Lesson 1: Tessellations of the plane

Goals

- Create and describe patterns with specific polygons that fill the plane.

Lesson Narrative

This optional sequence of three lessons can be done any time after the first unit in the course. Students are introduced to tessellations of the plane. The activities in this lesson provide students a chance to go more deeply and apply year 9 geometry concepts to a mathematical context. The activities in this sequence of three lessons build on each other, so should be done in order. It is not necessary to do all three lessons to get some benefit, although more connections are made the further one gets.

As students progress through the activities, they gain an intuition for the variety of ways some shapes can be put together to build a tessellation and the restrictions on how some shapes can be put together to build a tessellation. They start with general tessellations, and then look at regular tessellations. Finally in the third lesson, they examine tessellations using other regular polygons.

Throughout these lessons, students make use of structure when building their tessellations. They reason abstractly and quantitatively when deciding which polygons can be used in the different types of tessellations and make viable arguments to convince each other that some polygons cannot be used to tessellate the plane.

Building On

- Draw, construct, and describe geometrical shapes and describe the relationships between them.

Addressing

- Understand congruence and similarity using physical models, transparencies, or geometry software.

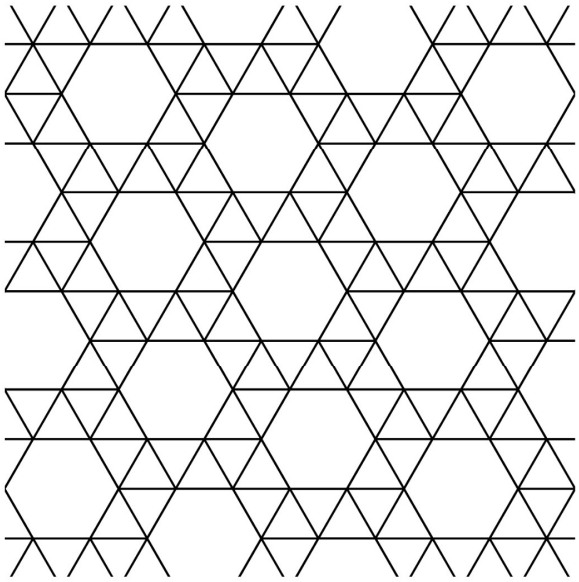
Instructional Routines

- Collect and Display
- Discussion Supports
- Notice and Wonder

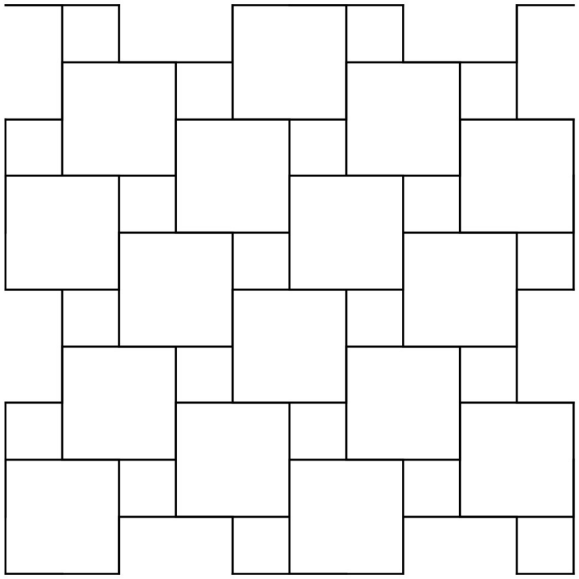
Required Materials

Pre-printed slips, cut from copies of the blackline master

Describing a Tessellation
Card A



Describing a Tessellation
Card B



Tracing paper

Required Preparation

If using the print version of the materials, all students require tracing paper. If using the applets in the digital version of the materials, students may not need tracing paper.

Prepare 1 pre-cut copy of the Describing a Tessellation blackline master for every 2 students. Each student needs one of the two slips showing an individual tessellation.

Student Learning Goals

Let's explore geometric patterns!

1.1 Notice and Wonder: Polygon Patterns

Optional: 5 minutes

This activity introduces students to patterns of polygons that cover the plane. Examples include:

- patterns using a single shape
- patterns using multiple shapes

Studying these patterns and understanding how and why they repeat to fill up the plane is expressing regularity in repeated reasoning. In this case, the repeated reasoning is continuing to lay out the shapes in the same pattern.

