 fluid ounces.

   a. Write an equation that relates the number of bottles of water (b) to the total volume of water (w) in fluid ounces.

   b. How much water is in 51 bottles?

   c. How many bottles does it take to hold 51 fluid ounces of water?

   Solution

   a. $w = 17b$ or $b = \frac{1}{17}w$

   b. 867 fluid ounces, because $17 \times 51 = 867$

   c. 3 bottles, because $51 \div 17 = 3$

3. **Problem 3 Statement**

   There are about 1.61 kilometres in 1 mile. Let x represent a distance measured in kilometres and y represent the same distance measured in miles. Write two equations that relate a distance measured in kilometres and the same distance measured in miles.

Solution

$x = 1.61y$ and $y = \frac{1}{1.61}x$ or $y = 0.62x$

4. **Problem 4 Statement**

In Canadian coins, 16 quarters is equal in value to 2 toonies.

number of quarters	number of toonies
1	
16	2
20	
24	

a. Complete the table.

b. What does the value next to 1 mean in this situation?

Solution

number of quarters	number of toonies
1	$\frac{1}{8}$
16	2
20	2.5
24	3

a. $\frac{1}{8}$ means that one-eighth of a toonie is worth the same as 1 quarter.

5. **Problem 5 Statement**

Each table represents a proportional relationship. For each table:

a. Fill in the missing parts of the table.

b. Draw a circle around the constant of proportionality.

x	y
2	10
	15
7	
1	

a	b
12	3
20	
	10
1	

m	n
5	3
10	
	18
1	

Solution

x	y
2	10
3	15
7	35
1	5

a	b
12	3
20	5
40	10
1	$\frac{1}{4}$

m	n
5	3
10	6
30	18
1	$\frac{3}{5}$

6. **Problem 6 Statement**

Describe some things you could notice in two polygons that would help you decide that they were not scaled copies.

Solution

If they were not the same shape (for example, if one was a triangle and one was a square), they could not be scaled copies. I could find an angle measure in one that was not an angle measure of the other. I could find that a different scale factor would have to be used on one part of the pair than on another.