Lesson 10: Using long division

Goals

- Interpret the long division method, and compare and contrast it (orally) with other methods for computing the quotient of whole numbers.
- Recognise and explain (orally) that long division is an efficient strategy for dividing numbers, especially with multi-digit dividends.
- Use long division to divide whole numbers that result in a whole-number quotient, and multiply the quotient by the divisor to check the answer.

Learning Targets

- I can use long division to find a quotient of two whole numbers when the quotient is a whole number.

Lesson Narrative

This lesson introduces students to **long division**. Students see that in long division the meaning of each digit is intimately tied to its place value, and that it is an efficient way to find quotients. In the partial quotients method, all numbers and their meaning are fully and explicitly written out. For example, to find $657 \div 3$ we write that there are at least 3 groups of 200, record a subtraction of 600, and show a difference of 57. In long division, instead of writing out all the digits, we rely on the position of any digit—of the quotient, of the number being subtracted, or of a difference—to convey its meaning, which simplifies the calculation.

In addition to making sense of long division and using it to calculate quotients, students also analyse some place-value errors commonly made in long division.

Building On

- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Addressing

- Fluently divide multi-digit numbers using the standard algorithm.

Building Towards

- Apply and extend previous understandings of arithmetic to algebraic expressions.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports

- Number Talk
- Think Pair Share

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits as they divide using long division and the partial quotients method. Consider having graph paper accessible throughout the lesson.

Student Learning Goals

Let's use long division.

10.1 Number Talk: Estimating Quotients

Warm Up: 5 minutes

This number talk prompts students to make reasonable estimates of quotients using their knowledge of numbers, division, and structures. Only two problems are given to allow more time for students to share their estimation strategies.

Making reasonable estimates helps to develop arithmetic fluency. Here, it relies on understanding the relationship between multiplication and division, and on the different properties of operations (commutative, associative, and distributive). For example, to find $500 \div 7$, students might think of the multiplication equation $7 \times ? = 500$. Since they know that $7 \times 100 = 700$ and $7 \times 30 = 210$, and that $500 = 700 - 200$, they could reason that 500 would be approximately $7 \times (100 - 30)$ or 7×70.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display each problem one at a time for all to see. Give students 1 minute of quiet think time and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

Students might try to calculate exact answers and take longer to produce an answer. Encourage them to approximate the actual answer by rounding the dividend or divisor or starting with friendlier numbers.

Student Task Statement

Estimate these quotients mentally.

$500 \div 7$

$1\,394 \div 9$

Student Response

Answers vary. Sample responses:

- 70, because $7 \times 7 = 49$, so $70 \times 7 = 490$, which is almost 500.
- 150, because $100 \times 9 = 900$ and $50 \times 9 = 450$, so $150 \times 9 = 1\,350$.

Activity Synthesis

Invite students to share their strategies. Record and display student explanations for all to see. Ask students to explain if or how the dividend or divisor impacted their choice of strategy. To involve more students in the conversation, consider asking:

- “Did anyone reason about the problem the same way but would explain it differently?”
- “Did anyone estimate in a different way?”
- “Does anyone want to add on to _____’s reasoning?”
- “Do you agree or disagree? Why?”

At the end of the discussion, if time permits, ask a few students to share a story problem or context that $1\,394 \div 9$ could represent.

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

10.2 Lin Uses Long Division

25 minutes

This activity introduces the use of **long division** to calculate a quotient of whole numbers. Students make sense of the process of long division by studying an annotated example and relating it to the use of partial quotients and base-ten diagrams. They begin to see that long division is a variant of the partial quotients method, but it is calculated and recorded differently.

In the partial quotients method, the division is done in instalments, resulting in a series of partial quotients. The size of each instalment can vary, but it is always a multiple of the divisor. Each partial quotient is written above the dividend and stacked; the sum of all partial quotients is the quotient.

In long division, the division is performed digit by digit, from the largest place to the smallest, so the resulting quotient is also recorded one digit at a time. In each step, one more digit of the quotient is calculated. Students notice that although only one digit of the quotient is written down at a time, the value that it represents is communicated through its placement.

To become proficient in long division requires time, encounters with a variety of division problems, and considerable practice. Students will have opportunities to study the algorithm more closely and to use it to divide increasingly more challenging numbers over several upcoming lessons.

