# Lesson 3: Equations for functions

## Goals

- Calculate the output of a function for a given input using an equation in two variables, and interpret (orally and in writing) the output in context.
- Create an equation that represents a function rule.
- Determine (orally and in writing) the independent and dependent variables of a function, and explain (orally) the reasoning.

## Learning Targets

- I can find the output of a function when I know the input.
- I can name the independent and dependent variables for a given function and represent the function with an equation.

## Lesson Narrative

So far we have used input-output diagrams and descriptions of the rules to describe functions. This is the first of five lessons that introduces and connects the different ways in which we represent functions in mathematics: verbal descriptions, equations, tables, and graphs. In this lesson students transition from input-output diagrams and descriptions of rules to equations.

This lesson also introduces the use of **independent** and **dependent variables** in the context of functions. For an equation that relates two quantities, it is sometimes possible to write either of the variables as a function of the other. For example, in the activity Dimes and Quarters, we can choose either the number of quarters or the number of dimes to be the independent variable. If we know the number of quarters and have questions about the number of dimes, then this would be a reason to choose the number of quarters as the independent variable.

### Addressing

- Define, evaluate, and compare functions.
- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in KS3.

### Building Towards

- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

### Instructional Routines

- Three Reads
- Discussion Supports
- Think Pair Share

### Student Learning Goals

Let's find outputs from equations.

## 3.1 A Square's Area

**Warm Up: 5 minutes**

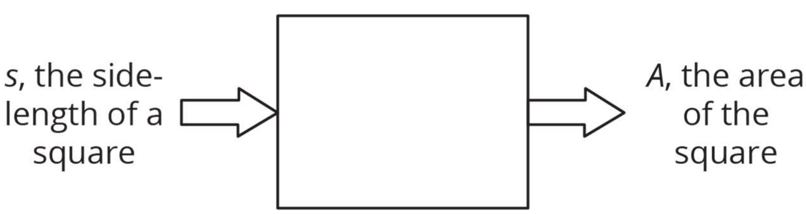
The purpose of this warm-up is for students to use repeated reasoning to write an algebraic expression to represent a rule of a function. The whole-class discussion should focus on the algebraic expression in the final row, however the numbers in the table give students an opportunity to also practise calculating the square of numbers written in fraction and decimal form.

### Launch

Arrange students in groups of 2. Give students 1–2 minutes of quiet work time and then time to share their algebraic expression with their partner. Follow with a whole-class discussion.

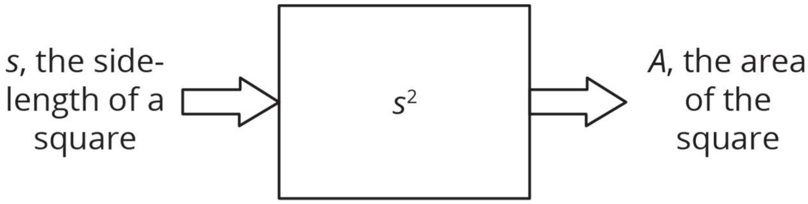
### Student Task Statement

Fill in the table of input-output pairs for the given rule. Write an algebraic expression for the rule in the box in the diagram.




| input | output |
|---|---|
| 8 | |
| 2.2 | |
| $12\frac{1}{4}$ | |
| $s$ | |

### Student Response




| input | output |
|---|---|
| 8 | 64 |
| 2.2 | 4.84 |
| $12\frac{1}{4}$ | $150\frac{1}{16}$ |
| $s$ | $s^2$ or $A$ |

### Activity Synthesis

Select students to share how they found each of the outputs. After each response, ask the class if they agree or disagree. Record and display responses for all to see. If both responses are not mentioned by students for the last row, tell students that we can either put $s^2$ or $A$ there. Tell students we can write the equation $A = s^2$ to represent the rule of this function.

End the discussion by telling students that while we've used the terms input and output so far to talk about specific values, when a letter is used to represent any possible input we call it the **independent variable** and the letter used to represent all the possible outputs is the **dependent variable**. In this case, $s$ is the independent variable and $A$ the dependent variable, and we say "$A$ depends on $s$."