Instructional Routines

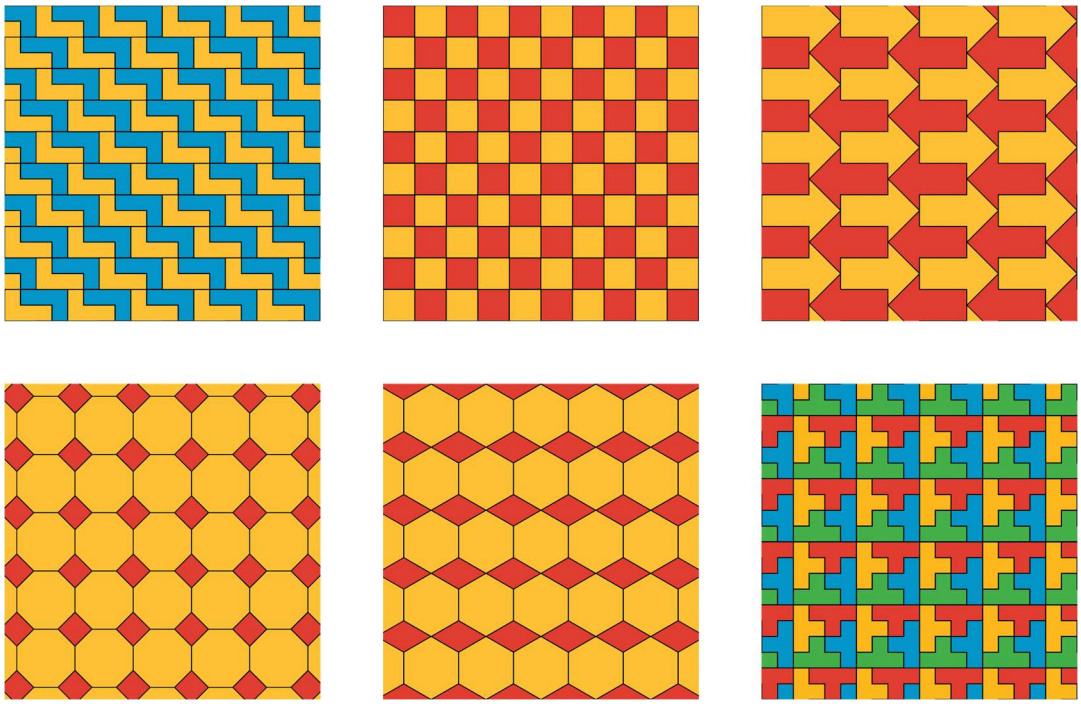
- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at a set of images, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the images for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- Four of the patterns use only one shape. Two of them use two different shapes.
- The pattern could continue to the left and right or up and down.

Things students may wonder:

- Are the colours of the shapes important?

- Do the arrows count as one shape or two, since they are pointing in different directions?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the images. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time.

Important ideas that can come up are the following:

- The shapes in a tessellation cover the plane (or would if the pattern continues) without gaps.
- Sometimes there is only one shape. Sometimes there are several shapes in the tessellation.
- With the square one, all of the squares are complete. In the other patterns, the shapes on the boundary are cut off.

If time allows, ask:

- "How do I know what happens as the tessellation continues to grow off of the page?" (There is often a pattern, but we would have to indicate that the pattern continues.)
- "Are the colours of the shapes important?" (They are important for identifying patterns and making the pictures prettier, but they could be changed and the pattern would still be the same.)

1.2 Tessellations

Optional: 20 minutes (there is a digital version of this activity)

The goal of this task is to introduce the notion of a tessellation and then carefully examine how to create a tessellation with each pattern block shape (square, rhombus, equilateral triangle, isosceles trapezium). For hexagons, there is only one way to fit them together because there will be gaps unless three hexagons meet at each vertex. The other shapes offer much more flexibility, and students have an opportunity to use their artistic creativity.

Students look for and make use of structure, both when they try to put copies of the shape together to build a tessellation and when they examine whether or not it is possible to construct a different tessellation.

Instructional Routines

- Discussion Supports

Launch

Introduce the definition of a **tessellation** of the plane by polygons: a tessellation covers the plane with copies of the shape with no gaps and no overlaps.

Show the students the image from the previous notice and wonder activity, and ask them if these are tessellations. (Yes.)

Demonstrate how to use tracing paper to create a tessellation.

Arrange students in groups of 2. Each pair uses the same shape for their tessellations.