Instructional Routines

- Clarify, Critique, Correct
- Think Pair Share

Launch

Tell students that they will now consider a third method—called long division—for solving the same division problem that Elena and Andre had calculated using base-ten diagrams and the partial quotients method. Encourage students to refer to their work on those activities, or display Elena and Andre's methods for all to see. Ask a couple of students to briefly explain how Elena and Andre calculated $657 \div 3$.

Arrange students in groups of 2. Give students 2–3 minutes of quiet think time to make sense of the annotated example of long division, and then 3–4 minutes to discuss the first set of questions with a partner. Follow with a whole-class discussion before students use long division to answer the second set of questions.

Display Lin's method for all to see and ask a student to explain what Lin had done in his or her own words. Then, discuss students' responses to the first set of questions and these questions:

- In the first step, Lin divided 6 by 3 to get 2. Did it matter where Lin wrote the 2? Why did she put it over the 6? (Yes, it mattered. Because the 6 represents 600, she was really dividing 600 by 3, which is 200. The 2 needs to be written in the hundreds place to tell us its actual value.)
- After writing down the 2, Lin subtracted 6. Why? And why was the result of the subtraction not 651 (since $657 - 6 = 651$)? (Though she wrote a subtraction of 6, she actually subtracted 600. Because she had just divided 600 by 3, that portion of 657 is already accounted for.)
- Could Lin have written the full amounts being subtracted instead of just the non-zero digit (e.g., subtracting by 600, 50 and 7, instead of subtracting by 6, 5, and 7 after

aligning them to certain places)? (Yes, it would involve more writing, but it works just as well.)

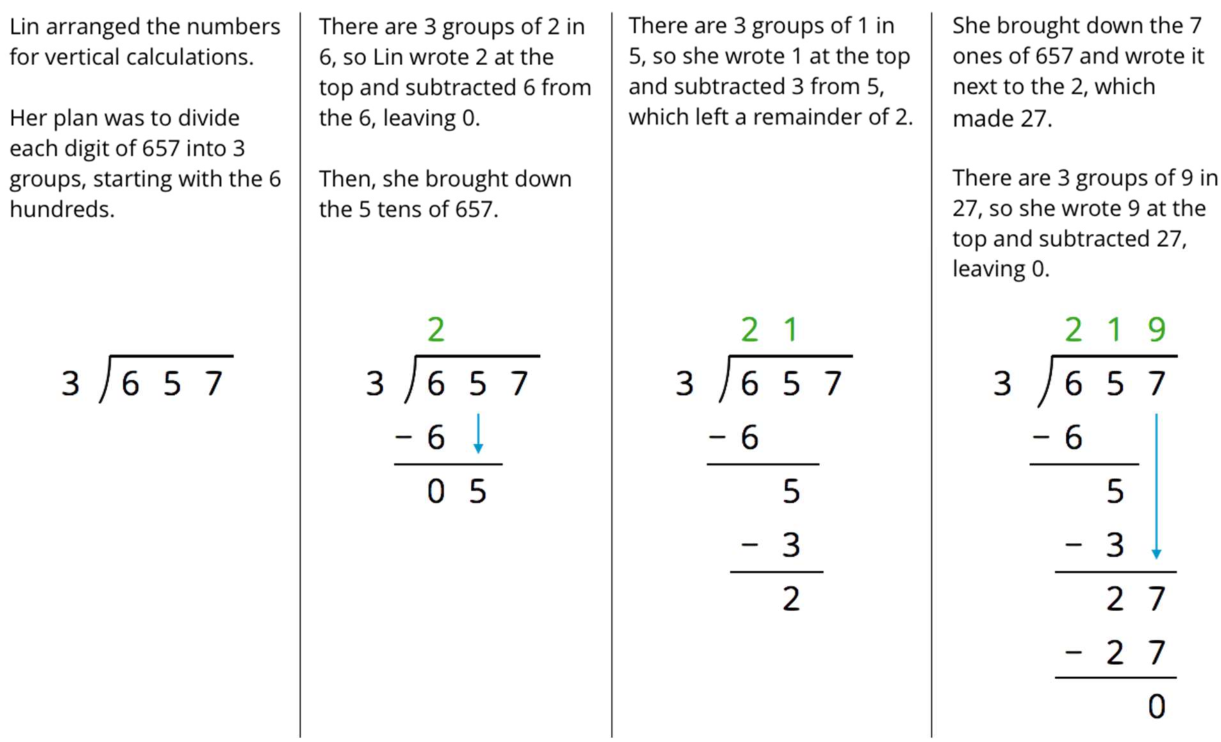
- How is this process similar to and different than the partial quotients method? (It is a similar idea of taking out a certain multiple of 3 at a time, but in long division we do it digit by digit and in the order of place value—from the largest unit to the smallest.)