## 3.2 Diagrams, Equations, and Descriptions

**15 minutes**

The purpose of this activity is for students to make connections between different representations of functions and start transitioning from input-output diagrams to other representations of functions. Students match input-output diagrams to descriptions and come up with equations for each of those matches. Students then calculate an output given a specific input and determine the independent and dependent variables.

### Instructional Routines

- Discussion Supports
- Think Pair Share

## Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet work time and time to share their responses with their partner and come to agreement on their answers. Follow with whole-class discussion.

## Student Task Statement

Record your answers to these questions in the table provided.

1. Match each of these descriptions with a diagram:

   a. the circumference, $C$, of a circle with **radius**, $r$

   b. the distance in miles, $d$, that you would travel in $t$ hours if you drive at 60 miles per hour

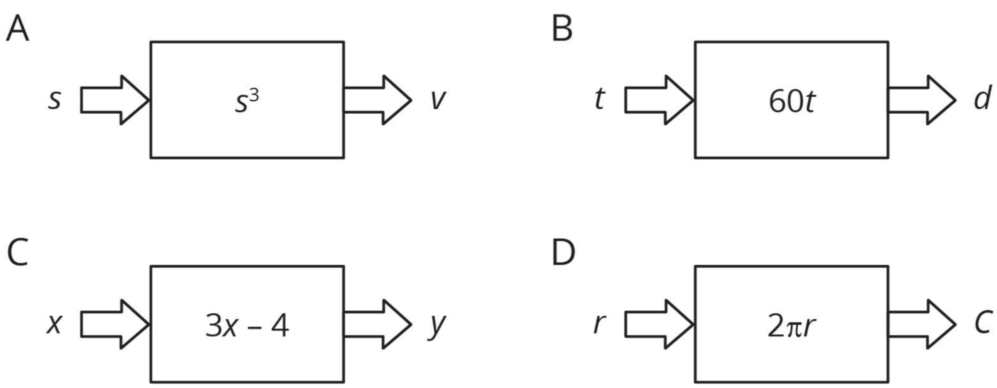
   c. the output when you triple the input and subtract 4

   d. the volume of a cube, $v$ given its edge length, $s$

2. Write an equation for each description that expresses the output as a function of the input.

3. Find the output when the input is 5 for each equation.

4. Name the **independent** and **dependent** variables of each equation.




| description | a | b | c | d |
|---|---|---|---|---|
| diagram | | | | |
| equation | | | | |
| input = 5<br>output = ? | | | | |
| independent variable | | | | |

| | | | | |
|---|---|---|---|---|
| dependent variable | | | | |

### Student Response

| description | a | b | c | d |
|---|---|---|---|---|
| diagram | D | B | C | A |
| equation | $C = 2\pi r$ | $d = 60t$ | $y = 3x - 4$ | $v = s^3$ |
| input = 5 output = ? | $10\pi \approx 31.4$ | 300 | 11 | 125 |
| independent variable | $r$ | $t$ | $x$ | $s$ |
| dependent variable | $C$ | $d$ | $y$ | $v$ |

### Are You Ready for More?

Choose a 3-digit number as an input.

Apply the following rule to it, one step at a time:

- Multiply your number by 7.
- Add one to the result.
- Multiply the result by 11.
- Subtract 5 from the result.
- Multiply the result by 13
- Subtract 78 from the result to get the output.

Can you describe a simpler way to describe this rule? Why does this work?

### Student Response

If we apply the steps to a generic 3-digit number $x$, the result is

$13(11(7x + 1) - 5) - 78 = 1,001x$

For any 3-digit number $x$, the number $1\,001x$ is just that number repeated twice. This works since $1\,001x = 1\,000x + x$, so for example,

$$\begin{aligned} 1\,001 \times 314 &= 1\,000 \times 314 + 1 \times 314 \\ &= 314\,000 + 314 \\ &= 314\,314 \end{aligned}$$

## Activity Synthesis

The goal of this discussion is for students to describe the connections they see between the different entries for the 4 descriptions. Display the table for all to see and select different groups to share the answers for a column in the table. As groups share their answers, ask:

- "How did you know that this diagram matched with this description?" (We remembered the formula for the circumference of a circle, so we knew description A went with diagram D.)
- "Where in the equation do you see the rule that is in the diagram?" (The equation is the dependent variable set equal to the rule describing what happens to the independent variable in the diagram.)
- "Explain why you chose those quantities for your independent and dependent variables." (We know the independent variable is the input and the dependent variable is the output, so we matched them up with the input and output shown in the diagram.)