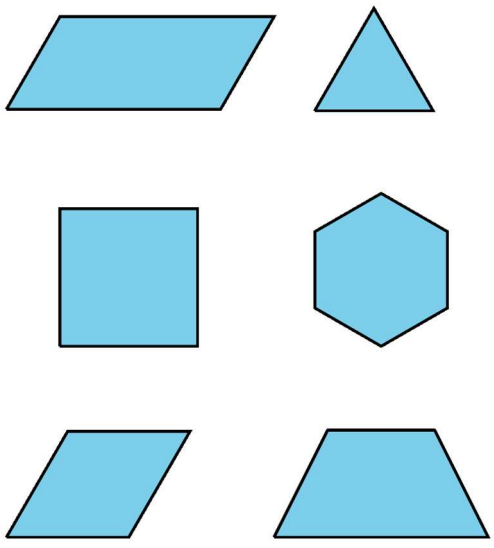
Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following term and maintain the display for reference throughout the lesson: tessellation. Consider providing step-by-step directions for students to use tracing paper to create a tessellation. Invite students to suggest language or diagrams to include that will support their understanding.
Supports accessibility for: Memory; Language Speaking: Discussion Supports. Amplify students' uses of mathematical language to communicate about tessellations; edge to edge, vertex, symmetrical, plane, rigid motion, etc. Encourage students to use these words, revoicing students' ideas as necessary. Ask students to chorally repeat the phrases that include these words in context.
Design Principle(s): Support sense-making; Optimise output (for explanation)

Anticipated Misconceptions

Tessellations do not need to be edge to edge. That is, pieces do not need to fit together with edges of the same lengths matching exactly.

Tessellations do not need to have symmetric, repeating patterns, though sometimes the shape forces it (as with the regular hexagons).

Student Task Statement



1. Pick one of the shapes. Create a **tessellation** by tracing copies of your shape. Make sure to use the same shape as your partner.

2. Compare your tessellation to your partner's. How are they similar? How are they different?

3. If possible, make a third tessellation of the plane with your shape (different from the ones you and your partner already created). If not possible, explain why it is not possible.

Student Response

1. Answers vary.

2. Answers vary.

3. It depends on the shape. It is not possible for the hexagon: Three hexagons have to come together at each vertex: once the first hexagon of the pattern is placed, everything else has no flexibility. It is possible for the other shapes: Triangles can be built into hexagons, or they can make rows that can be translated and stacked on top of one another. Parallelograms can also be made into hexagons or rows. Trapeziums can be made into hexagons or rows. Rhombuses can be made into rows and translated. Squares can also be made into rows and translated.

Activity Synthesis

Important questions to address include:

- "Were you able to make different tessellations with your shape?" (It depends on the shape.)
- "If not, why not?" (Three hexagons have to come together at each vertex: once the first hexagon of the pattern is placed, everything else has no flexibility.)
- "If so how?" (Triangles can be built into hexagons, or they can make rows that can be translated and stacked on top of one another. Parallelograms can also be made into hexagons or rows. Trapeziums can be made into hexagons or rows. Rhombuses can be made into rows and translated. Squares can also be made into rows and translated.)
- "What does it look like to *not* define a tessellation?" (Two octagons can be put together sharing a vertex, but there is a gap that is not large enough for a third octagon.)

Make sure, as students share their ideas for tessellations, to use the language of rigid motions to describe the tessellations. For example, if a student has built a tessellation with parallelograms, choose two parallelograms and ask:

- "How can I use transformations to move one of these parallelograms into the position of the other?"
- "Are there other ways I could do this?"

1.3 Describing a Tessellation

Optional: 20 minutes

The goal of this activity is to accurately describe a tessellation of the plane. While students do not need to use the words translation, rotation, or reflection, their understanding of transformations of the plane will play a key role in explaining (and interpreting) where to place each shape in a tessellation. Communicating a geometric pattern clearly in words fully engages students in attending to precision.

As a warm-up, each student will describe a tessellation while the partner identifies a picture of the tessellation. After a brief discussion of what language was most helpful, students then take turns describing a tessellation as their partner attempts to build the tessellation. They switch roles and then reflect on any misinterpretations that happened and how they may have been related to the language used to describe the tessellations.