Consider demonstrating the long-division process with another example such as $912 \div 4$ before asking students to complete the rest of the task. Provide access to graph paper. Tell students that the grid could help them line up the digits.

Representation: Access for Perception. Read Lin's method for calculating the quotient of $657 \div 3$ aloud. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language

Student Task Statement

Lin has a method of calculating quotients that is different from Elena's method and Andre's method. Here is how she found the quotient of $657 \div 3$:



1. Discuss with your partner how Lin's method is similar to and different from drawing base-ten diagrams or using the partial quotients method.

   – Lin subtracted 3×2, then 3×1, and lastly 3×9. Earlier, Andre subtracted 3×200, then 3×10, and lastly 3×9. Why did they have the same quotient?

- In the third step, why do you think Lin wrote the 7 next to the remainder of 2 rather than adding 7 and 2 to get 9?

2. Lin's method is called **long division**. Use this method to find the following quotients. Check your answer by multiplying it by the divisor.

   a. $846 \div 3$

   b. $1816 \div 4$

   c. $768 \div 12$

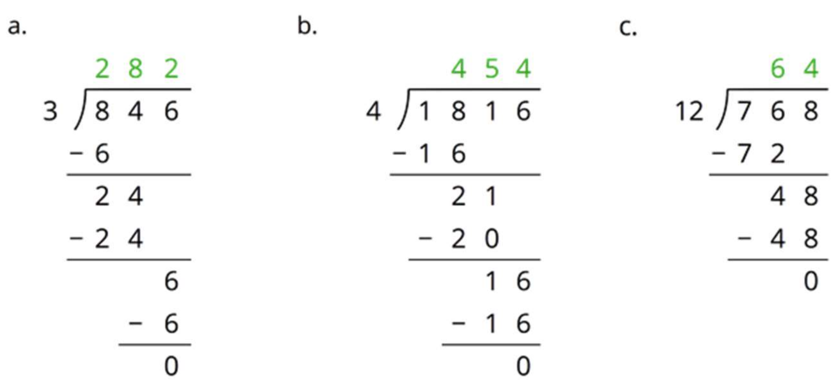
Student Response

1.

- The 3×2 Lin subtracted from 6 represents 3×200 subtracted from 600, since the 6 and the 2 are both in the hundreds place. Similarly, the 3×1 Lin subtracted from 5 represents 3×10 subtracted from 5 tens (or 50). Lin's work shows the same steps Andre took without writing out as many digits in the calculations.

- The 2 represents two tens or 20. The 7 represents 7 ones. So Lin is adding 2 tens to 7 ones, and the result is 27.

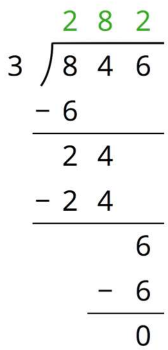
2.



Activity Synthesis

Display the worked-out long divisions for all to see. Select a student to explain the steps for at least one of the division problems. Highlight two ideas about long division: 1) we start by dividing the largest base-ten units and work toward smaller units, and 2) the placement of each digit of the quotient matters because it conveys the value of the digit.

a.



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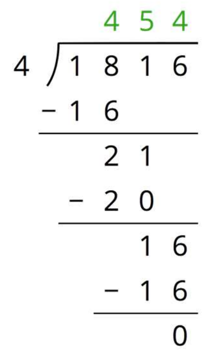
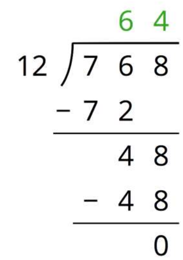


c.



Draw students' attention to the second problem (1 816 ÷ 4) or third problem (768 ÷ 12), in which the first digit of the dividend is smaller than the divisor. Select 1–2 students to share how they approached these situations. If not brought up in students' explanation, discuss how we could reason about these.