*Speaking: Discussion Supports.* As students describe the connections they noticed in the table across from the different entries for the four descriptors, revoice student ideas to demonstrate mathematical language use. In addition, press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. This will help students to produce and make sense of the language needed to communicate their own ideas about functions and independent and dependent variables. *Design Principle(s): Support sense-making; Optimise output (for explanation)*

# 3.3 Dimes and Quarters

**15 minutes**

The purpose of this activity is for students to work with a function where either variable could be the independent variable. Knowing the total value for an unknown number of dimes and quarters, students are first asked to consider if the number of dimes could be a function of the number of quarters and then asked if the reverse is also true. Since this isn't always the case when students are working with functions, the discussion should touch on reasons for choosing one variable vs. the other, which can depend on the types of questions one wants to answer.

Identify students who efficiently rewrite the original equation in the third problem and the last problem to share during the discussion.

## Instructional Routines

- Three Reads

## Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet work time followed by partner discussion for students to compare their answers and resolve any differences. Follow with a whole-class discussion.

*Representation: Internalise Comprehension.* Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.
*Supports accessibility for: Language; Conceptual processing Reading: Three Reads.* Use this routine to support reading comprehension and to set students up to interpret the representations of the situation provided in the task statement (an equation) and discussion (function diagrams). In the first read, students read the information with the goal of comprehending the situation (e.g., Jada has dimes and quarters). In the second read, ask students to identify important quantities. Listen for, and amplify, naming of the quantities that vary in relation to each other in this situation: number of dimes, total value of dimes, number of quarters, total value of quarters. After the third read, ask students to discuss possible strategies to answer the questions that follow, paying attention to the different coin values, and how each question is phrased. This will help students comprehend the problem and make sense of important quantities and variables when working with a function in which either variable could be the independent variable.
*Design Principle(s): Support sense-making*

## Anticipated Misconceptions

Some students may be unsure how to write rules for the number of dimes as a function of the number of quarters and vice versa. Prompt them to use the provided equation and what they know about keeping equations equal to create the new equations.

## Student Task Statement

Jada had some dimes and quarters that had a total value of $12.50. The relationship between the number of dimes, $d$, and the number of quarters, $q$, can be expressed by the equation $0.1d + 0.25q = 12.5$. A quarter is worth 25 cents, a dime is worth 10 cents and a dollar is worth 100 cents.

1. If Jada has 4 quarters, how many dimes does she have?
2. If Jada has 10 quarters, how many dimes does she have?
3. Is the number of dimes a function of the number of quarters? If yes, write a rule (that starts with $d =$...) that you can use to determine the output, $d$, from a given input, $q$. If no, explain why not.
4. If Jada has 25 dimes, how many quarters does she have?
5. If Jada has 30 dimes, how many quarters does she have?

6. Is the number of quarters a function of the number of dimes? If yes, write a rule (that starts with $q =$...) that you can use to determine the output, $q$, from a given input, $d$. If no, explain why not.

### Student Response

1. 115. If $q = 4$, then the equation tells us that $0.1d + (0.25) \times 4 = 12.5$. Subtracting 1 from both sides gives $0.1d = 11.5$, so $d = 115$.

2. 100. If $q = 10$, then the equation tells us that $0.1d + (0.25) \times 10 = 12.5$. Subtracting 2.5 from both sides gives $0.1d = 10$, so $d = 100$.

3. Yes. If you know the number of quarters, then you can determine the number of dimes from the equation. We can even write the equation in a way that shows this: $d = 125 - 2.5q$. The expression $125 - 2.5q$ represents the output—it is the rule that determines the output $d$ from a given input $q$.

4. 40. If $d = 25$, then the equation tells us that $0.1(25) + (0.25)q = 12.5$. Subtracting 2.5 from both sides gives $.25q = 10$, so $q = 40$.

5. 38. If $d = 30$, then the equation tells us that $0.1(30) + (0.25)q = 12.5$. Subtracting 3 from both sides gives $.25q = 9.5$, so $q = 38$.