Instructional Routines

- Collect and Display

Launch

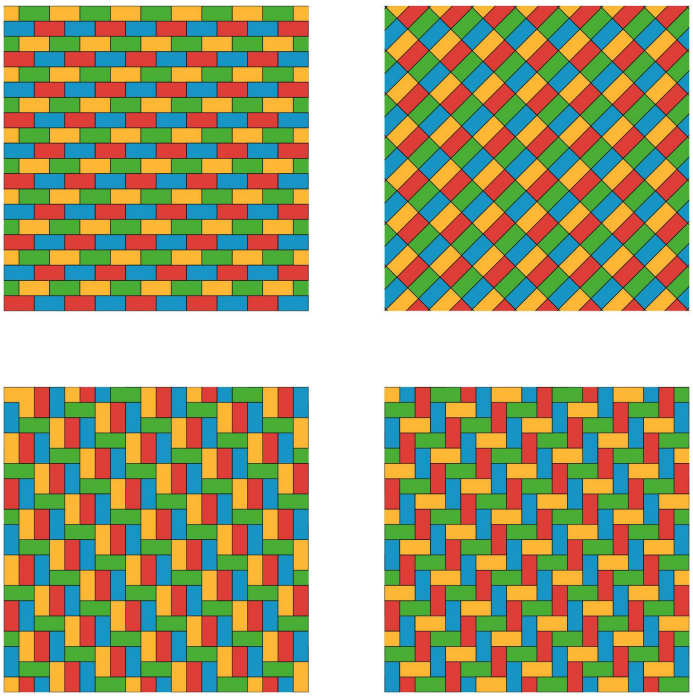
Students work in pairs. Each pair requires a set of 2 cards for Part 2, one card for each student.

Stop students after they have completed Part 1 of the task for the first part of the discussion given in the synthesis.

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organisation Speaking, Listening, Representing: Collect and Display. While students work with their partners during this activity, use this to record initial language used to describe their tessellations. Since students are not required to use specific language like rotation or translation yet, record the language students use as they construct and compare their tessellations. Display the recording language throughout the activity, and refer to it during the synthesis. Add or edit language as students respond to each question.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

1. Pick one of the designs and describe the tessellation. Your partner will identify which tessellation you are describing. Then swap roles so your partner describes the tessellation and you identify the design.



2. You and your partner each have a card with a tessellation. Describe what is on your card so that your partner can produce the tessellation (this should be done so that you cannot see your partner's work until it is complete).

3. Check together to see if your partner's tessellation agrees with your card and discuss any differences.

4. Change roles so your partner describes a tessellation, which you attempt to produce.

5. Check the accuracy of your construction and discuss any discrepancies.

Student Response

Part 1: Answers vary. Sample response (for bottom left design): This tessellation has groups of three rectangles next to one another, sharing the long sides: the long sides are vertical. In addition to these groups of three rectangles, there are single rectangles with the long side lying horizontally.

Part 2: Answers vary. Sample response (for card A): Draw a hexagon. Use each of the hexagon's sides as one side of an equilateral triangle drawn outside the hexagon. The hexagon with its 6 triangles should look like a star. Now we make more stars, with an edge of one triangle from each new star sharing one side with a triangle on a previous star. If we draw new stars consistently on the same side of existing triangles (in this case, always on the clockwise side), we get the image in card A. Sample response (for card B): Draw a square with horizontal and vertical sides. Use the midpoint of the right side as the top left vertex of a second square the same size. Repeat for the other three midpoints of the original square: The midpoint of the top should be the bottom left of a new square, the midpoint of the left should be the bottom right of a new square, and the midpoint of the

bottom should be the top right of a new square. Now repeat this process for each square arising to get the design in card B.

Activity Synthesis

For the discussion after students complete Part 1:

- "In what ways was describing the tessellation difficult?" (Finding words to communicate how the rectangles are aligned with one another.)
- "Did you use the words translate, rotate, or reflect?" (Instead of "translate" students may use words like "move." Similarly, students may describe rotations with words like "turn.")
- "What was challenging about describing or identifying the tessellation?"

For the discussion after students complete Part 2, focus on the use of the words translate, reflect, and rotate (or equivalents).

- "When you used a translation, did you specify the direction and how far?" (Maybe, but they may also use the language "put next to" or equivalent.)
- "When a rotation was involved, did you specify the number of degrees of the rotation?" (Probably not. They will more likely say "rotate until the sides match. . ." or "turn upside down.")
- "When a rotation was involved, did you specify the centre of rotation?" (Answers vary. Probably not.)
- "Did you use any reflections?" (Answers vary.)