- "Let's take 1 816 ÷ 4 as an example. If we were using base-ten diagrams, we would have 1 piece representing a thousand. How would we divide that piece into 4 groups?" (We would ungroup it into 10 hundreds, add them to the 8 pieces representing 8 hundreds, and then distribute the 18 hundreds into 4 groups.)
- "How can we apply the same idea to long division? If there is not enough thousands to divide into 4 groups, what can we do?" (We can think of the 1 thousand and 8 hundreds as 18 hundreds and divide that value instead.)
- "How many hundreds would go into each group if we divide 18 hundreds into 4 groups?" (4 hundreds, with a remainder of 2 hundreds.)
- "Where should we write the 4? Why?" (In the hundreds place, because it represents 4 hundreds.)
- "How do we deal with the 2 hundreds?" (Since there is not enough to distribute into 4 groups, we can ungroup them into 20 tens, combine them with the 1 ten, and divide 21 tens by 4.)

Writing, Speaking: Clarify, Critique, and Correct. Use this routine to support whole-class discussion before students share their answer to 1 816 ÷ 4. Display a calculation that shows incorrect placement of the 4 in the hundreds digit of the quotient above the 1 in the thousands digit of the dividend. Ask pairs to clarify and critique by asking, "What error was made? Why does the placement of each digit matter?" Give students 1–2 minutes to write a brief response. Look for students who addresses the placement of each digit by specifying the process they used, and invite these students to share with the class. This will help students clearly describe how to find quotients using long division.
Design Principle(s): Optimise output (for justification); Cultivate conversation

10.3 Dividing Whole Numbers

Optional: 10 minutes

In this activity, students continue to practise using long division to find quotients. Here, the presence of 0's in the dividend and the quotient presents an added layer of complexity, prompting students to really make sense of the meaning of each digit in numbers they are dealing with.

Instructional Routines

- Discussion Supports

Launch

Give students 6–7 minutes of quiet work time. Follow with a whole-class discussion. Provide access to graph paper.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as "First, I _____ because…," "Why did you…?," and "How did you get…?"
Supports accessibility for: Language; Social-emotional skills

Student Task Statement

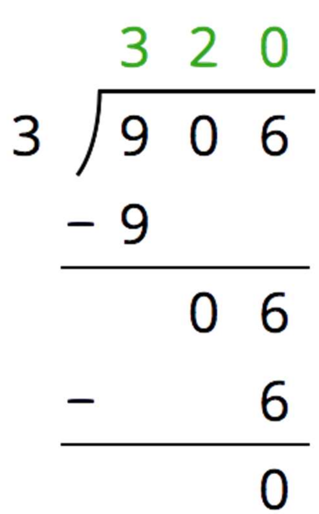
1. Find each quotient.

    a. $633 \div 3$

    b. $1\,001 \div 7$

    c. $2\,996 \div 14$

2. Here is Priya's calculation of $906 \div 3$.



a. Priya wrote 320 for the value of 906 ÷ 3. Check her answer by multiplying it by 3. What product do you get and what does it tell you about Priya's answer?

b. Describe Priya's mistake, then show the correct calculation and answer.

Student Response

1.

a.

```
    2 1 1
 3 )6 3 3
  - 6
    ---
      3
    - 3
    ---
        3
      - 3
      ---
        0
```

b.

```
      1 4 3
 7 )1 0 0 1
   - 7
   -------
     3 0
   - 2 8
   -------
       2 1
     - 2 1
     -----
         0
```

c.

```
        2 1 4
 1 4 )2 9 9 6
     - 2 8
     -------
         1 9
       - 1 4
       -----
           5 6
         - 5 6
         -----
             0
```

2.

a. $320 \times 3 = 960$. Priya made a mistake. When the result of $906 \div 3$ is multiplied by 3, it should equal 906.

b. Priya made a mistake when she placed the 2 of the quotient in the tens place, above the 0 in 906. This would mean that she is taking away 3 times 20, or 60. Since there are only 6 ones remaining, she should have taken away 3 times 2, not 3 times 20. Instead of putting the 2 in the tens place of 906, she should have placed a 0 there and placed the 2 in the ones place (over the 6). The correct answer is 302.

Activity Synthesis

Focus class discussion on attending to the meaning of each digit in performing division. Discuss:

- How did you deal with the 0's in 1 001? Would they cause any difficulty when doing long division? (I brought down the first 0 and then performed division like I would have done with any other digit. After subtracting one 7 from 10, I was left with 3. Putting a 0 after the 3 changes the value to 30. So even though the 0 alone has no value, it changes the value of the numbers in front of it).
- How can you check your answer to a division problem such as $1\,001 \div 147$? (We can check by multiplying the quotient by the divisor. If the division was done correctly, then $143 \times 7 = 1\,001$, which is true).
- What happens if you check Priya's answer for $906 \div 3$? ($320 \times 3 = 960$, so this tells us that Priya's answer is incorrect).