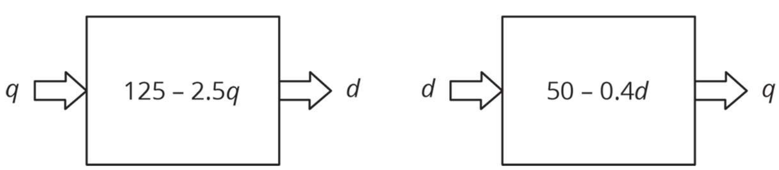
6. Yes. If you know the number of dimes, then you can determine the number of quarters from the equation. We can even write the equation in a way that shows this: $q = 50 - 0.4d$. The expression $50 - 0.4d$ represents the output—it is the rule that determines the output $q$ from a given input $d$.

### Activity Synthesis

Select previously identified students to share their rules for dimes as a function of the number of quarters and quarters as a function of the number of dimes, including the steps they used to rewrite the original equation.

Tell students that if we write an equation like $d = 125 - 2.5q$, this shows that $d$ is a function of $q$ because it is clear what the output (value for $d$) should be for a given input (value for $q$).

Display the diagrams for all to see:




When we have an equation like $0.1d + 0.25q = 12.5$, we can choose either $d$ or $q$ to be the independent variable. That means we are viewing one as depending on the other. If we know the number of quarters and want to answer a question about the number of dimes, it is helpful to write $d$ as a function of $q$. If we know the number of dimes and want to answer a question about the number of quarters, it is helpful to write $q$ as a function of $d$.

Ensure students understand that we can't always do this type of rearranging with equations and have it make sense because sometimes only one variable is a function of the other, and sometimes neither is a function of the other. For example, students saw earlier that while squaring values is a function, the reverse—that is, identifying what value was squared—is not. We will continue to explore when these different things happen in future lessons.

### Lesson Synthesis

Tell students that we often use **independent** and **dependent variables** to represent the inputs and outputs of functions. For some functions, we can describe the relationship between the variables with an equation. Sometimes we can choose, depending on the situation, which variable should be the independent and which should be the dependent variable. To help students think more about what independent and dependent variables represent and their use with functions, ask:

- "How can we describe the area of square $A$ of side length $s$ with an equation? Which is the independent and which is the dependent variable?" (We can write $s^2 = A$, where $s$ is the independent variable and $A$ is the dependent variable.)
- "The relationship between the number of 10p coins, $d$, and the number of 5p coins, $n$, that total £5 can be expressed by the equation $0.1d + 0.05n = 5$. When would it be useful to choose the number of 10p coins as the independent variable and rewrite the equation?" (If we knew the number of 10p coins and wanted to know the number of 5p coins, it would be useful to rewrite the equation so it looked like $n = ...$)

## 3.4 The Value of Some Quarters

**Cool Down: 5 minutes**

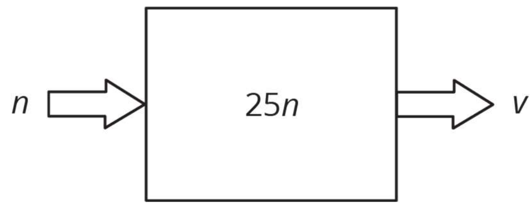
#### Student Task Statement

The value $v$ of your quarters (in cents) is a function of $n$, the number of quarters you have.

1. Draw an input-output diagram to represent this function.
2. Write an equation that represents this function.
3. Find the output when the input is 10.
4. Identify the independent and dependent variables.

#### Student Response

1. Here is the diagram:




2. $v = 25n$. This reflects the statement that the value (in cents) of your collection of quarters is always 25 times the number of quarters you have.

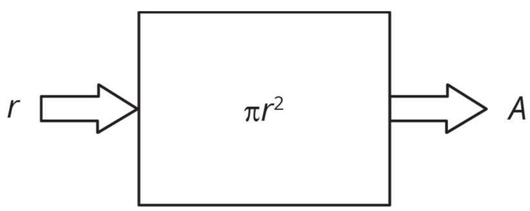
3. When the input is 10, the output is 250 (since $250 = 25 \times 10$).