Make sure students notice that although checking an answer can tell you that you have made a mistake, it will not necessarily identify *where* the mistake is. It only works if you perform the multiplication correctly.

Conversing: Discussion Supports. Before the class discussion, give students the opportunity to meet with a partner to share their ideas using the three reflection questions. Ask students to focus their discussion on ways to check an answer. Provide these sentence frames: "To check my answer, I ….", "I know my answer is correct/incorrect because ….", "Using Priya's work as an example, I ….". The listener should press for details by asking clarifying questions such as, "How do you know that method works to check your answer?" and "Could you explain that using $1\,001 \div 7$?" Give each student an opportunity to be the speaker and the listener. This will help students communicate their thinking around verifying answers to long division problems.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

Lesson Synthesis

Long division is another method for finding quotients. It follows similar lines of reasoning for dividing with base-ten diagrams or using the partial quotients method. All three methods rely on the structure of the base-ten number system.

- How is dividing using long division similar to dividing by drawing base-ten diagrams? (Even though one method involves drawing and the other involves using only numbers, they rely on the same principle of dividing base-ten units into equal-sized groups. In both methods, when there is not enough of a unit to divide equally into groups, we can ungroup the unit into the next smaller base-ten units.)
- How is long division similar to and different from the partial quotients method? (They are similar in that we divide in "instalments," but in the partial quotients method, we can decide on the size of each instalment or each group being subtracted from the

dividend. In long division, we follow a very specific order based on place value and we divide digit by digit—from left to right, and subtract as large a group as possible at any step. In long division, we also do not write out all the numbers in our calculations; we use one digit at a time and rely on its place in the base-ten system to convey its value.)

- Which method for finding quotients do you think is the most efficient? (It depends on the numbers involved. If the numbers are large or long, drawing would be laborious and prone to error, and using partial quotients might mean a whole lot of steps. Long division might be simpler because we are reasoning with one digit and one place-value unit at a time.)

10.4 Dividing by 15

Cool Down: 5 minutes

Student Task Statement

Use long division to find the value of 1 875 ÷ 15.

Student Response

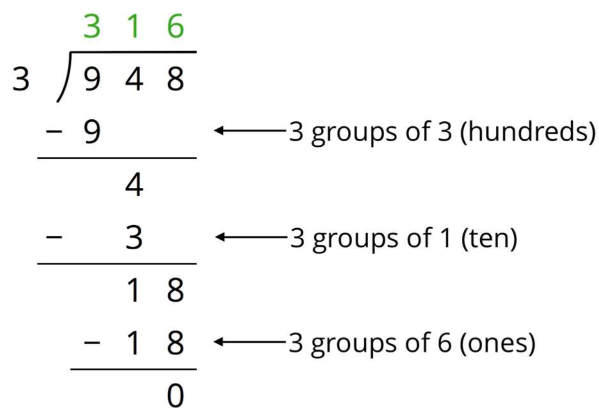
125

```
        1 2 5
15 / 1 8 7 5
   - 1 5
     -------
       3 7
     - 3 0
       -----
         7 5
       - 7 5
         ---
           0
```

Student Lesson Summary

Long division is another method for calculating quotients. It relies on place value to perform and record the division.

When we use long division, we work from left to right and with one digit at a time, starting with the leftmost digit of the dividend. We remove the largest group possible each time, using the placement of the digit to indicate the size of each group. Here is an example of how to find 948 ÷ 3 using long division.



- We start by dividing 9 hundreds into 3 groups, which means 3 hundreds in each group. Instead of writing 300, we simply write 3 in the hundreds place, knowing that it means 3 hundreds.
- There are no remaining hundreds, so we work with the tens. We can make 3 groups of 1 ten in 4 tens, so we write 1 in the tens place above the 4 of 948. Subtracting 3 tens from 4 tens, we have a remainder of 1 ten.
- We know that 1 ten is 10 ones. Combining these with the 8 ones from 948, we have 18 ones. We can make 3 groups of 6, so we write 6 in the ones place.

In total, there are 3 groups of 3 hundreds, 1 ten, and 6 ones in 948, so $948 \div 3 = 316$.

Glossary

- long division

Lesson 10 Practice Problems

1. **Problem 1 Statement**

   Kiran is using long division to find $696 \div 12$.