4. $n$ is the independent variable, and $v$ is the dependent variable.

## Student Lesson Summary

We can sometimes represent functions with equations. For example, the area, $A$, of a circle is a function of the radius, $r$, and we can express this with an equation: $A = \pi r^2$

We can also draw a diagram to represent this function:




In this case, we think of the radius, $r$, as the input, and the area of the circle, $A$, as the output. For example, if the input is a radius of 10 cm, then the output is an area of $100\pi$ cm$^2$, or about 314 square cm. Because this is a function, we can find the area, $A$, for any given radius, $r$.

Since it is the input, we say that $r$ is the **independent variable** and, as the output, $A$ is the **dependent variable**.

Sometimes when we have an equation we get to choose which variable is the independent variable. For example, if we know that

$10A - 4B = 120$

then we can think of $A$ as a function of $B$ and write

$A = 0.4B + 12$

or we can think of $B$ as a function of $A$ and write

$B = 2.5A - 30$

## Glossary

- dependent variable

- independent variable
- radius

## Lesson 3 Practice Problems

1. **Problem 1 Statement**

   Here is an equation that represents a function: $72x + 12y = 60$.

   Select **all** the different equations that describe the same function:

   a. $120y + 720x = 600$

   b. $y = 5 - 6x$

   c. $2y + 12x = 10$

   d. $y = 5 + 6x$

   e. $x = \frac{5}{6} - \frac{y}{6}$

   f. $7x + 2y = 6$

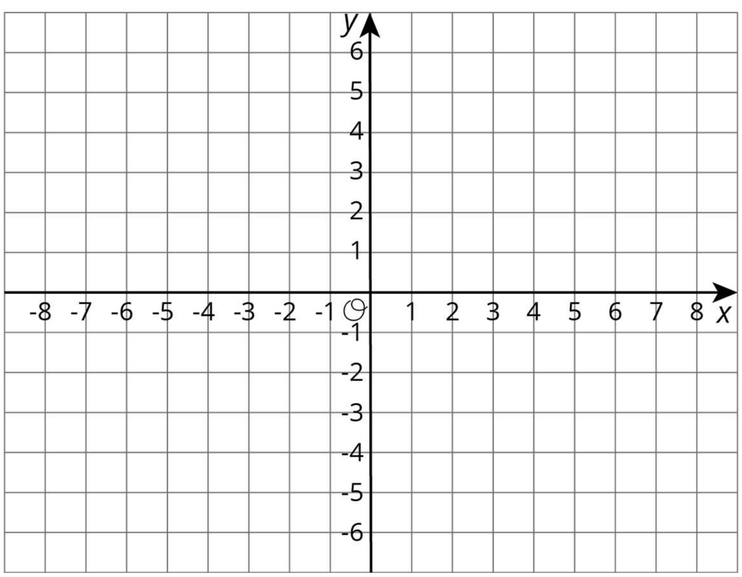
   g. $x = \frac{5}{6} + \frac{y}{6}$

   **Solution** ["A", "B", "C", "E"]

2. **Problem 2 Statement**

   a. Graph a system of linear equations with no solutions.

   b. Write an equation for each line you graph.



**Solution**

Answers vary. The graph could be any two lines that are parallel.

3. **Problem 3 Statement**

Brown rice costs £2 per pound, and beans cost £1.60 per pound. Lin has £10 to spend on these items to make a large meal of beans and rice for a potluck dinner. Let $b$ be the number of pounds of beans Lin buys and $r$ be the number of pounds of rice she buys when she spends all her money on this meal.

a. Write an equation relating the two variables.

b. Rearrange the equation so $b$ is the independent variable.

c. Rearrange the equation so $r$ is the independent variable.

**Solution**

a. $2r + 1.6b = 10$

b. $r = 5 - 0.8b$

c. $b = 6.25 - 1.25r$

4. **Problem 4 Statement**

Solve each equation and check your answer.

$2x + 4(3 - 2x) = \frac{3(2x+2)}{6} + 4$

$4z + 5 = -3z - 8$

$\frac{1}{2} - \frac{1}{8}q = \frac{q-1}{4}$

**Solution**

a. $x = 1$.

b. $z = \frac{-13}{7}$

c. $q = 2$