   1 2) 6 9 6

   He starts by dividing 69 by 12. In which place should Kiran place the first digit of the quotient (5)?

   a. Hundreds

   b. Tens

   c. Ones

   d. Tenths

   Solution B

2. **Problem 2 Statement**

Here is a long-division calculation of 917 ÷ 7.

$$\begin{array}{r} 131 \\ 7\overline{)917} \\ -7 \\ \hline 21 \\ -21 \\ \hline 7 \\ -7 \\ \hline 0 \end{array}$$

a. There is a 7 under the 9 of 917. What does this 7 represent?

b. What does the subtraction of 7 from 9 mean?

c. Why is a 1 written next to the 2 from $9 - 7$?

Solution

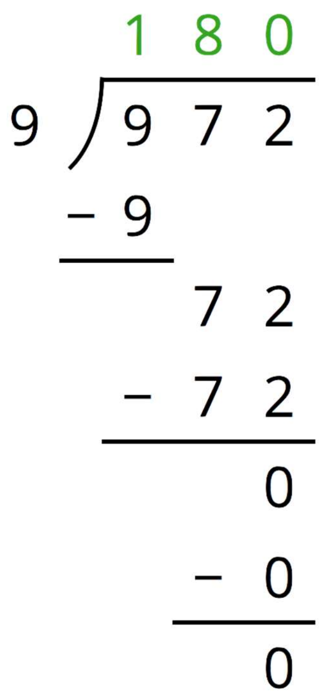
a. Answers vary. Sample response: The 7 under the 9 represents 700 (because it is written directly under the hundreds place of 917).

b. Answers vary. Sample response: It means a subtraction of 7 groups of 1 hundred from 9 hundreds.

c. Answers vary. Sample response: To represent the 10 in 917. There is 2 hundreds left after 7 hundreds are subtracted from 9 hundreds. The 2 hundreds is combined with the 1 ten from 917, which makes 21 tens.

3. **Problem 3 Statement**

Han's calculation of 972 ÷ 9 is shown here.



a. Find 180×9.

b. Use your calculation of 180×9 to explain how you know Han has made a mistake.

c. Identify and correct Han’s mistake.

Solution

a. $180 \times 9 = 1620$

b. If Han were correct, the product of 180 and 9 would be 972.

c. Answers vary. Sample response: Han’s mistake is that when he brought down the 7 from 972 and saw that 7 tens could not be divided into 9 groups (or 7 is not a multiple of 9), he did not write 0 above the 7 before bringing down the 2 ones. Here is the correct long division calculation:

$$\begin{array}{r|rrr} & 1 & 0 & 8 \\ \hline 9 & 9 & 7 & 2 \\ - & 9 & & \\ \hline & & 7 & \\ & - & 0 & \\ \hline & & 7 & 2 \\ & - & 7 & 2 \\ \hline & & & 0 \end{array}$$

4. **Problem 4 Statement**

Find each quotient.

a. $5\overline{)465}$

b. $12\overline{)924}$

c. $3\overline{)1107}$

Solution

a. 93

b. 77

c. 369

a.

$$\begin{array}{r|rrr} & & 9 & 3 \\ \hline 5 & 4 & 6 & 5 \\ - & 4 & 5 & \\ \hline & & 1 & 5 \\ & - & 1 & 5 \\ \hline & & & 0 \end{array}$$

b.

$$\begin{array}{r|rrr} & & 7 & 7 \\ \hline 12 & 9 & 2 & 4 \\ - & 8 & 4 & \\ \hline & & 8 & 4 \\ & - & 8 & 4 \\ \hline & & & 0 \end{array}$$

c.

$$\begin{array}{r|rrrr} & & 3 & 6 & 9 \\ \hline 3 & 1 & 1 & 0 & 7 \\ - & & 9 & & \\ \hline & & 2 & 0 & \\ & - & 1 & 8 & \\ \hline & & & 2 & 7 \\ & & - & 2 & 7 \\ \hline & & & & 0 \end{array}$$

5. **Problem 5 Statement**

   One ounce of a yogurt contains of 1.2 grams of sugar. How many grams of sugar are in 14.25 ounces of yogurt?

   a. 0.171 grams

   b. 1.71 grams

   c. 17.1 grams

   d. 171 grams

   Solution C

6. **Problem 6 Statement**

   The mass of one coin is 16.718 grams. The mass of a second coin is 27.22 grams. How much greater is the mass of the second coin than the first? Show your reasoning.

   Solution

   10.502 grams, because $27.22 - 16.718 = 10.502$